INTRODUCTION.

BHASCARA ACHARYA, the author of the following treatise, was born at Biddur, a city in the Deccan, in the year of Salivahana 1036, which corresponds with the year 1114 of the Christian era.* He wrote several astronomical and mathematical works, the most celebrated of which are the Lilawati, Bija Gannita, and Siromani. The two first, which relate to Arithmetic, Geometry, and Algebra, appear to have superseded entirely the more ancient treatises on these subjects, no other being in use, or, so far as we know, having even been seen, by astronomers of the present day.

The Bija Gannita treats of Algebra. It was translated into Persian in 1634 by Ata Allah Rashidi;† and from this version an account of the work has been published lately by Edward Strachey Esq. of the Bengal civil service, in what is termed "partly literal translation and partly abstract," accompanied with learned notes and illustrations.

The Siromani is a treatise on Astronomy. As it explains the science in a fuller and more perspicuous manner than the ancient and celebrated work called the Surya Siddhanta, it has a high repute among astronomers of the Deccan, and is often the only work which they peruse. It is divided into two *Adya*, or parts, named the *Cola Adya*, that which regards the globular form of the earth, and the *Gannita Adya*, that which relates to astronomical computations.

* As. Res. vol. 9. p. 351.

† Strachey's Bija Gannita, preface p. 4.

The Lilawati exhibits a regular, well connected, and, considering the period in which it was written, a profound system of arithmetic; and also contains many useful propositions in geometry and mensuration. It is the first work which is studied by Hindu astronomers, or rather astrologers; for in this country these two professions are always conjoined, and in general the former is considered subservient to the latter. The rules are written in verse in a very concise and even elliptical style, and possess in no slight degree the characteristic obscurity of Sanscrit compositions on science and philosophy.

By directions of the Emperor Acbar, whose liberal promotion of literature and science added glory to his conquests, it was translated into Persian in 1587 by Fyzi, the brother of Abu Fazil the Emperor's secretary. I procured a copy of this version from Mulla Firoz, a learned Parsi, who has studied with much success the astronomical system of the Arabians. Fyzi informs us that in making his translation he had the "assistance of men learned in the science, particularly of astrologers in the Deccan". His translation possesses that general accuracy which might be expected from a person of Fyzi's talents and knowledge, aided by such eminent mathematicians as his own situation, or the influence of his Royal Patron, could obtain. It is, however, often very obscure, and in several places there are considerable omissions, especially towards the end of the arithmetic, and in the geometrical operations which immediately precede the chapter on circles. The chapters on indeterminate problems and on transpositions are altogether omitted. Besides, the style is not only much more diffuse than what necessarily arises from the difference of the Persian and Sanscrit idioms, but the manner also of delivering the rules, and of detailing the operations, generally varies in a very considerable degree from that of the original text. This, indeed, is so remarkable as to induce a suspicion, that Fyzi contented himself with writing down the verbal explanation afforded by his assistants.

In the library of my very learned and estimable friend William Erskine Esq. there is a translation of the Lilawati into the language of Marwar which I have examined. It is of a date so late as 1762, and probably was made for the use of the Jaina priests, many of whom profess astrology. In consequence of the close affinity between the Marwari and Sanscrit languages, the translation is in general very literal, and also retains all the technical terms employed in the original In several chapters, however, there are important omissions, and the indeterminate problems and transpositions are left out altogether.

A curious account of the occasion of writing the Lilawati is given by Fyzi in the preface to his translation. The story he relates has not been confirmed to me by any native of this country, nor have I observed it mentioned by any Hindu author; but as it may amuse the reader, I shall here transcribe a translation of it made by Mr. Strachey, and published in Hutton's mathematical tracts.

" It is said that the composing the Lilawati was occasioned by the following circumstance. Lilawati was the name of the author's (Bhascara's) daughter, concerning whom it appeared, from the qualities of the Ascendant at her birth, that she was destined to pass her life unmarried, and to remain without children. The father ascertained a lucky hour for contracting her in marriage, that she might be firmly connected, and have children. It is said that when that hour approached, he brought his daughter and his intended son near him. He left the hour cup on the vessel of water, and kept in attendance a time-knowing astrologer, in order that when the cup should subside in the water, those two precious jewels should be united. But, as the intended arrangement was not according to destiny, it happened that the girl, from a curiosity natural to children, looked into the cup, to observe the water coming in at the hole; when by chance a pearl separated from her bridal dress, fell into the cup, and, rolling down to the hole, stopped the influx of the water. So the astrologer waited in expectation of the promised hour. When the operation of the cup had thus been delayed beyond all moderate time, the father was in consternation, and examining, he found that a small pearl had stopped the course of the water, and that the long-expected hour was passed. In short, the father, thus disappointed, said to his unfortunate daughter, I will write a book of your name, which shall remain to the latest times—for a good name is a second life, and the ground-work of eternal existence."

