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Bija Ganita, or, the Algebra of the Hindus

An important mathematician and astronomer in medieval India, Bhascara Acharya (1114–85) wrote treatises on arithmetic, algebra, geometry and astronomy. He is also believed to have been head of the astronomical observatory at Ujjain, which was the leading centre of mathematical sciences in India. Forming part of his Sanskrit magnum opus *Siddhānta Shiromani*, the present work is his treatise on algebra. It was first published in English in 1813 after being translated from a Persian text by the East India Company civil servant Edward Strachey (1774–1832). The topics covered include operations involving positive and negative numbers, surds and zero, as well as algebraic, simultaneous and indeterminate equations. Strachey also appends useful notes made by the orientalist Samuel Davis (1760–1819). Of enduring interest in the history of mathematics, this was notably the first work to acknowledge that a positive number has two square roots.

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Bija Ganita
or, the Algebra of the Hindus

Bhascara Acharya
Translated by Edward Strachey

Cambridge University Press

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BIJA GANITA,

&c. &c. &c.

BIJA GANITA:

OR THE

ALGEBRA

OF THE

HINDUS.

BY

EDWARD STRACHEY,

OF THE

EAST INDIA COMPANY's BENGAL CIVIL SERVICE.

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MR. DAVIS'S NOTES.

ERRATA.

Page 3, line 4, *for* Heilbronnen *read* Heilbronner.
5, 1, *for* undistinguished *read* undistinguishing.
11, 6, *after* notes *read* at the bottom of the pages.
13, 1, *dele* the inverted commas at the beginning of the line.
17, 5, (at the end), *for* number *read* colour.
29, 25, Suppose $\frac{ax + c}{b} = y$ where a, b, and c are known and x and y unknown.
93, 9, 10, and 11, the character prefixed to the numbers 1 and 2 is here लो which is the first letter of the word *Loheet* (see opposite page); but it should be a different character, viz. the first letter of the word *Roop* रू.

Note—In Mr. Davis's notes the word Ja, Roo, Bha, Ca, &c. which are frequently used, are contractions of Jabut, Roop, Bhady, Canist, &c. They should have been printed with points after them, thus, Ja. Roo. &c.

PREFACE.

It is known that there are Sanscrit books on Astronomy and Mathematics. Whether the Science they contain is of Hindoo Origin and of high antiquity, or is modern and borrowed from foreign sources, is a question which has been disputed. Some of the Advocates for the Hindoos have asserted their pretensions with a degree of zeal which may be termed extravagant; and others among their opponents have with equal vehemence pronounced them to be impostors, plagiaries, rogues, blind slaves, ignorant, &c. &c.

My object in the following paper is to support the opinion that the Hindoos had an original fund of Science not borrowed from foreign sources. I mean to infer also, because of the connexion of the sciences and their ordinary course of advancement, that the Hindoos had other knowledge besides what is established by direct proof to be theirs, and that much of what they had, must have existed in early times.

But with respect to the antiquity of the specimens which I am going to exhibit, nothing seems to be certainly known beyond this, that in form and substance as they are here, they did exist at the end of the 12th or the beginning of the 13th Century.

It is not my purpose to inquire here what parts of Indian science have already been ascertained to be genuine. I only wish to observe that the doubts which have been raised as to the pretensions of the Hindoos are of very recent birth, and that no such doubts have been expressed by persons who were perhaps as well able to judge of the matter as we are.*

* The Edinburgh Review, in criticising Mr. Bentley's Indian Astronomy, in the 20th number, ably contended for the antiquity and originality of Hindoo Science. The writer of that article however seems to have left the field; and his successor, in a Review of Delambre's History of Greek Arithmetic, has taken the other side of the question, with much zeal. This Critic is understood to be Mr. Leslie, who, in his Elements of Geometry, has again attacked the Hindoos. Mr. Leslie, after explaining the rule for constructing the sines by differences, which was given in the 2nd Volume of the Asiatic Researches by Mr. Davis, from the Surya Siddhanta, adds the followni̇g remarks.

