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The Algebra of Mohammed ben Musa

Mohammed ben Musa (c.780–c.850) was a Persian mathematician and astronomer. The word 'algebra' derives from his *Compendious Book on Calculation by Completion and Balancing*, which introduced modern algebraic methods. First published in 1831, this translation from Arabic into English was prepared by the German orientalist Friedrich August Rosen (1805–37). The key algebraic methods introduced are reduction, completion and balancing. To reduce an equation is to change an expression to a simpler form; completion is to remove a negative quantity from one side of the equation and add it to the other; and balancing is to cancel like terms on opposite sides of the equation. An account is also given of solving polynomial equations up to the second degree. Rosen's introduction and notes accompany the translation, which remains relevant in the history of mathematics.

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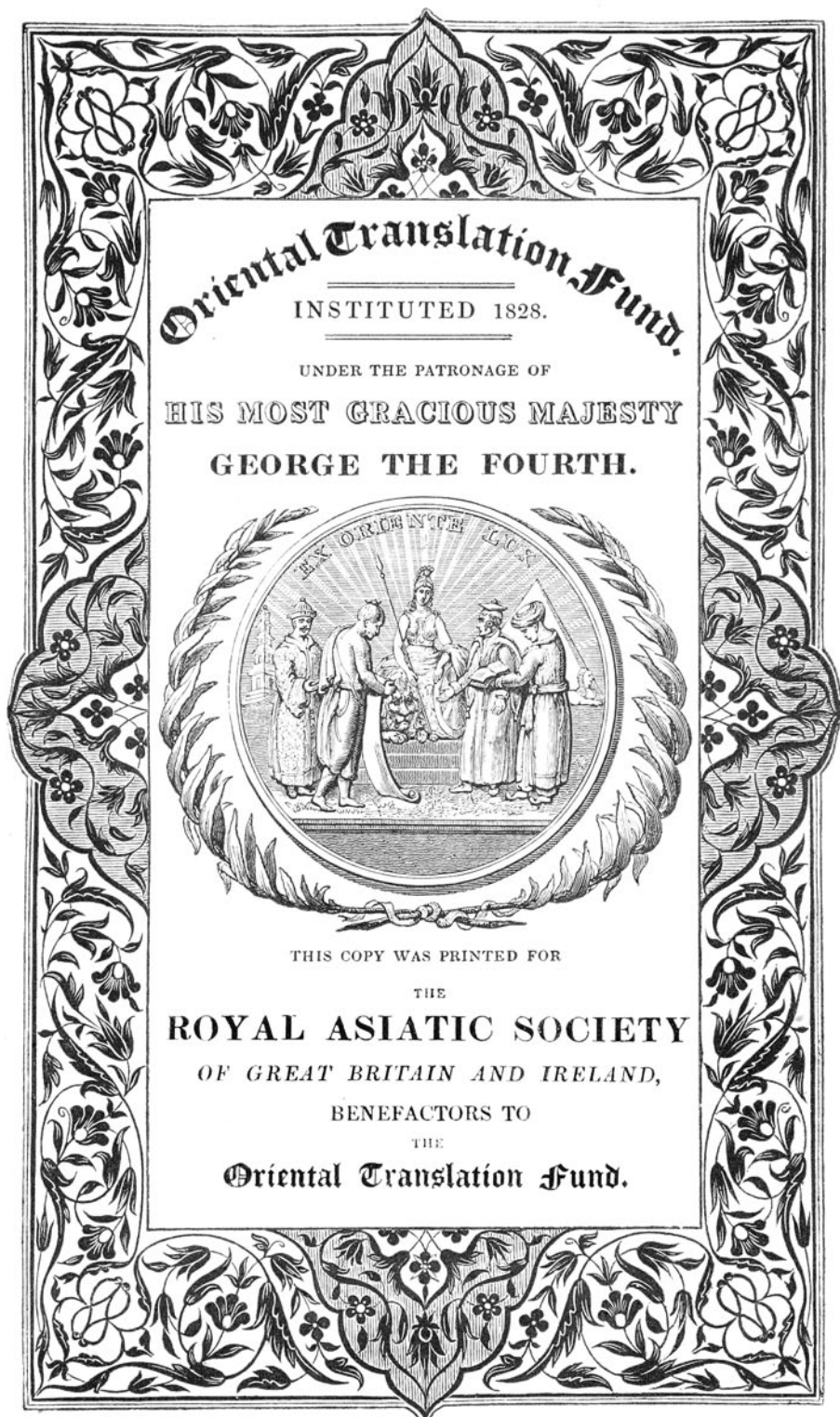
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P R E F A C E.