My object in the following translation is to furnish an authentic document, which, by exhibiting not only the actual degree of mathematical knowledge possessed by the Hindus in the 12th century, but also, by shewing their modes and principles of operation, may lead to a fair conculusion regarding their pretensions to originality in this department of science. When it is considered that a translation of the present work has been long a desideratum, and that an unsuccessful attempt to translate it was made by that excellent mathematician Mr. Burrow, it will readily be supposed that considerable difficulties must have occurred in the course of this undertaking. It is probable, indeed, that I should have failed entirely, had not most material assistance been derived from three commentaries which I had the good fortune to obtain. Two of these were procured for me at Nagpore by the kind exertions of George Sotheby Esq. assistant to the British Resident at that place. It will be seen that ample use has been made of these commentaries. The method of operating in the examples is, in almost every instance, illustrated from them at full length. I reckon it also a great advantage to have possessed three different copies of the original text. One of these copies was written in Guzerat in Samvut 1729, which corresponds with the Christian year 1673. This being the copy from which I have translated, it has been sent to England in order to be placed in the Library of the Hon'ble East India Company. The other two copies were written in the Deccan. Their dates are not put down, but they are evidently not so old as the copy from Guzerat. The rules and examples of the Lilawati are also contained in two of the commentaries; so that I have had an opportunity of consulting actually five copies of the original work; and all these correspond with a degree of accuracy which I have rarely found to exist in different copies of Sanscrit books. The chief difference consists in the transposition of a few of the rules; and in the copy from Guzerat there are several *kshepaka*, or interpolated rules, which are not contained in the two other copies.

Tho' the opportunities now adverted to were favourable to the success of the undertaking, I am still not so sanguine as to imagine that this translation will fully satisfy the wishes and expectations of mathematicians in Europe. Some will perhaps be of opinion, that the examples, besides being drawn out at length from the commentaries, ought also to have been put down in the mathematical language of the western world. It did not, however, appear to me that, in regard to many of the examples, this was peculiarly requisite after the illustration afforded by the notes from the commentaries. Tho' neither brief nor elegant, these notes shew the different steps of each operation in a manner pretty clear and intelligible, so that no mathematician, it is imagined, will experience any difficulty or trouble in comprehending the rules or examples; and therefore they can occasion no trouble to any person who is disposed to translate them into the mathematical language to which he has been accustomed. Being chiefly desirous of presenting to the reader the views of the Hindu author, and the train of thought and of operation in the original, without any desire of accommodating them to the technical phraseology of European science, which in some instances might have led the mind into another series of ideas rendered familiar to it by long habit; I may perhaps have appeared sometimes to have carried this conformity to an excess scarcely justified by the idom of our language. But to any one who lays aside his preconceived habits of thinking on these subjects, and who takes up the work as a Scholar, and allows his mind to be led by the author's train of thought, this can occasion no difficulty. Illustrations, however, of such rules as were considered most curious and important, will be found in the notes. For most of these illustrations, many of which will be found extremely ingenious and elegant, I am indebted to the kind assistance of my young friends Lieutenants Mecleod, and Tate of the Engineers. It adds in no small degree to my obligation to these gentlemen, that their observations were written during the short intervals of leisure allowed them from the very laborious official duties in which they were engaged. I have only cause to regret that I had not the benefit of their assistance at an earlier period of the undertaking.

A defect of much more importance than what I have just noticed, will be discerned in the want of historical notes, comparing the mathematical knowledge displayed in the Lilawati with that which existed in Europe at the same period; and also comparing the Hindu rules and modes of operation with those of the Grecian school and of modern Europe. Such a comparison, however, even had the translator's qualifications rendered him competent to undertake it, could not have been accomplished with much success in this part of the world, where most of the works which ought to have been consulted cannot be obtained. Besides, any attempt of this nature, by a person even the best qualified, must be very imperfect until the astronomical and mathematical knowledge of the Hindus be more fully laid open to our view, by accurate translations of the principal works on these subjects. But whenever a work of so much curiosity and interest in the history of mathematical science shall be undertaken, it is perhaps not presuming too far to say, that the present small publication will be found a curious and valuable document.