We are told that in early times Pythagoras and Democritus, who taught the Greeks astronomy and mathematics, learnt these sciences in India. The Arabians

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" Such is the detailed explication of that very ingenious mode which, in certain cases, the Hindoo Astro- " nomers employ for constructing the table of approximate sines. But totally ignorant of the principles of " the operation, those humble calculators are content to follow blindly a slavish routine. The Brahmins must " therefore have derived such information from people farther advanced than themselves in science, and of " a bolder and more inventive genius. Whatever may be the pretensions of that passive race, their know- " ledge of trigonometrical computation has no solid claim to any high antiquity. It was probably, before " the revival of letters in Europe, carried to the East, by the tide of victory. The natives of Hindustan " might receive instruction from the Persian Astronomers, who were themselves taught by the Greeks of " Constantinople, and stimulated to those scientific pursuits by the skill and liberality of their Arabian con- " querors."—(Leslie's Elements, p. 485.)

When scientific operations are detailed, and most of the theorems on which they depend are given in the form of rules, surely it is not to be inferred because the demonstrations do not always accompany the rules, therefore that they were not known; on the contrary, the presumption in such a case is that they were known. So it is here, for the Hindoos certainly had at least as much trigonometry as is assumed by Mr. Leslie to be the foundation of their rule. Mr. Leslie, after inferring that the Hindoos must have derived their science from people farther advanced than themselves, proceeds to shew the sources from which they might have borrowed, namely, the Persians, the Greeks, and the Arabians. Now as for the Persians as a nation, we do not know of any science of theirs except what was originally Greek or Arabian. This indeed Mr. Leslie would seem not to deny; and as for the Greeks and Arabians it is enough to say that the Hindoos could not borrow from them what they never had. They could not have borrowed from them this *slavish routine* for the sines, which depends on a principle not known even to the modern Europeans till 200 years ago. In short the tide of victory could not have carried that which did not exist.

It appears from Mr. Davis's paper that the Hindoos knew the distinctions of sines, cosines, and versed sines. They knew that the difference of the radius and the cosine is equal to the versed sine; that in a right-angled triangle if the hypothenuse be radius the sides are sines and cosines. They assumed a small arc of a circle as equal to its sine. They constructed on true principles a table of sines, by adding the first and second differences.

From the Bija Ganita it will appear that they knew the chief properties of right-angled and similar triangles.

In Fyzee's Lilavati I find the following rules:

(The hypothenuse of a right-angled triangle being h, the base b, and the side s.)

Assume any large number p, then $\frac{\sqrt{((b^2+s^2)p^2)}}{p} = h$.

$$b = \sqrt{(h^2 - s^2)} \text{ and } s = \sqrt{(h^2 - b^2)}.$$
$$\sqrt{((b-s)^2 + 2bs)} = h.$$
$$(h+b)(h-b) = h^2 - b^2.$$

b being given to find h and s in any number of ways; let p be any number; then $\frac{2pb}{p^2-1} = s$, and $ps - b = h$.

$$\frac{\frac{b^2}{p} - p}{2} = s, \text{ and } \frac{\frac{b^2}{p} + p}{2} = h.$$

h being given, $\frac{2ph}{p^2+1} = s$, and $ps - h = b$.

always considered the Indian astrology and astronomy as different from theirs and the Greeks. We hear of Indian astronomy known to them in the time of the Çaliph Al Mamun. (See d'Herbelot). Aben Asra is said to have compared the Indian sphere with the Greek and Persian spheres. (Heilbronnen Hist. Math. p. 456). We know that the Arabians ascribe their numeral figures to the

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Let p and q be any numbers; then

$$2pq = s,\ p^2 - q^2 = b,\ \text{and}\ p^2 + q^2 = h.$$

Given $a = h \pm s$; then $\frac{a - \frac{b^2}{a}}{2} = s$, and $\frac{a + \frac{b^2}{a}}{2} = h$.

Given $a = b + s$; then $\frac{a - \sqrt{(2h^2 - a^2)}}{2} = b$, and $\frac{a + \sqrt{(2h^2 - a^2)}}{2} = s$.

There are also rules for finding the areas of triangles, and four-sided figures; among others the rule for the area of a triangle, without finding the perpendicular.

