INDIAN

Arithmetic and Algebra.

CHAPTER I.

INTRODUCTION.

1. HAVING bowed to the deity, whose head is like an elephant's;[1] whose feet are adored by gods; who, when called to mind, relieves his votaries from embarrassment; and bestows happiness on his worshippers; I propound this easy process of computation,[2] delightful by its elegance,[3] perspicuous with words concise, soft and correct, and pleasing to the learned.

AXIOMS.

[CONSISTING IN DEFINITIONS OF TECHNICAL TERMS.]

[*Money by Tale.*]

2. Twice ten cowry shells[4] are a *cácini*; four of these are a *pana*; sixteen of which must be here considered as a *dramma*; and in like manner, a *nishca*, as consisting of sixteen of these.

[1] GANÉSA, represented with an elephant's head and human body.
[2] *Pátí-ganita*; *pátí*, *paripátí*, or *vyacta-ganita*, arithmetic.
[3] *Lílávatí* delightful: an allusion to the title of the book. See notes on § 13 and 277.
[4] Cypræa moneta. *Sans.* Varátaca, capardí; *Hind.* Caurí.

[*Weights.*]

3. A *gunja*[1] (or seed of Abrus) is reckoned equal to two barley-corns; a *valla*, to two *gunjas;* and eight of those are a *d'harańa;* two of which make a *gadyáńaca*. In like manner one *d'háṭaca* is composed of fourteen *vallas*.

4. Half ten *gunjas* are called a *másha*,[2] by such as are conversant with the use of the balance: a *carsha* contains sixteen of what are termed *máshas;* a *pala*, four *carshas*. A *carsha* of gold is named *suverńa*.

[*Measures.*]

5—6. Eight breadths of a barley-corn[3] are here a finger; four times six fingers, a cubit;[4] four cubits, a staff;[5] and a *crósa* contains two thousand of these; and a *yójana*, four *crósas*.

So a bambu pole consists of ten cubits; and a field (or plane figure) bounded by four sides, measuring twenty bambu poles, is a *nivartana*[6]

7. A cube,[7] which in length, breadth and thickness measures a cubit, is termed a solid cubit: and, in the meting of corn and the like, a measure,

[1] A seed of Abrus precatorius: black or red; the one called *crishńala;* the other *racti*, *racticá* or *rattica;* whence *Hind.* ratti.

[2] Physicians reckon seven *gunjas* to the *másha;* lawyers, seven and a half. The same weight is intended; and the difference of description arises only from counting by heavier or lighter seeds of Abrus: in like manner as the earth is the same, whether rated at 3300 *yójanas;* or, with the *Śirómańi*, 4967; or, according to others, 6522. GAŃ.

[3] Eight barley-corns *(yava)* by breadth, or three grains of rice by length, are equal to one finger *(angula)*. GAŃ.

[4] *Hasta*, *cara* and synonyma of hand or fore arm. According to the commentator GAŃÉSA, this intends the practical cubit as received by artisans, and vulgarly called *gaj* [or *gaz*]. It is nearer to the yard than to the true cubit: but the commentator seems to have no sufficient ground for so enlarging the cubit.

[5] *Danda*, a staff: directed to be cut nearly of man's height. MENU, 2. 46.

[6] A superficial measure or area containing 400 square poles. SUR.

[7] *Dwádaśáśri*, lit. dodecagon, but meaning a parallelopipedon; the term *asra*, corner or angle, being here applied to the edge or line of incidence of two planes. See CHATURVÉDA on BRAHMEGUPTA, § 6.

which contains a solid cubit, is a *c'hárí* of *Magad'ha*[1] as it is denominated in science.

8. A *dróńa* is the sixteenth part of a *c'hárí;* an *âd'haca* is a quarter of a *dróńa;* a *prast'ha* is a fourth part of an *âd'haca;* and a *cùdaba*[2] is by the ancients[3] termed a quarter of a *prast'ha*.[4]

The rest of the axioms, relative to time[5] and so forth, are familiarly known.[6]

[1] The country or province situated on the *Śónebhadrá* river.—GAŃ. It is South Bihar. See, concerning other *c'hárí* measures, a note on § 236.

