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Carl Gustav Jacob Jacobi (1804–51) was one of the nineteenth century's greatest mathematicians, as attested by the diversity of mathematical objects named after him. His early work on number theory had already attracted the attention of Carl Friedrich Gauss, but his reputation was made by his work on elliptic functions. Elliptic integrals had been studied for a long time, but in 1827 Jacobi and Niels Henrik Abel realised independently that the correct way to view them was by means of their inverse functions – what we now call the elliptic functions. The next few years witnessed a flowering of the subject as the two mathematicians pushed ahead. Adrien-Marie Legendre, an expert on the old theory, wrote: 'I congratulate myself that I have lived long enough to witness these magnanimous conflicts between two equally strong young athletes'. This Latin work, first published in 1829, is Jacobi's pioneering account of the new theory.

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Carl Gustav Jacob Jacobi
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Carl Gustav Jacob Jacobi

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FUNDAMENTA NOVA
THEORIAE
FUNCTIONUM ELLIPTICARUM

AUCTORE

D. CAROLO GUSTAVO IACOBO IACOBI,

PROF. ORD. IN UNIV. REGIOM.

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PROOEMIUM.

Ante biennium fere, cum theoriam functionum ellipticarum accuratius examinare placuit, incidi in quaestiones quasdam gravissimas, quae et theoriae illi novam faciem creare, et universam artem analyticam insigniter promovere videbantur. Quibus ad exitum felicem et propter difficultatem rei vix expectatum perductis, prima earum momenta breviter et sine demonstratione, mox cum vehementius illa desiderari, et invento novo vix fidem tribui videretur, addita demonstratione, cum Geometris communicavi. Urgebar simul, ut systema completum quaestionum a me susceptarum in publicum ederem. Cui desiderio ut ex parte saltem satisfacerem, fundamenta, quibus quaestiones meae superstructae sunt, in publicum edere constitui. Quae fundamenta nova theoriae functionum ellipticarum iam indulgentiae Geometrarum commendamus.

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Ut a typographorum mendis, quantum fieri potuit, mundus evaderet liber, Cl. SCHERK curare voluit, cui ea de re valde me obstrictum esse profiteor. Quae emendanda restant, ad calcem adiecta sunt.

Scribebam m. Febr. a. 1829
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