For the circle there are these rules (C being the circumference, D the diameter, c the chord, v the versed sine, a the arch,)

$$C : D :: 22 : 7;\ \text{and}\ C : D :: 3927 : 1250.$$ (Also see Ayeen Akbery, vol. 3, p. 32.)

$$\frac{D - \sqrt{((D + c)(D - c))}}{2} = v.$$

$$2\sqrt{(D - v)} \times v = c.$$

$$\frac{4aD(C - a)}{\frac{5}{4}C^2 - (C - a)a} = c,\ \text{and}\ \frac{C}{2} - \sqrt{\left(\frac{\frac{C^2}{4} - \frac{5}{4}C^2c}{4D + c}\right)} = a.$$

Also formulæ for the sides of the regular polygons of 3, 4, 5, 6, 7, 8, 9 sides inscribed in a circle. There are also rules for finding the area of a circle, and the surface and solidity of a sphere. It will be seen also that Bhascara is supposed to have given these two rules, viz—the sine of the sum of two arcs is equal to the sum of the products of the sine of each multiplied by the cosine of the other, and divided by the radius; and the cosine of the sum of two arcs is equal to the difference between the products of their sines and of their cosines divided by radius.

Is it to be doubted that the Hindoos applied their rule for the construction of the sines, to ascertain the ratio of the diameter of a circle to its circumference?—thus the circumference of a circle being divided into 360 degrees, or 21600 minutes, the sine of 90 degrees which is equal to the radius would be found by the rule 3438. This would give the ratio of the diameter to the circumference $7 : 21\frac{567}{573}$ and $1250 : 3926\frac{402}{573}$, and assuming, as the Hindoos commonly do, the nearest integers, the ratio would be 7 : 22 or 1250 : 3927.

It is not to be denied that there are some remarkable coincidences between the Greek and the Hindoo science; for example, among many which might be given it may be suggested that the contrivances ascribed to Antiphon and Bryso, and that of Archimedes, for finding the ratio of the diameter of a circle to its circumference might have been the foundation of the Hindoo method; that Diophantus's speculations on indeterminate problems might be the origin of the Hindoo Algebra. But there are no truths in the history of science of which we are better assured than that the Surya Siddhanta rule for the sines, with the ratio of the diameter of a circle to its circumference 1250 : 3927; and the Bija Ganita rules for indeterminate problems were not known to the Greeks. Such are the stumbling blocks which we always find in our way when we attempt to refer the Hindoo science to any foreign origin.

Indians; and Massoudi refers Ptolemy's astronomy to them. (See Bailly's preface to his Indian astronomy, where is cited M. de Guignes Mem. Acad. Ins. T. 36, p. 771). Fyzee, who doubtless was conversant with Greco-arabian learning, and certainly knew the Hindoos well, has never started any doubt of the originality of what he found among them. The preface to the Zeej Mahommed Shahry, or Astronomical Tables, which were published in India in 1728, speaks of the European, the Greek, the Arabian, and the Indian systems as all different. That work was compiled with great learning by persons who were skilled in the sciences of the West, as well as those of the East*. More examples might be given—but to proceed.

The Bija Ganita is a Sanscrit treatise on algebra, by Bhascara Acharya, a celebrated Hindoo Astronomer and Mathematician.

Fyzee†, who, in 1587, translated the Lilavati, a work of his on arithmetic, mensuration, &c. speaks of an astronomical treatise of Bhascara s, dated in the 1105th year of the Salibahn, which answers to about 1183 of our æra; but Fyzee also says, it was 373 years before 995 Hegira, which would bring it down to A.D. 1225. So that Bhascara must have written about the end of the 12th century, or beginning of the 13th.

A complete translation of the Bija Ganita is a great desideratum; so it has been for more than 20 years, and so it seems likely to remain.

It will be seen however that we have already means of learning, with sufficient accuracy, the contents of this work. I have a Persian translation of the Bija Ganita, which was made in India in 1634, by Ata Allah Rusheedee. The Persian does not in itself afford a correct idea of its original, as a translation should do; for it is an

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* See Asiatic Researches, 5th vol. on the Astronomical Labours of Jy Singh.