[2] 'In the *Cuṫapa*, the depth is a finger and a half; the length and breadth, each, three.' ŚRÍD'HARA ÁCHÁRYA cited by GANGÁD'HARA and SÚRYADÁSA. 'The *cuṫapa* or *cùdaba* is a wooden measure containing 13½ cubic *angulas;* the *prast'ha*, (four times as many) 54; the *âd'haca*, 216; the *dróńa*, 864; the *c'hárí*, 13824.'—GANG. and SÚR. See As. Res. vol. 5, p. 102.

[3] By ŚRÍD'HARA and the rest. SUR.

[4] Another stanza, (an eighth, on the subject of weights and measures,) occurs in one copy of the text; and that number is indicated in the *Manóranjana*. But the commentaries of GAŃÉŚA and SÚRYADÁSA specify seven, and GANGÁD'HARA alone expounds the additional stanza. It is therefore to be rejected as spurious, and interpolated: not being found in other copies of the text. The subject of it is the *mańa (mań)* of forty *śéṫas (śér)*; which, as a measure of corn by weight, is ascribed to the *Turushcas* or Muhammedans of India; the people of *Yavana-déśa*, as the commentator terms them.

"The *śéṫa** is here reckoned at twice seven *tancas*, each equal to three-fourths of a *gadyánaca:* and a *mańa*, at forty *śéṫas*. The name is in use among the *Turushcas*, for a weight of corn and like articles." See notes on § 97 and 236.

[5] The author has himself explained the measures of time in the astronomical part of his treatise. *(Sidd'hánta-śirómańi*, § 16-18.) GANG. and SÚR.

[6] Concerning weights and measures, see *Gańita-sára* of ŚRÍD'HARA, §4—8; and PRÍT'HÚDACA SWÁMÍ CHATURVÉDA on BRAHMEGUPTA's arithmetic, § 10-11.

* The copy of GANGÁD'HARA's commentary writes *śaura*. But the exemplar of the text, containing the passage, has *śéṫa*.

CHAPTER II.

SECTION I.

Invocation.[1]

9. SALUTATION to GAṄÉŚA, resplendent as a blue and spotless lotus; and delighting in the tremulous motion of the dark serpent, which is perpetually twining within his throat.

Numeration.

10—11. Names of the places of figures have been assigned for practical use by ancient writers,[2] increasing regularly[3] in decuple proportion: namely, unit, ten, hundred, thousand, myriad, hundred thousands, million, ten millions, hundred millions, thousand millions, ten thousand millions, hundred thousand millions, billion, ten billions, hundred billions, thousand billions, ten thousand billions, hundred thousand billions.[4]

[1] A reason of this second introductory stanza is, that the foregoing definitions of terms are not properly a part of the treatise itself; none such having been premised by ÁRYA-BHAṬṬA and other ancient authors to their treatises of arithmetic. GAṄ. and *Manó.*

[2] According to the *Hindus*, numeration is of divine origin; 'the invention of nine figures *(anca)*, with the device of places to make them suffice for all numbers, being ascribed to the beneficent Creator of the universe,' in BHÁSCARA's *Vásaná* and its gloss; and in CRĬSHNA's commentary on the *Vija-gaṅita*. Here nine figures are specified; the place, when none belongs to it, being shown by a blank *(śúnya)*; which, to obviate mistake, is denoted by a dot or small circle.

[3] From the right, where the first and lowest number is placed, towards the left hand. GAṄ. &c.

[4] Sans. *éca, daśa, śata, sahasra, ayuta, lacsha, prayuta, cóṭi, arbuda, abja* or *padma, c'harva, nic'harva, mahápadma, śancu, jalad'hi* or *samudra, antya, mad'hya, parárd'ha.*

A passage of the *Véda*, which is cited by SÚRYA-DÁSA, contains the places of figures. 'Be these the milch kine before me, one, ten, a hundred, a thousand, ten thousand, a hundred thousand, a million, ... Be these milch kine my guides in this world.'

GAṄÉŚA observes, that numeration has been carried to a greater number of places by ŚRÍD'HARA and others; but adds, that the names are omitted on account of the numerous contradictions and the little utility of those designations. The text of the *Gaṅita-sára* or abridgment of ŚRÍD'HARA does not correspond with this reference: for it exhibits the same eighteen places, and no more. *Gaṅ-sár.* § 2—3.)

SECTION II.

Eight Operations[1] *of Arithmetic.*

12. Rule of addition and subtraction:[2] half a stanza.

The sum of the figures according to their places is to be taken in the direct or inverse order:[3] or [in the case of subtraction] their difference.

