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A Synopsis of Elementary Results in Pure and Applied Mathematics

When George Shoobridge Carr (1837–1914) wrote his *Synopsis of Elementary Results* he intended it as an aid to students preparing for degree-level examinations such as the Cambridge Mathematical Tripos, for which he provided private tuition. He would have been startled to see the two volumes, first published in 1880 and 1886 respectively, reissued more than a century later. Notably, in 1903 the work fell into the hands of the Indian prodigy Srinivasa Ramanujan (1887–1920) and greatly influenced his mathematical education. It is the interaction between a methodical teaching aid and the soaring spirit of a self-taught genius which gives this reissue its interest. Volume 1, presented here in its 1886 printing, contains sections on mathematical tables, algebra, the theory of equations, plane trigonometry, spherical trigonometry, elementary geometry and geometrical conics.

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A Synopsis of Elementary Results in Pure and Applied Mathematics

Containing Propositions, Formulae, and Methods of Analysis, with Abridged Demonstrations

VOLUME 1

GEORGE SHOOBRIDGE CARR

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OF

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IN

PURE AND APPLIED MATHEMATICS:

CONTAINING

PROPOSITIONS, FORMULÆ, AND METHODS OF ANALYSIS,

WITH

ABRIDGED DEMONSTRATIONS.

BY

G. S. CARR, M.A.

VOL. I.

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PREFACE TO PART I.

THE work, of which the part now issued is a first instalment, has been compiled from notes made at various periods of the last fourteen years, and chiefly during the engagements of teaching. Many of the abbreviated methods and mnemonic rules are in the form in which I originally wrote them for my pupils.

The general object of the compilation is, as the title indicates, to present within a moderate compass the fundamental theorems, formulæ, and processes in the chief branches of pure and applied mathematics.

The work is intended, in the first place, to follow and supplement the use of the ordinary text-books, and it is arranged with the view of assisting the student in the task of revision of book-work. To this end I have, in many cases, merely indicated the salient points of a demonstration, or merely referred to the theorems by which the proposition is proved. I am convinced that it is more beneficial to the student to recall demonstrations with such aids, than to read and re-read them. Let them be read once, but recalled often. The difference in the effect upon the mind between reading a mathematical demonstration, and originating one wholly or partly, is very great. It may be compared to the difference between the pleasure experienced, and interest aroused, when in the one case a traveller is passively conducted through the roads of a novel and unexplored country, and in the other case he discovers the roads for himself with the assistance of a map.

In the second place, I venture to hope that the work, when completed, may prove useful to advanced students as an *aide-mémoire* and book of reference. The boundary of mathematical science forms, year by year, an ever widening circle, and the advantage of having at hand some condensed statement of results becomes more and more evident.

To the original investigator occupied with abstruse researches in some one of the many branches of mathematics, a work which gathers together synoptically the leading propositions in all, may not therefore prove unacceptable. Abler hands than mine undoubtedly, might have undertaken the task of making such a digest; but abler hands might also, perhaps, be more usefully employed,—and with this reflection I have the less hesitation in commencing the work myself. The design which I have indicated is somewhat comprehensive, and in relation to it the present essay may be regarded as tentative. The degree of success which it may meet with, and the suggestions or criticisms which it may call forth, will doubtless have their effect on the subsequent portions of the work.

With respect to the abridgment of the demonstrations, I may remark, that while some diffuseness of explanation is not only allowable but very desirable in an initiatory treatise, conciseness is one of the chief requirements in a work intended for the purposes of revision and reference only In order, however, not to sacrifice clearness to conciseness, much more labour has been expended upon this part of the subject-matter of the book than will at first sight be at all evident. The only palpable result being a compression of the text, the result is so far a negative one. The amount of compression attained is illustrated in the last section of the present part, in which more than the number of propositions usually given in treatises on Geometrical Conics are contained, together with the figures and demonstrations, in the space of twenty-four pages.

The foregoing remarks have a general application to the work as a whole. With the view, however, of making the earlier sections more acceptable to beginners, it will be found that, in those sections, important principles have sometimes been more fully elucidated and more illustrated by examples, than the plan of the work would admit of in subsequent divisions.

A feature to which attention may be directed is the uniform system of reference adopted throughout all the sections. With the object of facilitating such reference, the articles have been numbered progressively from the commencement in large Clarendon figures; the breaks which will occasionally be found in these numbers having been purposely made, in order to leave room for the insertion of additional matter, if it should be required in a future edition, without disturbing the original numbers and references. With the same object, demonstrations and examples have been made subordinate to enunciations and formulæ, the former being printed in small, the latter in bold type. By these aids, the interdependence of propositions is more readily shown, and it becomes easy to trace the connexion between theorems in different branches of mathematics, without the loss of time which would be incurred in turning to separate treatises on the subjects. The advantage thus gained will, however, become more apparent as the work proceeds.