† I will here translate a part of Fyzee's preface:—"By order of king Akber, Fyzee translates into Persian, "from the Indian language, the book Lilavati, so famous for the rare and wonderful arts of calculation and "mensuration. He (Fyzee) begs leave to mention that the compiler of this book was Bhascara Acharya, whose "birth place, and the abode of his ancestors was the city of Biddur, in the country of the Deccan. Though "the date of compiling this work is not mentioned, yet it may be nearly known from the circumstance, that the "author made another book on the construction of Almanacks, called Kurrun Kuttohul, in which the date of "compiling it is mentioned to be 1105 years from the date of the Salibahn, an æra famous in India. From "that year to this, which is the 32d Ilahi year, corresponding with the lunar year 995, there have passed "373 years."

As the Ilahi began in the Hegira (or lunar) year, 992, (see Ayeen Akbery) the date 32 of Ilahi is of course an error. It is likely too that there is an error in the number 373.

Mr. Colebrooke, in the 9th vol. of Asiatic Researches, gives, on Bhascara's own authority, the date of his birth, viz. 1063 Saca. In 1105 Salibahn (or Saca) that is, about A.D. 1183, he was 42 years old.

undistinguished mixture of text and commentary, and in some places it even refers to Euclid. But to infer at once from this that every thing in the book was derived from Greek or Arabian writers, or that it was *inseparably* mixed, would be reasoning too hastily. A little patience will discover evidence of the algebra which it contains, being purely Hindoo*.

The following paper consists of an account of this translation, and some notes which I shall now mention:

Mr. Davis, the well-known author of two papers on Indian Astronomy in the Asiatic Researches, made, many years ago, in India, some abstracts and translations from the original Sanscrit Bija Ganita †, and it is greatly to be regretted that he did not complete a translation of the whole. The papers which contain his notes had long since been mislaid and forgotten. They have been but very lately found, and I gladly avail myself of Mr Davis's permission to make use of them here. The chief part of them is inserted at the end of my account of the Persian translation. To prevent misconception about these notes, it is proper for me to observe that in making them Mr. Davis had no other object than to inform himself generally of the nature of the Bija Ganita; they were not intended probably to be seen by any second person; certainly they were never proposed to convey a perfect idea of the work, or to be exhibited before the public in any shape. Many of them are on loose detached pieces of paper, and it is probable that, from the time they were written till they came into my hands, they were never looked at again. But nevertheless it will be seen that they do, without doubt, describe accurately a considerable portion of the most curious parts of the Bija Ganita; and though they may seem to occupy but a secondary place here, they will be found of more importance than all the rest of this work together.

They shew positively that the main part of the Persian translation is taken from

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* The late Mr. Reuben Burrow in one of his papers in the Asiatic Researches says, he made translations of the Bija Ganita and Lilavati. Those translations he left to Mr. Dalby. They consist of fair copies in Persian of Ata Allah's and Fyzee's translations, with the English of each word written above the Persian. The words being thus translated separately, without any regard to the meaning of complete sentences, not a single passage can be made out. It is plain, from many short notes which Mr. Burrow has written in the margin of his Bija Ganita, that he made his verbal translation by the help of a Moonshee, and that he had the original Sanscrit at hand, with some opportunity of consulting it occasionally. I am much obliged to Mr. Dalby for allowing me the use of Mr. Burrow's copy which has enabled me to supply deficiencies in mine; and it is otherwise interesting, because it shews that Mr. Burrow had access to the original Sanscrit (probably by means of a Moonshee and a Pundit) and compared it with the Persian.

† It is to be remarked that they were made from the Sanscrit only. Mr. Davis never saw the Persian translation.

the Sanscrit work, and that the references to Euclid are interpolations of the Persian translator they give most of the Hindoo Algebraic notation* which is wanting in the Persian, and they shew that the Astronomy of the Hindoos was connected with their Algebra.

I must however confess, that even before I saw these notes the thing was to my mind quite conclusive. For I found (as will be seen) in this Persian translation of 1634, said to be from the Sanscrit, a perfectly connected structure of science, comprehending propositions, which in Europe were invented successively by Bachet de Mezeriac, Fermat, Euler, and De La Grange†.

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* The Hindoos have no mark for +, they only separate the quantities to be added by a vertical line thus | or ||, as they separate their slocas or verses.

Their mark for *minus* is a dot over the quantity to be subtracted.

