THE AUTHOR'S PREFACE.

IN THE NAME OF GOD, GRACIOUS AND MERCIFUL!

This work was written by MOHAMMED BEN MUSA, of KHOWAREZM. He commences it thus:

Praised be God for his bounty towards those who deserve it by their virtuous acts: in performing which, as by him prescribed to his adoring creatures, we express our thanks, and render ourselves worthy of the continuance (of his mercy), and preserve ourselves from change: acknowledging his might, bending before his power, and revering his greatness! He sent MOHAMMED (on whom may the blessing of God repose!) with the mission of a prophet, long after any messenger from above had appeared, when justice had fallen into neglect, and when the true way of life was sought for in vain. Through him he cured of blindness, and saved through him from perdition, and increased

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through him what before was small, and collected through him what before was scattered. Praised be God our Lord! and may his glory increase, and may all his names be hallowed—besides whom there is no God; and may his benediction rest on MOHAMMED the Prophet and on his descendants!

The learned in times which have passed away, and among nations which have ceased to exist, were constantly employed in writing books on the several departments of science and on the various branches of knowledge, bearing in mind those that were to come after them, and hoping for a reward proportionate to their ability, and trusting that their endeavours would meet with acknowledgment, attention, and remembrance—content as they were even with a small degree of praise; small, if compared with the pains which they had undergone, and the difficulties which they had encountered in revealing the secrets and obscurities of science.

(2) Some applied themselves to obtain information which was not known before them, and left it to posterity; others commented upon the difficulties in the works left by their predecessors, and defined the best method (of study), or rendered the access (to science) easier or

placed it more within reach; others again discovered mistakes in preceding works, and arranged that which was confused, or adjusted what was irregular, and corrected the faults of their fellow-labourers, without arrogance towards them, or taking pride in what they did themselves.

That fondness for science, by which God has distinguished the Imam al Mamun, the Commander of the Faithful (besides the caliphat which He has vouchsafed unto him by lawful succession, in the robe of which He has invested him, and with the honours of which He has adorned him), that affability and condescension which he shows to the learned, that promptitude with which he protects and supports them in the elucidation of obscurities and in the removal of difficulties, —has encouraged me to compose a short work on Calculating by (the rules of) Completion and Reduction, confining it to what is easiest and most useful in arithmetic, such as men constantly require in cases of inheritance, legacies, partition, law-suits, and trade, and in all their dealings with one another, or where the measuring of lands, the digging of canals, geometrical computation, and other objects of various sorts and kinds are concerned—relying on the goodness of my intention therein, and hoping that the learned will reward it, by obtaining (for me) through their prayers the excellence of the Divine mercy: in requital of which, may the choicest blessings and the abundant bounty of God be theirs! My confidence rests with God, in this as in every thing, and in Him I put my trust. He is the Lord of the Sublime Throne. May His blessing descend upon all the prophets and heavenly messengers!

MOHAMMED BEN MUSA'S

COMPENDIUM

ON CALCULATING BY

COMPLETION AND REDUCTION.

WHEN I considered what people generally want in calculating, I found that it always is a number. (3)

I also observed that every number is composed of units, and that any number may be divided into units.

Moreover, I found that every number, which may be expressed from one to ten, surpasses the preceding by one unit: afterwards the ten is doubled or tripled, just as before the units were: thus arise twenty, thirty, &c., until a hundred; then the hundred is doubled and tripled in the same manner as the units and the tens, up to a thousand; then the thousand can be thus repeated at any complex number; and so forth to the utmost limit of numeration.

I observed that the numbers which are required in calculating by Completion and Reduction are of three kinds, namely, roots, squares, and simple numbers relative to neither root nor square.

A root is any quantity which is to be multiplied by itself, consisting of units, or numbers ascending, or fractions descending.*

A square is the whole amount of the root multiplied by itself.

A simple number is any number which may be pronounced without reference to root or square.

A number belonging to one of these three classes may be equal to a number of another class; you may say, for instance, "squares are equal to roots," or "squares are equal to numbers," or "roots are equal to numbers."†

(4) Of the case in which *squares are equal to roots,* this is an example. "A square is equal to five roots of the same;"‡ the root of the square is five, and the square is twenty-five, which is equal to five times its root.

So you say, "one third of the square is equal to four roots;"§ then the whole square is equal to twelve roots; that is a hundred and forty-four; and its root is twelve.

Or you say, "five squares are equal to ten roots;"‖ then one square is equal to two roots; the root of the square is two, and its square is four.

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* By the word root, is meant the simple power of the unknown quantity.

† $cx^2 = bx \qquad cx^2 = a \qquad bx = a$

‡ $x^2 = 5x \qquad \therefore x = 5$

§ $\frac{x^2}{3} = 4x \qquad \therefore x^2 = 12x \qquad \therefore x = 12$

‖ $5x^2 = 10x \qquad \therefore x^2 = 2x \qquad \therefore x = 2$

In this manner, whether the squares be many or few, (*i. e.* multiplied or divided by any number), they are reduced to a single square; and the same is done with the roots, which are their equivalents; that is to say, they are reduced in the same proportion as the squares.