IN the study of history, the attention of the observer is drawn by a peculiar charm towards those epochs, at which nations, after having secured their independence externally, strive to obtain an inward guarantee for their power, by acquiring eminence as great in science and in every art of peace as they have already attained in the field of war. Such an epoch was, in the history of the Arabs, that of the Caliphs AL MANSUR, HARUN AL RASHID, and AL MAMUN, the illustrious contemporaries of CHARLEMAGNE; to the glory of which era, in the volume now offered to the public, a new monument is endeavoured to be raised.

ABU ABDALLAH MOHAMMED BEN MUSA, of Khowarezm, who it appears, from his preface, wrote this Treatise at the command of the Caliph AL MAMUN, was for a long time considered as the original inventor of Algebra. “*Hæc ars olim a MAHOMETE, MOSIS Arabis filio, initium sumpsit: etenim hujus rei locuples testis LEO-*

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NARDUS PISANUS.” Such are the words with which HIERONYMUS CARDANUS commences his *Ars Magna*, in which he frequently refers to the work here translated, in a manner to leave no doubt of its identity.

That he was not the inventor of the Art, is now well established; but that he was the first Mohammedan who wrote upon it, is to be found asserted in several Oriental writers. HAJI KHALFA, in his bibliographical work, cites the initial words of the treatise now before us,* and

* I am indebted to the kindness of my friend Mr. GUSTAV FLUEGEL of Dresden, for a most interesting extract from this part of HAJI KHALFA'S work. Complete manuscript copies of the كشف الظنون are very scarce. The only two which I have hitherto had an opportunity of examining (the one bought in Egypt by Dr. EHRENBERG, and now deposited in the Royal Library at Berlin—the other among RICH'S collection in the British Museum) are only abridgments of the original compilation, in which the quotation of the initial words of each work is generally omitted. The prospect of an edition and Latin translation of the complete original work, to be published by Mr. FLUEGEL, under the auspices of the Oriental Translation Committee, must under such circumstances be most gratifying to all friends of Asiatic literature.

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states, in two distinct passages, that its author, MOHAMMED BEN MUSA, was the first Mussulman who had ever written on the solution of problems by the rules of completion and reduction. Two marginal notes in the Oxford manuscript—from which the text of the present edition is taken—and an anonymous Arabic writer, whose *Bibliotheca Philosophorum* is frequently quoted by CASIRI,* likewise maintain that this production of MOHAMMED BEN MUSA was the first work written on the subject† by a Mohammedan.

* تاريخ الحكماء, written in the twelfth century. CASIRI *Bibliotheca Arabica Escurialensis*, T. I. 426. 428.

† The first of these marginal notes stands at the top of the first page of the manuscript, and reads thus : هذا اول كتاب وضع في الجبر والمقابلة في الاسلام ولهذا ذكر فيه من كتاب وضع في الجبر والمقابلة في الاسلام في الجبر والمقابلة “ This is the first book written on (the art of calculating by) completion and reduction by a Mohammedan : on this account the author has introduced into it rules of various kinds, in order to render useful the very rudiments of Algebra.” The other scholium stands farther on : it is the same to which I have referred in my notes to the Arabic text, p. 177.

