Cambridge University Press
978-1-108-05601-4 - Bija Ganita or, the Algebra of the Hindus
Bhascara Acharya Translated by Edward Strachey
Excerpt
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BIJA GANITA.

"After the usual invocations and compliments, the Persian translator begins thus: "By the Grace of God, in the year 1044 Hegira" (or A. D. 1634) "being the "eighth year of the king's reign, I, Ata Alla Rasheedee, son of Ahmed Nadir, "have translated into the Persian language, from Indian, the book of Indian "Algebra, called Beej Gunnit (Bija Ganita), which was written by Bhasker Acharij "(Bhascara Acharya) the author of the Leelawuttee (Lilavati). In the science of "calculation it is a discoverer of wonderful truths and nice subtilties, and it con-"tains useful and important problems which are not mentioned in the Leelawuttee, "nor in any Arabic or Persian book. I have dedicated the work to Shah Jehan, and "I have arranged it according to the original in an introduction and five books."

INTRODUCTION.

"The introduction contains six chapters, each of which has several sections."

CHAPTER I.

On Possession (مال)* and Debt (دين).

"Know that whatever is treated of in the science of calculation is either "affirmative or negative; let that which is affirmative be called *mal*, and that "which is negative *dein*. This chapter has five sections."

Sect. I.

On Addition and Subtraction, that is, to encrease and diminish.

"If an affirmative is taken from an affirmative, or a negative from a negative, "the subtrahend is made contrary; that is to say, if it is affirmative suppose it "negative, and if negative suppose it affirmative, and proceed as in addition.

"The rule of addition is, that if it is required to add two affirmative quantities,

* Most of the technical terms here used are Arabic.

"or two negative quantities together, the sum is the result of the addition. If "they are affirmative call the sum affirmative: if negative call the sum negative. "If the quantities are of different kinds take the excess; if the affirmative is "greater, the remainder is affirmative; if the negative is greater, the remainder "is negative; and so it is in subtraction." (Here follow examples).

SECT. II.

On Multiplication *.

"If affirmative is multiplied by affirmative, or negative by negative, the product "is affirmative and to be included in the product. If the factors are contrary "the product is negative, and to be taken from the product. For example, let us "multiply two affirmative by three affirmative, or two negative by three negative, "the result will be six affirmative; and if we multiply two affirmative by three "negative, or the contrary, the result will be six negative."

SECT. III.

On Division.

"The illustration of this is the same as what has been treated of under multi- "plication, that is to say, if the dividend and the divisor are of the same kind "the quotient will be affirmative, and if they are different, negative. For "example, if 8 is the dividend and 4 the divisor, and both are of the same kind, "the quotient will be 2 affirmative; if they are different, 2 negative."

SECT. IV.

On Squares †.

"The squares of affirmative and negative are both affirmative; for to find the

* In the Persian translation the product of numbers is generally called the rectangle.

† I had a Persian treatise on Algebra in which there was this passage—"Any number which is to be multiplied "by itself is called by arithmeticians root (جذر), by measurers of surfaces side (ضلع), and by alge- "braists thing (شي). And the product is called by arithmeticians square (مجذور), by measurers "of surfaces square (مربع), and by algebraists possession (مال)." مال is also used for *plus*, and its opposite debt (دين) for *minus*. These terms, all of which are Arabic, are used in the Persian translation of the Bija Ganita, the geometrical more frequently than their corresponding arithmetical or algebraical ones.

"square of 4 affirmative we multiply 4 affirmative by 4 affirmative, and by the rules "of multiplication, as the factors are of the same kind, the product must be 16 "affirmative, and the same applies to negative."

Sect. V.

On the Square Root.

"The square root of affirmative is sometimes affirmative and sometimes nega-"tive, according to difference of circumstances. The square of 3 affirmative or "of 3 negative is 9 affirmative; hence the root of 9 affirmative is sometimes 3 "affirmative and sometimes 3 negative, according as the process may require. "But if any one asks the root of 9 negative I say the question is absurd, for there "never can be a negative square as has been shown."

CHAP. II.

On the Cipher.

"It is divided into four sections."

Sect. I.

On Addition and Subtraction.

"If cipher is added to a number, or a number is added to cipher, or if cipher "is subtracted from a number, the result is that number: and if a number is sub-"tracted from cipher, if it is affirmative it becomes negative, and if negative it "becomes affirmative. For example, if 3 affirmative is subtracted from cipher "it will be 3 negative, and if 3 negative is subtracted it will be 3 affirmative."

Sect. II.

On Multiplication.

"If cipher is multiplied by a number, or number by cipher, or cipher by cipher, "the result will be cipher. For example, if we multiply 3 by cipher, or con-"versely, the result will be cipher"

SECT. III.

On Division.

" If the dividend is cipher and the divisor a number the quotient will be cipher. " For example, if we divide cipher by 3 the quotient will be cipher, for multi- " plying it by the divisor the product will be the dividend, which is cipher: " and if a number is the dividend and cipher the divisor the division is impossible; " for by whatever number we multiply the divisor, it will not arrive at the divi- " dend, because it will always be cipher."