13. Example. Dear intelligent LÍLÁVATÍ,[4] if thou be skilled in addition and subtraction, tell me the sum of two, five, thirty-two, a hundred and ninety-three, eighteen, ten, and a hundred, added together; and the remainder, when their sum is subtracted from ten thousand.

Statement, 2, 5, 32, 193, 18, 10, 100.
[Answer.] Result of the addition, 360.
Statement for subtraction, 10000, 360.
[Answer.] Result of the subtraction, 9640.[5]

14—15. Rule of multiplication:[6] two and a half stanzas.

Multiply the last[7] figure of the multiplicand by the multiplicator, and

[1] *Paricarmáshṭaca*, eight operations, or modes of process: logistics or algorism.

[2] *Sancalana, sancalita, miśrana, yuti, yóga*, &c. summation, addition. *Vyavacalana, vyavacalita, sód'hana, patana*, &c. subtraction. *Antara*, difference, remainder.

[3] From the first on the right, towards the left; or from the last on the left, towards the right. GANG.

[4] Seemingly the name of a female to whom instruction is addressed. But the term is interpreted in some of the commentaries, consistently with its etymology, "Charming."—See § 1. and 277.

[5] Mode of working addition as shown in the *Manóranjana:*

Sum of the units,	2, 5, 2, 3, 8, 0, 0, ...	20
Sum of the tens,	3, 9, 1, 1, 0, ...	14
Sum of the hundreds,	1, 0, 0, 1, ...	2
Sum of the sums		360

[6] *Gunana, abhyása;* also *hanana* and any term implying a tendency to destroy. It is denominated *pratyutpanna* by BRAHMEGUPTA, § 3; and by ŚRÍD'HARA, § 15—17.

Gunya multiplicand. *Gunaca* multiplicator. *Ghata* product.

[7] The digit standing last towards the left. The work may begin either from the first or the last digit, according to ŚRÍD'HARA. *Ganita-sára*, § 15.

next the penult, and then the rest, by the same repeated. Or let the multiplicand be repeated under the several parts of the multiplicator, and be multiplied by those parts: and the products be added together. Or the multiplier being divided by any number which is an aliquot part of it, let the multiplicand be multiplied by that number and then by the quotient, the result is the product. These are two methods of subdivision by form. Or multiply separately by the places of figures, and add the products together. Or multiply by the multiplicator diminished or increased by a quantity arbitrarily assumed; adding, or subtracting, the product of the multiplicand taken into the assumed quantity.[1]

16. Example. Beautiful and dear LI'LA'VATI', whose eyes are like a fawn's! tell me what are the numbers resulting from one hundred and thirty-five, taken into twelve? if thou be skilled in multiplication by whole or by parts, whether by subdivision of form or separation of

[1] The author teaches six methods, according to the exposition of SU'RYADA'SA, &c.; but seven, as interpreted by GANGA'D'HARA: and those, combined with the four of SCANDASE'NA and SRI'D'HARA, (one of which at the least is unnoticed by BHA'SCARA,) make eight distinct ways. The mode of multiplication by parts (*c'handa-pracára*) is distinguished into *rúpa-vibhága* and *st'hána-vibhága*, or subdivision of the form and severance of the digits: the first is again divided into multiplication by integrant or by aliquot parts: the second in like manner furnishes two ways, according as the digits of the multiplier or of the multiplicand are severed. These then are four methods, deduced from two of SCANDASE'NA and SRI'D'HARA; to which two others are added by BHA'SCARA, consisting in the increase or decrease of the multiplier by an arbitrary quantity, and taking the sum or difference of the products. To those six must be joined the *Tatst'ha* of the older authors, and their *Capátasand'hi*; if indeed this be not (conformably with GANGA'D'HARA's opinion,) intended by BHA'SCARA's first method. It is wrought by repeating or moving the multiplier over (according to GANGA'D'HARA, or under, as directed by the *Manóranjana*,) every digit of the multiplicand; and, according to the explanation of GANE'SA, it proceeds obliquely, joining products along compartments. The *tatst'ha*, so named because the multiplier is stationary, appears from GANE'SA's gloss to be cross multiplication. 'After setting the multiplier under the multiplicand,' he directs to 'multiply unit by unit, and note the result underneath. Then, as in cross multiplication,* multiply unit by ten, and ten by unit, add together, and set down the sum in a line with the foregoing result. Next multiply unit by hundred, and hundred by unit, and ten by ten; add together, and set down the result as before: and so on, with the rest of the digits. This being done, the line of results is the product of the multiplication.' The commentator considers this method as 'difficult, and not to be learnt by dull scholars without oral instruction.' He adds, that 'other modes may be devised by the intelligent.' See Arithm. of BRAHM. § 55, *Gan.-sár.* § 15—17.