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From the manner in which our author, in his preface, speaks of the task he had undertaken, we cannot infer that he claimed to be the inventor. He says that the Caliph AL MAMUN encouraged him to write a *popular* work on Algebra: an expression which would seem to imply that other treatises were then already extant. From a formula for finding the circumference of the circle, which occurs in the work itself (Text p. 51, Transl. p. 72), I have, in a note, drawn the conclusion, that part of the information comprised in this volume was derived from an Indian source; a conjecture which is supported by the direct assertion of the author of the *Bibliotheca Philosophorum* quoted by CASIRI (I. 426, 428). That MOHAMMED BEN MUSA was conversant with Hindu science, is further evident from the fact* that he abridged, at AL MAMUN'S request—but before the accession of that prince to the caliphate—the *Sindhind*, or

* Related by EBN AL ADAMI in the preface to his astronomical tables. CASIRI, I. 427, 428. COLEBROOKE, Dissertation, &c. p. lxiv. lxxii.

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astronomical tables, translated by MOHAMMED BEN IBRAHIM AL FAZARI from the work of an Indian astronomer who visited the court of ALMANSUR in the 156th year of the Hejira (A.D. 773).

The science as taught by MOHAMMED BEN MUSA, in the treatise now before us, does not extend beyond quadratic equations, including problems with an affected square. These he solves by the same rules which are followed by DIOPHANTUS*, and which are taught, though less comprehensively, by the Hindu mathematicians†. That he should have borrowed from DIOPHANTUS is not at all probable; for it does not appear that the Arabs had any knowledge of DIOPHANTUS' work before the middle of the fourth century after the Hejira, when ABU'L-WAFA BUZZJANI rendered it into Arabic‡. It

* See DIOPHANTUS, *Introd.* § 11. and *Book iv.* problems 32 and 33.

† *Lilavati*, p. 29, *Vijaganita*, p. 347, of Mr. COLEBROOKE'S translation.

‡ CASIRI *Bibl. Arab. Escur.* I. 433. COLEBROOKE'S *Dissertation*, &c. p. lxxii.

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is far more probable that the Arabs received their first knowledge of Algebra from the Hindus, who furnished them with the decimal notation of numerals, and with various important points of mathematical and astronomical information.

But under whatever obligation our author may be to the Hindus, as to the subject matter of his performance, he seems to have been independent of them in the manner of digesting and treating it: at least the method which he follows in expounding his rules, as well as in showing their application, differs considerably from that of the Hindu mathematical writers. BHASKARA and BRAHMAGUPTA give dogmatical precepts, unsupported by argument, which, even by the metrical form in which they are expressed, seem to address themselves rather to the memory than to the reasoning faculty of the learner: MOHAMMED gives his rules in simple prose, and establishes their accuracy by geometrical illustrations. The Hindus give comparatively few examples, and are fond of investing the statement of their problems in

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rhetorical pomp : the Arab, on the contrary, is remarkably rich in examples, but he introduces them with the same perspicuous simplicity of style which distinguishes his rules. In solving their problems, the Hindus are satisfied with pointing at the result, and at the principal intermediate steps which lead to it : the Arab shows the working of each example at full length, keeping his view constantly fixed upon the two sides of the equation, as upon the two scales of a balance, and showing how any alteration in one side is counterpoised by a corresponding change in the other.

Besides the few facts which have already been mentioned in the course of this preface, little or nothing is known of our Author's life. He lived and wrote under the caliphate of AL MAMUN, and must therefore be distinguished from ABU JAFAR MOHAMMED BEN MUSA*,

* The father of the latter, MUSA BEN SHAKER, whose native country I do not find recorded, had been a robber or bandit in the earlier part of his life, but had afterwards found means to attach himself to the court of the Caliph AL-MAMUN ; who, after MUSA's death, took care of

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likewise a mathematician and astronomer, who flourished under the Caliph AL MOTADED (who reigned A.H. 279-289, A.D. 892-902).