SECT. IV.

On Squares, &c.

" The square, cube, square root, and cube root of cipher, are all cipher; the " reason of which is plain."

CHAP. III.

ON COLOURS.

" Whatever is unknown in examples of calculation, if it is one, call it thing, " (شي), and unknown (مجهول); and if it is more call the second black, " and the third blue, and the fourth yellow, and fifth red. Let these be termed " colours, each according to its proper colour. This chapter has five sections.

SECT. I.

On Addition and Subtraction of Colours.

" When we would add one to another, if they are of the same kind add the " numbers* together; if they are of two or more kinds, unite them as they are, " and that will be the result of the addition." Here follows an example.

" If we wish to subtract, that is to take one from the other, let the subtrahend " be reversed. If then two terms of the same kind are alike in this, that they are " both affirmative or both negative, let their sum be taken, otherwise their dif- " ference, and whatever of the kind cannot be got from the minuend, must be

* Meaning here the co-efficients.

" subtracted from cipher. Then let it be reversed, and this will be the result " exactly." (Here follows an example).

Sect. II.

On Multiplication of Colours.

" If a colour is multiplied by a number the product will be a number*, $x \times x$ " will be x^2, whether the number is the same or different, and the product multi- " plied by x will be x^3. If the colours are different multiply the numbers of both " together, and call the product the rectangle of those two colours." The following is given as a convenient method of multiplying:

	$+ 3x$	$+ 2$
$+5x$	$+ 15x^2$	$+ 10x$
-1	$- 3x$	$- 2$
Product	$+ 15x^2 + 7x - 2$	

which shews the product of $(5x - 1) \times (3x + 2.)$ (Here follow examples).

Sect. III.

On Division of Colours.

" Write the dividend and divisor in one place, find numbers or colours or both, " such that when they are multiplied by the divisor, the product subtracted from " the dividend will leave no remainder. Those numbers or colours will be the " quotient."

* In the Persian translation there is no algebraic notation, I mean to translate " the unknown" by x, " the black" by y, and so on. And in like manner I have used the marks of multiplication, &c. instead of writing the words at length as they are in the Persian.

SECT. IV.

On the square of Colours:

"That is to say, the product arising from any thing multiplied by itself." Examples.

SECT. V.

On the Square Root of Colours.

"To know the square root of a colour, find that which when it is multiplied by "itself the product subtracted from the colour whose root is required, will leave no "remainder. The rule is the same if there are other colours or numbers with "that colour."

Example. Required the square root of $16x^2+36-48x$. The roots of $16x^2$ and 36 are $4x$ and 6, and as $48x$ is $-$ these two roots must have different signs. Suppose one + and the other $-$, multiply them and the product will be $-24x$; twice this is $-48x$ which was required. The root then is $+4x-6$, or $+6-4x$.

Another Example. Required the square root of $9x^2+4y^2+z^2+12xy-6xz-4yz-6x-4y+2z+1$. Take the root of each square; we have $3x$, $2y$, z, and 1. Multiply these quantities and dispose the products in the cells of a square.

	$3x$	$2y$	z	1
$3x$	$9x^2$	$6xy$	$3xz$	$3x$
$2y$	$6xy$	$4y^2$	$2yz$	$2y$
z	$3xz$	$2yz$	z^2	z
1	$3x$	$2y$	z	1

To find what sort of quantities these are: The product of x and y is +, therefore the factors are like, suppose them both —. The product of x and z is —, therefore the former having been supposed — the latter must be + because the factors must be different. $3x$ is the product of $3x$ and 1; and x being —, 1 must be +. The sorts thus found are to be placed in the cells accordingly. The sum of the products is the square whose root was required. If x had been supposed + the sorts would have been contrary, the reason of which is plain.

CHAP. IV.

On Surds.

Containing five sections.

Sect. I.

On Addition and Subtraction.

To find the sum or difference of two surds; $\sqrt{a}$ and $\sqrt{b}$ for instance.

Rule. Call $a+b$ the greater surd; and if $a \times b$ is rational call $2\sqrt{ab}$ the less surd. The sum will be $\sqrt{(a+b+2\sqrt{ab})}$ *, and the difference $\sqrt{(a+b-2\sqrt{ab})}$. If $a \times b$ is irrational the addition and subtraction are impossible.

Example. Required the sum of $\sqrt{2}$ and $\sqrt{8}$; $2+8=10$ the greater surd. $2\times8=16$, $\sqrt{16}=4$, $4\times2=8$ the less surd. $10+8=18$ and $10-8=2$. $\sqrt{18}$ then will be the sum and $\sqrt{2}$ the difference. If one of the numbers is rational take its square and proceed according to the rule, and this must be attended to in multiplication and division, for on a number square with a number not square the operation cannot be performed.

Another Rule. Divide a by b and write $\sqrt{\frac{a}{b}}$ in two places In the first place add 1, and in the second subtract 1; then we shall have $\sqrt{((\sqrt{\frac{a}{b}}+1)^2 \times b)}$ $=\sqrt{a}+\sqrt{b}$ and $\sqrt{((\sqrt{\frac{a}{b}}-1)^2\times b)}=\sqrt{a}-\sqrt{b}$. If $\frac{a}{b}$ is irrational the addition can only be made by writing the surds as they are, and the subtraction by writing the greater number + and the less —.