* *Vajrábhyása.* See *Víja-ganita*, § 77.

digits.[1] Tell me, auspicious woman, what is the quotient of the product divided by the same multiplier?

Statement, Multiplicand 135. Multiplicator 12.

Product (multiplying the digits of the multiplicand successively by the multiplicator) 1620.

Or, subdividing the multiplicator into parts, as 8 and 4; and severally multiplying the multiplicand by them; adding the products together: the result is the same, 1620.

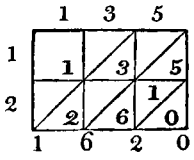
Or, the multiplicator 12 being divided by three, the quotient is 4; by which, and by 3, successively multiplying the multiplicand, the last product is the same, 1620.

Or, taking the digits as parts, viz. 1 and 2; the multiplicand being multiplied by them severally, and the products added together, according to the places of figures, the result is the same, 1620.

Or, the multiplicand being multiplied by the multiplicator less two, viz. 10, and added to twice the multiplicand, the result is the same, 1620.

Or, the multiplicand being multiplied by the multiplicator increased by eight, viz. 20, and eight times the multiplier being subtracted, the result is the same, 1620.

[1] The following scheme of the process of multiplication is exhibited in GANÉŚA's commentary.



Or the process may be thus ordered, according to GANGÁD'HARA,

12	12	12
1	3	5
12		60
	36	
	1620	

Or, in this manner,

135	135
1	2
	270
	135
	1620

Or in the subjoined modes taken from CHATURVÉDA, &c.

135	1	135
135	2	270
		1620

135	8	1080
135	4	540
		1620

135	20	2700
135	8	1080
		1620

17. Rule of division.[1] One stanza.

That number, by which the divisor being multiplied balances the last digit of the dividend [and so on[2]], is the quotient in division: or, if practicable, first abridge[3] both the divisor and dividend by an equal number, and proceed to division.

[Example.] Statement of the number produced by multiplication in the foregoing example, and of its multiplicator for a divisor: Dividend 1620. [Divisor 12.]

Quotient 135; the same with the original multiplicand.[4]

Or both the dividend and the divisor, being reduced to least terms by the common measure three, are 540 and 4; or by the common measure four, they become 405 and 3. Dividing by the respective reduced divisors, the quotient is the same, 135.

18—19. Rule for the square[5] of a quantity: two stanzas.

The multiplication of two like numbers together is the square. The square of the last[6] digit is to be placed over it; and the rest of the digits, doubled and multiplied by that last, to be placed above them respectively; then repeating the number, except the last digit, again [perform the like operation]. Or twice the product of two parts, added to the sum of the squares of the parts, is the square [of the whole number].[7] Or the product of the sum and

[1] *Bhága-hára, bhájana, haraṇa, ch'hédana:* division. *Bhájya,* dividend. *Bhájaca, hara,* divisor. *Labd'hi,* quotient.

[2] Repeating the divisor for every digit, like the multiplier in multiplication. GANG.

[3] *Apavartya,* abridging. See note on § 249.

[4] The process of long division is exhibited in the *Manóranjana* thus: The highest places of the proposed dividend, 16, being divided by 12, the quotient is 1; and 4 over. Then 42 becomes the highest remaining number, which divided by 12 gives the quotient 3, to be placed in a line with the preceding quotient (1): thus 13. Remains 60, which, divided by 12, gives 5: and this being carried to the same line as before, the entire quotient is exhibited: viz. 135. *Manór.*

[5] *Varga, crĭti,* a square number.

[6] The process may begin with the first digit: as intimated by the author, § 24.

[7] Let the portions, or quantities comprising the first and last figures, be represented by the first letters of the alphabet, says the commentator on the *Vásaná:* Then, proceeding by the rule of multiplication, there results *a v* 1, *a . á g* 1, *a . á g* 1, *á v* 1; and, adding together like terms, *a v* 1, *a . á g* 2, *á v* 1. RANG.

difference of the number and an assumed quantity, added to the square of the assumed quantity, is the square.[1]

20. Example. Tell me, dear woman, the squares of nine, of fourteen, of three hundred less three, and of ten thousand and five, if thou know the method of computing the square.