Tho I have aimed at rendering this version as literal as possible, still it is considerably more diffuse than the original text. The compact brevity of Sanscrit is incompatible perhaps with the idiom of the English language; but besides this, the rules in the Lilawati are delivered in a style so very elliptical and obscure, that often they would have been quite unintelligible had not the meaning been expanded in the translation. The two books on the *Kutaka*, or indeterminate problems, and on transpositions, being seldom studied, are peculiarly dark and doubtful. Before the rules could be comprehended, it was necessary, in most of the cases, to go over the examples, as they are exhibited both in the text and in the commentaries. Greater freedom, therefore, has been used in translating these chapters than in any other part of the work.

With the exception of the Sanscrit technical terms which have been retained, the words printed in *italics* are not in the original. In the geometrical operations it will be observed that by root is always meant the square root.

The statements, positions of the figures, and modes of operation, are given in the translation of the text precisely as they are exhibited in the original; and tho' this exact, or rather servile imitation, may sometimes cause a little perplexity to the reader, it corresponds with the rule which I mentioned to have laid down to myself in the translation; and is an evil of less magnitude than that of adopting methods which might lead to misinterpretation, and to erroneous conclusions.

In the notes taken from the commentaries I have also usually given the exact modes of expression, and of putting down the results. At the same time, in order to exhibit the operation in a shorter space, I have occasionally employed the marks used in European treatises. But it should be recollected that all these marks are foreign to the Hindu system. I shall have occasion to notice immediately, that the only mark employed by the Hindus, is that of minus.

The arithmetical operations of the *Hindus* are performed on a board about 12 inches long and 8 broad. A white ground being first formed with a kind of pipe clay, the board is covered with sand, or *gulal*, which is flour died of a purple colour. The forms of the figures or letters are traced with a wooden style, which displacing the sand or coloured flour, leaves the white ground exposed By drawing the finger over the sand, these forms are easily obliterated, and the board is prepared for receiving new impressions. This is a matter of great convenience; for as the figures, in order to be distinct, must be written large, the board cannot contain the individual steps of even a short calculation. Hence the practice is to obliterate, in succession, the intermediate results, so that when the operation is finished, the general result only stands on the board.

The Hindu methods of operating in the four fundamental rules of addition, subtraction, multiplication, and division, are detailed at some length in the notes; but as these operations are not found in books, and can be learned only by seeing the processes gone through, a few more illustrations in this place may not be deemed superfluous.

In ADDITION they usually begin at the column on the right hand and proceed towards the left, in the same manner as is done in Europe. There is, however, another method, named inverse or retrograde, which is performed by beginning at the column on the left hand, and proceeding towards the right. According to this method the column of figures on the left hand is first added up, and the result put down beneath. The next column is then added up, and the units being put down, the tens are carried to the preceding result, which is blotted out, and the new result is set down in its place. The following example will illustrate this mode of operation.

3527685
9278346
5316925
8437207
4925624
―――――
29353667
3148578

The result of the operation, however, does not stand in this manner on the board; but, as each preceding result is rubbed out, and the new product put down in its place, the total amount stands thus;

31485787

To prove addition, a line is drawn below the uppermost number, or above the lowermost, which is then supposed to be cut off, and the rest of the numbers being added up, the uppermost line, or the lowermost, as the case may be, is added to the sum; and if the total amount correspond with that found by the first addition, the operation is considered correct.

The Hindus seem to be entirely ignorant of the method of proving addition by rejecting the nines; a fact which is deserving of notice, because as this method is familiar to those who follow the Arabian system, it shews how slow the Hindus are to borrow from their neighbours, even after a long period of free and unreserved communication.