* For $\sqrt{(a+b\pm2\sqrt{ab})}=\sqrt{a}\pm\sqrt{b}$.

SECT. II.

On Multiplication.

Proceed according to the rules already given; but if one of the factors has numbers as dirhems or dinars, take their squares and go on with the operation.

Example. Multiply $\sqrt{3}+5$ by $\sqrt{2}+\sqrt{3}+\sqrt{8}$ As 5 is of the square sort take its square, and arrange in a table thus:

	$\sqrt{2}$	$\sqrt{3}$	$\sqrt{8}$
$\sqrt{3}$	$\sqrt{6}$	$\sqrt{9}$	$\sqrt{24}$
$\sqrt{25}$	$\sqrt{50}$	$\sqrt{75}$	$\sqrt{200}$
Product	$3+\sqrt{54}+\sqrt{450}+\sqrt{75}$		

In summing the terms of the product, if any square number is found, take its root. Here 9 is found and its root is 3. The rest of the terms being irrational, add such as can be added. $\sqrt{6}+\sqrt{24}=\sqrt{54}$. If this last were a square number its root should be extracted.

Again, $\sqrt{50}+\sqrt{200}=\sqrt{450}$. No further addition is possible; the complete product therefore is $3+\sqrt{54}+\sqrt{450}+\sqrt{75}$.

Another rule to be observed is, if any of the terms which compose the factors can be added, take their sum and write it in the table instead of the terms of which it is formed. Thus in the last example $\sqrt{2}$ and $\sqrt{8}$ may be added. Write $\sqrt{18}$ which is their sum in the table, and we shall have

	$\sqrt{3}$	$\sqrt{18}$
$\sqrt{3}$	$\sqrt{9}$	$\sqrt{54}$
$\sqrt{25}$	$\sqrt{75}$	$\sqrt{450}$
$3+\sqrt{75}+\sqrt{54}+\sqrt{450}$		

and the result is the same as before.

Another Example. Multiply $\sqrt{3}+\sqrt{25}$ by $\sqrt{3}+\sqrt{12}-5$. Instead of $\sqrt{3}$ and $\sqrt{12}$ write their sum $\sqrt{27}$. Take the square of 5 it is 25, and this is negative notwithstanding the rule which says that whether the root is negative or affirmative the square shall be affirmative. Here the square must be of the same sort as the root. Multiply $\sqrt{27}-\sqrt{25}$ by $\sqrt{3}+\sqrt{25}$.

	$+\sqrt{27}$	$-\sqrt{25}$
$+\sqrt{3}$	$+\sqrt{81}$	$-\sqrt{75}$
$+\sqrt{25}$	$+\sqrt{675}$	$-\sqrt{625}$
	$-16+\sqrt{300}$	

$\sqrt{81}=9$ and $\sqrt{625}=25$. 25 being negative and 9 affirmative their sum is -16, and the sum of $+\sqrt{675}$ and $-\sqrt{75}$ is $+\sqrt{300}$. Therefore $(\sqrt{3}+\sqrt{25})\times(\sqrt{3}+\sqrt{12}-5)=-16+\sqrt{300}$.

SECT. III.

On Division.

Divide the dividend by the divisor, and if the quotient is found without a remainder the division is complete. When this cannot be done proceed as follows:

When in the divisor there are both affirmative and negative terms, if there are more of the former make one of them negative; if more of the latter make one of them affirmative. When all the terms are affirmative make one negative, and when all are negative make one affirmative. When the number of affirmative terms is equal to that of the negative, it is optional to change one of them or not. Multiply the divisor (thus prepared) by the original divisor, and add the products rejecting such quantities as destroy each other. Multiply the prepared divisor by the dividend, and divide the product of this multiplication by that of the former the result will be the quotient required.

Example. Let the dividend be that which was the product in the first example under the rule for multiplication, viz. $3+\sqrt{54}+\sqrt{450}+\sqrt{75}$, and the divisor $\sqrt{18}+\sqrt{3}$.

$$\frac{75}{3}=25,\ \frac{450}{18}=25,\ 3^2=9,\ \frac{9}{3}=3,\ \frac{54}{18}=3,\ \sqrt{25}=5,$$

the quotient then is $5+\sqrt{3}$.

Another Example. Divide $\sqrt{9}+\sqrt{54}+\sqrt{450}+\sqrt{75}$ by $5+\sqrt{3}$. Make $\sqrt{3}$ negative, and multiply 5 (or $\sqrt{25}$) $-\sqrt{3}$ by the divisor $\sqrt{25}+\sqrt{3}$.

	$+\sqrt{25}$	$-\sqrt{3}$
$+\sqrt{3}$	$+\sqrt{75}$	$-\sqrt{9}$
$+\sqrt{25}$	$+\sqrt{625}$	$-\sqrt{75}$