As to the case in which *squares are equal to numbers*; for instance, you say, "a square is equal to nine;"* then this is a square, and its root is three. Or "five squares are equal to eighty;"† then one square is equal to one-fifth of eighty, which is sixteen. Or "the half of the square is equal to eighteen;"‡ then the square is thirty-six, and its root is six.

Thus, all squares, multiples, and sub-multiples of them, are reduced to a single square. If there be only part of a square, you add thereto, until there is a whole square; you do the same with the equivalent in numbers.

As to the case in which *roots are equal to numbers;* for instance, "one root equals three in number;"§ then
the root is three, and its square nine. Or "four roots (5)
are equal to twenty;"‖ then one root is equal to five, and the square to be formed of it is twenty-five. Or "half the root is equal to ten;"¶ then the

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* $x^2=9 \qquad x=3$

† $5x^2=80 \therefore x^2=\frac{80}{5}=16$

‡ $\frac{x^2}{2}=18 \therefore x^2=36 \therefore x=6$

§ $x=3$

‖ $4x=20 \qquad \therefore x=5$

¶ $^{x}=10 \qquad \therefore x=20$

whole root is equal to twenty, and the square which is formed of it is four hundred.

I found that these three kinds; namely, roots, squares, and numbers, may be combined together, and thus three compound species arise;* that is, "squares and roots equal to numbers;" "squares and numbers equal to roots;" "roots and numbers equal to squares."

Roots and Squares are equal to Numbers;† for instance, "one square, and ten roots of the same, amount to thirty-nine dirhems;" that is to say, what must be the square which, when increased by ten of its own roots, amounts to thirty-nine? The solution is this: you halve the number‡ of the roots, which in the present instance yields five. This you multiply by itself; the product is twenty-five. Add this to thirty-nine; the sum is sixty-four. Now take the root of this, which is eight, and subtract from it half the number of the roots, which is five; the remainder is three. This is the root of the square which you sought for; the square itself is nine.

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* The three cases considered are,

1st. $cx^2 + bx = a$

2d. $cx^2 + a = bx$

3d. $cx^2 = bx + a$

† 1st case: $cx^2 + bx = a$

Example $x^2 + 10x = 39$

$$x = \sqrt{\left[\left(\tfrac{10}{2}\right)^2 + 39\right]} - \tfrac{10}{2}$$

$$= \sqrt{64} - 5$$

$$= 8 - 5 = 3$$

‡ *i. e.* the coefficient.

The solution is the same when two squares or three, or more or less be specified;* you reduce them to one single square, and in the same proportion you reduce also the roots and simple numbers which are connected therewith.

For instance, "two squares and ten roots are equal to forty-eight dirhems;"† that is to say, what must be (6)
the amount of two squares which, when summed up and added to ten times the root of one of them, make up a sum of forty-eight dirhems? You must at first reduce the two squares to one; and you know that one square of the two is the moiety of both. Then reduce every thing mentioned in the statement to its half, and it will be the same as if the question had been, a square and five roots of the same are equal to twenty-four dirhems; or, what must be the amount of a square which, when added to five times its root, is equal to twenty-four dirhems? Now halve the number of the roots; the moiety is two and a half. Multiply that by itself; the product is six and a quarter. Add this to twenty-four; the sum is thirty dirhems and a quarter. Take the root of this; it is five and a half. Subtract from this the moiety of the number of the roots, that is two and a half; the

* $cx^2+bx=a$ is to be reduced to the form $x^2+\frac{b}{c}x=\frac{a}{c}$

† $2x^2+10x=48$

$$x^2+5x=24$$
$$x=\sqrt{[(\tfrac{5}{2})^2+24]}-\tfrac{5}{2}$$
$$=\sqrt{[6\tfrac{1}{4}+24]}-2\tfrac{1}{2}$$
$$=5\tfrac{1}{2}-2\tfrac{1}{2}=3$$

remainder is three. This is the root of the square, and the square itself is nine.

The proceeding will be the same if the instance be, "half of a square and five roots are equal to twenty-eight dirhems;"* that is to say, what must be the amount of a square, the moiety of which, when added to the equivalent of five of its roots, is equal to twenty-eight dirhems? Your first business must be to complete your square, so that it amounts to one whole square. This you effect by doubling it. Therefore double it, and double also that which is added to it, as well as what is equal to it. Then you have a square and ten roots, equal to fifty-six dirhems. Now halve the roots; the moiety is five. Multiply this by itself; the product is twenty-five. Add this to fifty-six; the sum is eighty-one. Extract the root of this; it is nine. Subtract from this the moiety of the number of roots, which is five; the remainder is four. This is the root of the square which you sought for; the square is sixteen, and half the
(7) square eight.

Proceed in this manner, whenever you meet with squares and roots that are equal to simple numbers: for it will always answer.

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* $\frac{x^2}{2}+5x=28$

$$x^2+10x=56$$

$$x=\sqrt{[(\tfrac{10}{2})^2+56]}-\tfrac{10}{2}$$

$$=\sqrt{25+56}-5$$

$$=\sqrt{81}-5$$

$$=9-5=4$$