The Algebra section was printed some years ago, and does not quite correspond with the succeeding ones in some of the particulars named above. Under the pressure of other occupations, this section moreover was not properly revised before going to press. On that account the table of errata will be found to apply almost exclusively to errors in that section; but I trust that the list is exhaustive. Great pains have been taken to secure the accuracy of the rest of the volume. Any intimation of errors will be gladly received.

I have now to acknowledge some of the sources from which the present part has been compiled. In the Algebra, Theory of Equations, and Trigonometry sections, I am largely indebted to Todhunter's well-known treatises, the accuracy and completeness of which it would be superfluous in me to dwell upon.

In the section entitled Elementary Geometry, I have added to simpler propositions a selection of theorems from Townsend's Modern Geometry and Salmon's Conic Sections.

In Geometrical Conics, the line of demonstration followed agrees, in the main, with that adopted in Drew's treatise on the subject. I am inclined to think that the method of that author cannot be much improved. It is true that some important properties of the ellipse, which are arrived at in

Drew's Conic Sections through certain intermediate propositions, can be deduced at once from the circle by the method of orthogonal projection. But the intermediate propositions cannot on that account be dispensed with, for they are of value in themselves. Moreover, the method of projection applied to the hyperbola is not so successful; because a property which has first to be proved true in the case of the equilateral hyperbola, might as will be proved at once for the general case. I have introduced the method of projection but sparingly, always giving preference to a demonstration which admits of being applied in the same identical form to the ellipse and to the hyperbola. The remarkable analogy subsisting between the two curves is thus kept prominently before the reader.

The account of the C. G. S. system of units given in the preliminary section, has been compiled from a valuable contribution on the subject by Professor Everett, of Belfast, published by the Physical Society of London.* This abstract, and the tables of physical constants, might perhaps have found a more appropriate place in an after part of the work. I have, however, introduced them at the commencement, from a sense of the great importance of the reform in the selection of units of measurement which is embodied in the C. G. S. system, and from a belief that the student cannot be too early familiarized with the same.

The Factor Table which follows is, to its limited extent, a reprint of Burckhardt's "*Tables des diviseurs*," published in

---

* "Illustrations of the Centimetre-Gramme-Second System of Units." London: Taylor and Francis. 1875.

1814–17, which give the least divisors of all numbers from 1 to 3,036,000. In a certain sense, it may be said that this is the only sort of purely mathematical table which is absolutely indispensable, because the information which it gives cannot be supplied by any process of direct calculation. The logarithm of a number, for instance, may be computed by a formula. Not so its prime factors. These can only be arrived at through the tentative process of successive divisions by the prime numbers, an operation of a most deterrent kind when the subject of it is a high integer.

A table similar to and in continuation of Burckhardt's has recently been constructed for the fourth million by J. W. L. Glaisher, F.R.S., who I believe is also now engaged in completing the fifth and sixth millions. The factors for the seventh, eighth, and ninth millions were calculated previously by Dase and Rosenberg, and published in 1862–65, and the tenth million is said to exist in manuscript. The history of the formation of these tables is both instructive and interesting.*

As, however, such tables are necessarily expensive to purchase, and not very accessible in any other way to the majority of persons, it seemed to me that a small portion of them would form a useful accompaniment to the present volume. I have, accordingly, introduced the first eleven pages of Burckhardt's tables, which give the least factors of the first 100,000 integers nearly. Each double page of the table here printed is

* See "*Factor Table for the Fourth Million.*" By James Glaisher, F.R.S. London: Taylor and Francis. 1880. Also *Camb. Phil. Soc. Proc.*, Vol. III., Pt. IV., and *Nature*, No. 542, p. 462.

an exact reproduction, in all but the type, of a single quarto page of Burckhardt's great work.

It may be noticed here that Prof. Lebesque constructed a table to about this extent, on the plan of omitting the multiples of seven, and thus reducing the size of the table by about one-sixth.* But a small calculation is required in using the table which counterbalances the advantage so gained.

The values of the Gamma-Function, pages 30 and 31, have been taken from Legendre's table in his "*Exercices de Calcul Intégral*," Tome I. The table belongs to Part II. of this Volume, but it is placed here for the convenience of having all the numerical tables of Volume I. in the same section.

In addition to the authors already named, the following treatises have been consulted—Algebras, by Wood, Bourdon, and Lefebure de Fouroy; Snowball's Trigonometry; Salmon's Higher Algebra; the Geometrical Exercises in Potts's Euclid; and Geometrical Conics by Taylor, Jackson, and Renshaw.