Statement, 9, 14, 297, 10005.

[Answer.] Proceeding as directed, the squares are found: 81, 196, 88209, 100,100,025.

Or, put 4 and 5, parts of nine. Their product doubled 40, added to the sum of their squares 41, makes 81.

So, taking 10 and 4, parts of fourteen. Their product 40, being doubled, is 80; which, added to 116, the sum of the squares 100 and 16; makes the entire square, 196.

Or, putting 6 and 8. Their product 48, doubled, is 96; which, added to the sum of the squares 36 and 64, viz. 100, makes the same, 196.

Again, 297, diminished by three, is 294; and, in another place, increased by the same, is 300. The product of these is 88200; to which adding the square of three 9, the sum is as before the square, 88209.

21. Rule for the square-root:[2] one stanza.

Having deducted from the last of the odd digits[3] the square number,

The proposed quantity may be divided into three parts instead of two; and the products of the first and second, first and third, and second and third, being added together and doubled, and added to the sum of the squares of the parts, the total is the square sought. GAN.

[1] Another method is hinted in the author's note on this passage; consisting in adding together the product of the proposed quantity by any assumed one, and its product by the proposed less the assumed one. RANG.

[2] *Varga-múla* root of the square: *Múla*, *pada*, and other synonyma of root.

[3] Every uneven place is to be marked by a vertical line, and the intermediate even digits by a horizontal one. But, if the last place be even, it is joined with the contiguous odd digit. Ex. $\overset{1}{8}\,\overset{-}{8}\,\overset{1}{2}\,\overset{-}{0}\,\overset{1}{9}$.

From the last uneven place 8, deduct the square 4, remains $4\,\overset{-}{8}\,\overset{1}{2}\,\overset{-}{0}\,\overset{1}{9}$. Double the root 2, and divide by that (4) the subsequent even digit $4\overset{-}{8}$: quotient nine [a higher one cannot be taken for the root of the foregoing digit would become greater than 2:] the remainder is $1\,2\,\overset{1}{2}\,\overset{-}{0}\,\overset{1}{9}$. From the uneven place [with the residue] $1\,2\,\overset{1}{2}$, subtract the square of the quotient 9, viz. 81, the remainder is $4\,1\,\overset{-}{0}\,\overset{1}{9}$. The double of the quotient 18 is to be placed in a line with the former double number

double its root; and by that dividing the subsequent even digit, and subtracting the square of the quotient from the next uneven place, note in a line [with the preceding double number] the double of the quotient. Divide by the [number as noted in a] line the next even place, and deduct the square of the quotient from the following uneven one, and note the double of the quotient in the line. Repeat the process [until the digits be exhausted.] Half the [number noted in the] line is the root.

22. Example. Tell me, dear woman, the root of four, and of nine, and those of the squares before found, if thy knowledge extend to this calculation.

Statement, 4, 9, 81, 196, 88209, 100100025.
Answer. The roots are 2, 3, 9, 14, 297, 10005.

23—25. Rule for the cube[1]: three stanzas.

The continued multiplication of three like quantities is a cube. The cube of the last [digit] is to be set down; and next the square of the last multiplied by three times the first; and then the square of the first taken into the last and tripled; and lastly the cube of the first: all these, added together according to their places, make the cube. The proposed quantity [consisting of more than two digits] is distributed into two portions, one of which is then taken for the last [and the other for the first]; and in like manner repeatedly [if there be occasion.[2]] Or the same process may be begun from the first place of figures, either for finding the cube, or the square. Or three times the proposed number, multiplied by its two parts, added to the sum of the cubes of those parts, give the cube. Or the square root of the proposed number being cubed, that, multiplied by itself, is the cube of the proposed square.[3]

4; thus, 58. By this divide the even place 41$\bar{0}$; the quotient is 7, and remainder 4$\bar{9}$; to which uneven digit the square of the quotient 49 answers without residue. The double of the quotient 14 is put in a line with the preceding double number 58, making 594. The half of which is the root sought, 297. *Manó.* and GANG.

[1] *G'hana*, a cube. (*Lit.* solid.)

[2] The subdivision is continued until it comes to single digits.

GAṄÉŚA confines it to the places of figures *(st'hána-vibhága,)* not allowing the portioning of the number *(rúpa-vibhága;)* because the addition is to be made according to the places.

[3] This carries an allusion to the raising of quantities to higher powers than the cube. GAṄÉŚA