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978-1-108-05068-5 - A Synopsis of Elementary Results in Pure and Applied Mathematics: Containing Propositions, Formulae, and Methods of Analysis, with Abridged Demonstrations: Volume 2

George Shoobridge Carr

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