Instead of a mark for multiplication they write the factors together as we do, thus, ab for $a \times b$.

Division they mark as we do by a horizontal line drawn between the dividend and divisor, the lower quantity being the divisor.

For unknown quantities they use letters of the alphabet as we do. They use the first letters of the words signifying colours.

The known quantity (which is always a number) has the word roop (form) or the first letter of the word prefixed.

The square of the unknown quantity is marked by adding to the expression of the simple quantity the first letter of the word which means square, and in like manner the cube.

The sides of an equation are written one above another; every quantity on one side is expressed again directly under it on the other side. Where there is in fact no corresponding quantity, the form is preserved by writing that quantity with the co-efficient 0.

The methods of prefixing a letter to the known number, and using the first letter of the words square and cube are the same as those of Diophantus. I mention it as a curious coincidence; perhaps some people may attach more importance to it than I do.

† The propositions which I here particularly allude to are these:—

1. A general method of solving the problem $\frac{ax \pm c}{b} = y$, a, b and c being given numbers and x and y indeterminate. The solution is founded on a division like that which is made for finding the greatest common measure of two numbers. The rules comprehend every sort of case, and are in all respects quite perfect.

2. The problem $am^2 + 1 = n^2$, (a being given and m and n required) with its solution.

3. The application of the above to find any number of values of $ax^2 + b = y^2$ from one known case.

4. To find values of x and y in $ax^2 + b = y^2$ by an application of the problem $\frac{ax \pm c}{b} = y$. It is unnecessary for me *here* to give any detail of the Hindoo methods.

The first question about this extraordinary matter is, what evidence have we that it is not all a forgery? I answer, shortly, that independently of its being now found in the Sanscrit books, it is ascertained to have been there in 1634 and 1587, that is to say, in times when it could not have been forged.

The following extract from a paper of De La Grange, in the 24th volume of the Memoirs of the Berlin Academy, for the year 1768, contains a summary of that part of the history of Algebra which is now alluded to. As for the 4th of the points abovementioned, the method in detail (however imperfect in some respects) is, as far as I know, new to this day. The first application of the principle in Europe is to be sought in the writings of De La Grange himself.

To maintain that the Bija Ganita rules for the solution of indeterminate problems might have been had from any Greek or Arabian, or any modern European writers before the Mathematicians just named, would be as absurd as to say that the Newtonian Astronomy might have existed in the time of Ptolemy. It is true that Bachet wrote a few years before 1634, but this is no sort of objection to the argument, for that part which might be questioned as a mere copy of Bachet's method, namely, the rules for indeterminate problems of the first degree, is closely connected with matters of latter invention in Europe, and is in Mr. Dalby's copy of Fyzee's translation of the Lilavati, which I have before said was made in 1587; and Mr. Davis's notes shew that it is in the Sanscrit Bija Ganita, which was

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" La plupart des Géometres qui ont cultivé l'analyse de Diophante se sont, a l'exemple de cet illustre inven- " tuer, uniquement appliqués à eviter les valeurs irrationelles; et tout l'artifice de leurs méthodes se reduit à " faire en sorte que les grandeurs inconnues puissent se déterminer par des nombres commensurables.

" L'art de resoudre ces sortes de questions ne demande gueres d'autres principes que ceux de l'analyse " ordinaire: mais ces principes deviennent insuffisant lors-qu'on ajoute la condition que les quantités cher- " chées soient non seulement commensurables mais encore égales à des nombres entiers.