Subtraction is performed either from right to left, or from left to right, the less number in both cases, being placed above the greater. In subtracting from right to left, when the number in the minuend is less than the one above it in the subtrahend, ten is borrowed upon the next minuend figure (which represents tens, hundreds, &c. according to its place); and this operation is denoted by a perpendicular stroke placed opposite the figure upon which the ten is borrowed; the subtrahend figure being then subtracted from the borrowed ten, and the remainder added to the figure in the minuend, the result is put down in the line below; after which, one is added to the preceding figure in the subtrahend, and the same process is repeated as before.*

EXAMPLE.

```
 '''|''
 263543
7652432
-------
7388889
```

The operation is thus expressed in words; 3 cannot be subtracted from 2, therefore borrow 10 upon the next minuend figure 3 (which here represents tens); then subtract 3 from 10, and there remains 7; 7 and 2 are 9, put down 9 below the minuend, and carry or add 1 to the subtrahend figure 4; this makes 5; then say 5 cannot be subtracted from 3, therefore borrow ten upon the next minuend figure 4, and subtract 5 from 10, there remains 5, which added to 3 makes 8; put this down below the minuend; and thus proceed thro' all the figures. This method differs from ours in placing the subtrahend above the minuend, and in subtracting the subtrahend figure from the borrowed 10, and adding the remainder to the minuend figure, instead of adding the borrowed 10 to the minuend figure, and subtracting the subtrahend figure from the sum.

As the method of subtracting from left to right is explained in the note p. 7 it will be sufficient in this place merely to give an example by way of illustration.

EXAMPLE.

```
16747358
42353437
--------
26616189
25 0 07
--------
25606079
```

Which is thus expressed in words: 1 from 4, there remains 3; 6 cannot be subtracted from 2, therefore take 10 from the preceding remainder 3 (which represents tens, hundreds, &c. according to its place) and adding it to the subtrahend figure 2, say 6 from 12, there remains 6; and thus proceed thro' all figures.

* In note D p. 6 and 7, a mistake has occurred in stating this mode of subtraction; but it will be easily corrected by looking at the operation given above.

Under the head of MULTIPLICATION, the first thing which probably will be noticed, is the want of a multiplication table. The table given in European treatises, is considered of high antiquity, being generally ascribed to Pythagoras, who is supposed by some to have studied arithmetic in India. The omission of such a table in the Lilawati, may have arisen from that work being designed not for children, but for grown up persons who have received the rudiments of education at school, where several very extensive multiplication tables are taught.

No less than five methods of multiplication are given in the text, and the commentaries add one or two more ; but the most common method is that comprehended under the first rule. It is performed in the following manner :

Write down the multiplicand, and below it the multiplier, so that the first figure on the right of the multiplier shall be immediately below the last figure on the left of the multiplicand ; and multiply this last figure on the left by all the figures of the multiplier ; then having put down the products according to their places above the multiplicand, move the multiplier forward one place, and multiply the second figure of the multiplicand in the same manner as before ; thus continuing to repeat the operation until all the figures in the multiplicand are gone through.

EXAMPLE.

131
2150
100112
126298
25508624
multiplicand .... 52436 .... 28000824 product.
multiplier 534
534
534
534
534

The individual results, however, are not set down in this manner, but the tens in each succeeding product are carried to the preceeding one, which is then rubbed out, and the new result is set down in its place. At the end of the operation, therefore, the whole product stands in one line, with the multiplier under it; thus

28000824
534

The obliterated figures, and the new results, may be presented in the following manner:

00
128
0971
81111
19860
72156
101112
670158
126298
25508624

multiplicand .. 52436 ...... 28000824 product
multiplier .. 534

The figures that are not crossed out form the product.

Though this method must appear very complicated and confused to an European arithmetician, it is the one which is most generally practised by the Jyotishis, or astronomers; and when we recollect that their manner of writing on the board does not admit of the individual steps being exhibited together, the above method, notwithstanding its apparent perplexity, may perhaps be considered the easiest, and most simple, which could be adopted under these circumstances. To perceive this, it is only necessary to attempt the multiplication of the above sum by proceeding from right to left, according to the European mode; at the same time blotting out each individual product in succession, and bringing the tens from the succeeding product to form a new result, in the place of the preceeding one, and thus exhibiting at once the whole product, without the intermediate operations.

It may be considered a coincidence worthy of notice, that the Greeks performed multiplication from left to right, probably for the same reasons which still influence Hindu arithmeticians.

The only method which the Hindus have of proving multiplication is by division. They seem entirely unacquainted with the method of proving it by casting the nines out of the sum of the figures in each of the factors, a method, however, which is given in Arabian treatises, and is denominated *tarazu*, or the balance.