the education of his three sons, MOHAMMED, AHMED, and AL HASSAN. (ABILFARAGII *Histor. Dyn.* p. 280. CASIRI, I. 386. 418). Each of the sons subsequently distinguished himself in mathematics and astronomy. We learn from ABULFARAJ (*l. c.* p. 281) and from EBN KHALLIKAN (art. ثابت بن قرة) that THABET BEN KORRAH, the well-known translator of the Almagest, was indebted to MOHAMMED for his introduction to AL MOTADED, and the men of science at the court of that caliph. EBN KHALLIKAN's words are: فخرج من حران ونزل كفرتوثا واقام بها: مدة الي ان قدم محمد بن موسى من بلاد الروم راجعا الي بغداد فاجتمع به فراه فاضلا فصيحاً فاستصحبه الي بغداد وانزله في داره ووصله بالخليفة فادخله في جملة المنجمين (THABET BEN KORRAH) left Harran, and established himself at Kafratutha, where he remained till MOHAMMED BEN MUSA arrived there, on his return from the Greek dominions to Bagdad. The latter became acquainted with THABET and on seeing his skill and sagacity, invited THABET to accompany him to Bagdad, where MOHAMMED made him lodge at his own house, introduced him to the Caliph, and procured him an appointment in the body of astronomers." EBN KHALLIKAN here speaks of MOHAMMED BEN MUSA as of a well-known individual: he has however devoted no special article to an account of his life. It is possible

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The manuscript from whence the text of the present edition is taken—and which is the only copy the existence of which I have as yet been able to trace—is preserved in the Bodleian collection at Oxford. It is, together with three other treatises on Arithmetic and Algebra, contained in the volume marked *CMXVIII. Hunt. 214, fol.*, and bears the date of the transcription A.H. 743 (A.D. 1342). It is written in a plain and legible hand, but unfortunately destitute of most of the diacritical points: a deficiency which has often been very sensibly felt; for though the nature of the subject matter can but seldom leave a doubt as to the general import of a sentence, yet the true reading of some passages, and the precise interpretation of others, remain involved in obscurity. Besides, there occur several omissions of words, and even of entire sentences; and also instances of words or short passages writ-

that the tour into the provinces of the Eastern Roman Empire here mentioned, was undertaken in search of some ancient Greek works on mathematics or astronomy.

ten twice over, or words foreign to the sense introduced into the text. In printing the Arabic part, I have included in brackets many of those words which I found in the manuscript, the genuineness of which I suspected, and also such as I inserted from my own conjecture, to supply an apparent hiatus.

The margin of the manuscript is partially filled with *scholia* in a very small and almost illegible character, a few specimens of which will be found in the notes appended to my translation. Some of them are marked as being extracted from a commentary (شرح) by AL MOZAIHAFI*, probably the same author, whose full name is JEMALEDDIN ABU ABDALLAH MOHAMMED BEN OMAR AL JAZA'I† AL MOZAIHAFI, and whose "Introduction to Arithmetic," (مقدمة في الحساب) is contained in the same volume with MOHAMMED'S work in the Bodleian library.

Numerals are in the text of the work always

* Wherever I have met with this name, it is written without the diacritical points المرزحفي, and my pronunciation rests on mere conjecture.

† المرزاعي (?)

expressed by words : figures are only used in some of the diagrams, and in a few marginal notes.

The work had been only briefly mentioned in URIS' catalogue of the Bodleian manuscripts. Mr. H. T. COLEBROOKE first introduced it to more general notice, by inserting a full account of it, with an English translation of the directions for the solution of equations, simple and compound, into the notes of the "*Dissertation*" prefixed to his invaluable work, "*Algebra, with Arithmetic and Mensuration, from the Sanscrit of Brahme Gupta and Bhascara.*" (London, 1817, 4to. pages lxxv-lxxix.)

The account of the work given by Mr. COLEBROOKE excited the attention of a highly distinguished friend of mathematical science, who encouraged me to undertake an edition and translation of the whole : and who has taken the kindest interest in the execution of my task. He has with great patience and care revised and corrected my translation, and has furnished the commentary, subjoined to the text, in the form of common algebraic notation. But my

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obligations to him are not confined to this only ; for his luminous advice has enabled me to overcome many difficulties, which, to my own limited proficiency in mathematics, would have been almost insurmountable.

In some notes on the Arabic text which are appended to my translation, I have endeavoured, not so much to elucidate, as to point out for further enquiry, a few circumstances connected with the history of Algebra. The comparisons drawn between the Algebra of the Arabs and that of the early Italian writers might perhaps have been more numerous and more detailed ; but my enquiry was here restricted by the want of some important works. MONTUCLA, COSSALI, HUTTON, and the Basil edition of CARDANUS' *Ars magna*, were the only sources which I had the opportunity of consulting.