" M. Bachet de Mezeriac, auteur d'un excellent commentarie sur Diophante et de différens autres ouvrages " est, je crois, le premier quit ait tanté de soumettre cette condition au calcul. Ce savant a trouvê une méthode " générale pour resoudre en nombres entiers toutes les equations du premier degré a deux ou plusieurs incon- " nues, mais il ne paroit pas avoir été plus loin; et ceux qui aprés lui se sont occupés du même objet, ont " aussi presque tous bornè leurs recherches aux equations indéterminées du premier degré; leurs efforts se " sont réduits a varier les méthods qui peuvent servir a la resolution de ces sortes d'equations, et aucun, si " j'ose le dire, n'a donné une methode plus directe, plus générale, et plus ingenieuse que celle de M. Bachet " qui se trouve dans ses récréations mathématiques intituéles '*Problems plaisans et délectables qui se font par les* " *nombres.*' Il est a la vérité assez surprenant que M. de Fermat qui s'etoit si long tems et avec tant de " succés exercé sur la théorie des nombres entiers, n'ait pas cherché à resoudre généralement les problems " indeterminés du second degré, et des degrés superieurs comme M. Bachet avoit fait ceux du premier degré; " on a cependent lieu de croire qu'il s'etoit aussi appliqué a cette recherche, par le probleme qu'il proposa " comme une espece de défi à M. Wallis et à tous les Geometres Anglois, et qui consistoit à trouver deux " carrés entiers, dont l'un étant multiplié par un nombre entier donné non carré & ensuite retranché de l'autre, " le reste fut etre égal à l'unité, car, outre que ce probleme est un cas particulier des équations du second " dégré à deux inconnués il est comme la clef de la résolution génerale de ces équations. Mais soit que " M. de Fermat n'ait pas continué ses recherches sur cette matiere, soit qu'elle ne soit parvenue jusqu'à nous, " il est certain qu'on n'en trouve aucune trace dans ses ouvrages.

" Il paroit même que les Geometres Anglois qui ont résolu le probleme de M. de Fermat n'ont pas connu " toute l'importance dont il est pour la solution générale des problemes indéterminés du second degré, du " moins on ne voit pas qu'ils en ayant jamais fait usage, et Euler est si je ne me trompe, le prémier qui ait " fait voir comment à l'aide de ce probleme on peut trouver une infinité de solutions en nombres entiers de " toute équation du second dégré à deux inconnues, dont on connoit déja une solution.

" Il résulte de tout ce que nous venons de dire, que depuis l'ouvrage de M. Bachet que a paru en 1613, " jusqu'a prêsent, ou du moins jusqu'au mémoire que je donnai l'année passée sur la solution de problems " indéterminés du sécond dégré, la théorie de ces sortes de problemes n'avolt pas a proprement parler, éte " poussée au dela du premier dégré."

written four centuries before Bachet. Though we are not without direct proofs from the original, yet, as even the best Sanscrit copies of the Bija Ganita, or any number of such copies exactly corresponding, would still be open to the objection of interpolations, it is necessary in endeavouring to distinguish the possible and the probable corruptions of the text, from what is of Indian origin, to recur to the nature of the propositions themselves, and to the general history of the science. Indeed we have not data enough to reason satisfactorily on other principles. We cannot rely upon the perfect identity or genuineness of any book before the invention of printing, unless the manuscript copies are numerous, and of the same age as the original. Such is the nature of our doubt and difficulty in this case, for old mathematical Sanscrit manuscripts are exceedingly scarce; and our uncertainty is greatly increased by a consideration of this fact, that in latter times the Greek, Arabian, and modern European science has been introduced into the Sanscrit books.

Yet, in cases precisely parallel to this of the Hindoos, we are not accustomed to withhold our belief as to the authenticity of the reputed works of the ancients, and in forming our judgment we advert more to the contents of the book than to the state of the manuscript. When the modern Europeans first had Euclid, they saw it only through an Arabic translation. Why did they believe that pretended translation to be authentic? Because they found it contained a well connected body of science; and it would have been equally as improbable to suppose that the Arabian translator could have invented it himself as that he could have borrowed it from his countrymen. There are principles on which we decide such points. We must not look for mathematical proof, but that sort of probability which determines us in ordinary matters of history.

Every scrap of Hindoo science is interesting; but it may be asked why publish any which cannot be authenticated? I answer, that though this translation of Ata Allah's which professes to exhibit the Hindoo algebra in a Persian dress, does indeed contain some things which are not Hindoo, yet it has others which are certainly Hindoo. By separating the science from the book we may arrive at principles, which if cautiously applied, cannot mislead, which in some cases will shew us the truth, and will often bring us to the probability when certainty is not to be had. On this account I think the Persian translation at large interesting, notwithstanding it contains some trifling matters, some which are not intelligible, and others which are downright nonsense.

I have said that Mr. Davis's notes shew a connexion of the algebra of the

Hindoos with their astronomy. Mr. Davis informs me that in the astronomical treatises of the Hindoos, reference is often made to the algebra; and particularly he remembers a passage where Bhascara says " it would be as absurd for a person " ignorant of algebra to write about astronomy, as for one ignorant of grammar " to write poetry."

Bhascara, who is the only Hindoo writer on algebra whose works we have yet procured, does not himself pretend to be the inventor, he assumes no character but that of a compiler*. Fyzee never speaks of him but as a person eminently skilled in the sciences he taught. He expressly calls him the compiler of the Lilavati.

I understand from Mr. Davis, and I have heard the same in India, that the Bija Ganita was not intended by Bhascara as a separate unconnected work, but as a component part of one of his treatises on astronomy, another part of which is on the circles of the sphere.

I have found among Mr. Davis's papers, some extracts from a Sanscrit book of astronomy, which I think curious, although the treatise they were taken from is modern. Mr. Davis believes it to have been written in Jy Sing's time, when the European improvements were introduced into the Hindoo books. Two of these extracts I have added to the notes on the Bija Ganita. The first of the two shews that a method has been ascribed by Hindoo Astronomers to Bhascara of calculating sines and cosines by an application of the principles which solve indeterminate problems of the second degree. This suggestion is doubtless of Hindoo origin, for the principles alluded to were hardly known in Europe in Jy Sing's time†. I think it very probable that the second extract is also purely Hindoo, and that the writer knew of Hindoo authors who said the square root might be extracted by the cootuk; that is to say, the principle which effects the solution of indeterminate problems of the first degree. From this, and from what is in the Bija Ganita, one cannot but suspect that the Hindoos had continued fractions, and possibly some curious arithmetic of sines. On such matters however, let every one exercise his own judgment. ‡

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* " Almost any trouble and expence would be compensated by the possession of the three copious treatises " on algebra from which Bhascara declares he extracted his Bija Ganita, and which in this part of India are " supposed to be entirely lost."—As. Res. vol. iii. Mr. Davis " On the Indian Cycle of 60 years."

† Jy Sing reigned from 1694 to 1744.

‡ Mr. Reuben Burrow, who, by the bye, it must be confessed is very enthusiastic on these subjects, in a paper

We must not be too fastidious in our belief, because we have not found the works of the teachers of Pythagoras; we have access to the wreck only of their ancient learning; but when we see such traces of a more perfect state of knowledge, when we see that the Hindoo algebra 600 years ago had in the most interesting parts some of the most curious modern European discoveries, and when we see that it was at that time applied to astronomy, we cannot reasonably doubt the originality and the antiquity of mathematical learning among the Hindoos. Science in remote times we expect to find within very narrow limits indeed. its *history* is all we look to in such researches as these. Considering this, and comparing the contents of the Hindoo books with what they might have been expected to contain, the result affords matter of the most curious speculation.

May I be excused for adding a few words about myself. If my researches have not been so deep as might have been expected from the opportunities I had in India, let it be remembered that our labours are limited by circumstances. It is true I had at one time a copy of the original Bija Ganita, but I do not understand Sanscrit, nor had I then any means of getting it explained to me. Official avocations often prevented me from bestowing attention on these matters, and from seizing opportunities when they did occur. Besides, what is to be expected in this way from a *mere amateur*, to whom the simplest and most obvious parts only of such subjects are accessible?

E. S.

The following account of Ata Allah's Bija Ganita is partly literal translation, partly abstract, and partly my own.

The literal translation is marked by inverted commas; that part which consists of my own remarks or description will appear by the context, and all the rest is abstract.

I have translated almost all the rules, some of the examples entirely, and

in the appendix of the 2d vol. of the As. Res. speaks of the Lilavati and Bija Ganita, and of the mathematical knowledge of the Hindoos: He says, he was told by a Pundit, that some time ago there were other treatises of algebra, &c. (See the paper.)

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others in part; in short, whatever I thought deserving of particular attention, for the sake of giving a distinct idea of the book.

Perhaps some of the translated parts might as well have been put in an abstract; the truth is, that having made them originally in their present form I have not thought it worth while to alter them.

The notes are only a few remarks which I thought might be of use to save trouble and to furnish necessary explanation.