Articles 260, 431, 569, and very nearly all the examples, are original. The latter have been framed with great care, in order that they might illustrate the propositions as completely as possible.

G. S. C.

HADLEY, MIDDLESEX;
May 23, 1880.

* "Tables diverses pour la décomposition des nombres en leurs facteurs premiers." Par V. A. Lebesque. Paris. 1864.

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George Shoobridge Carr
Frontmatter
More information

ERRATA.

Art. 13,		*for*	$-a^2b^2$	*read*	$+a^2b^2$.
,, 56,	Line 1,	,,	3	,,	$\frac{3}{2}$.
,, 66,	,, 5,	,,	x	,,	x^2.
,, 90,	,, 4,	,,	numerators 1, 1, 1	,,	$1, a, a^2$.
,, 99,	,, 1,	,,	denominator $r-1$	,,	$n-1$.
,, 107,	,, 1,	,,	taken	,,	taken m at a time.
,, 108,	,, 2,	,,	(196)	,,	(360).
,, 131,	,, 1, 2,	,,	5	,,	6.
,, ,,	,, 5,	,,	$(-1)^5$	,,	$(-1)^{23}$.
,, 133,	,, 3, 6, 7,	,,	$6x$	,,	$3x$.
,, ,,	,, 8,	,,	4	,,	34.
,, ,,	,, 9,	,,	204, 459	,,	102, 306.
,, ,,	,, 10,	,,	459	,,	$9n$.
,, 138,	,, 4,	,,	$\frac{10.9.8}{1.2.3}$	,,	7.8.9.10.
,, 140,		,,	$(q+1)^k$	,,	$(q+1)^{(k)}$ Notation of (96).
,, 182,	,, 5,	,,	u_{n-1} in numerator	,,	u^2_{n-1}.
,, 191,	,, 4,	,,	(163)	,,	(164).
,, 220,	,, 6,	,,	$(x+y+z)^2$	,,	$2(x+y+z)^2$.
,, 221,	,, 4,	,,	(1)	,,	square of (1).
,, 237,	,, 11,	,,	$x^2=1$	,,	$x^2=-1$.
,, 238,	,, 5,	,,	(x^2-4x+8) on left side	,,	$(x^2-4x+8)^3$.
,, 239,	,, 11,		(234)		*Dele.*
,, 248,	,, 4,	,,	(29)	,,	(28).
,, 267,	,, 4,	,,	(267)	,,	(266).
,, 274,	,, 8,	,,	$\lfloor 11$	,,	$2\lfloor 11$.
,, 276,	,, 13,	,,	$p+2$	,,	$p+1$.
,, ,,	,, 14,	,,	$(p-1)$	,,	$\lfloor p-1$.
,, 283,	,, 3,	,,	$x=1$	,,	$a=b$.
,, 288,	,, 7,	,,	$n-1$	,,	$n+1$.
,, 289,	,, 4,	,,	$H(r, n-1)$	,,	$H(n, r-1)$.
,, 290,	,, 2,	,,	$H(r+1, n-1)$	,,	$H(n, r)$.
,, 325,	,, 17,	,,	P_2	,,	P.
,, ,,		,,	$P_1P_2P_3$, last line but one	,,	$Q_1Q_2Q_3$.
,, 333,	,, 3,	,,	$\left(\frac{a+b}{2}\right)$	,,	$\left(\frac{a+b}{2}\right)^m$.
,, 361,	,, 7,	,,	3528	,,	10284.
,, 481,	,, 6,	,,	$n-3$	,,	$n-1$.
,, 514,	,, 4,		applying Descartes' rule		*Dele.*
,, 517,	,, 3,	,,	a^3	,,	x^3.
,, 544,	,, 1,				Transpose F and f.
,, 551,	,, 1,	,,	B_1	,,	B.
,, ,,	,, 9,	,,	$\alpha-n$	,,	$\alpha-\kappa$.
,, 704,		,,	(11, 12)	,,	(9, 10, 1).
,, 729,		,,	(940)	,,	(960).

Article 112 should be as follows:—

$$\frac{1}{1+2\sqrt{3}-\sqrt{2}} = \frac{1+2\sqrt{3}+\sqrt{2}}{(1+2\sqrt{3})^2-2} = \frac{1+2\sqrt{3}+\sqrt{2}}{11+4\sqrt{3}} = \frac{(1+2\sqrt{3}+\sqrt{2})(11-4\sqrt{3})}{73}.$$

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George Shoobridge Carr
Frontmatter
More information

ERRATA CONTINUED FROM PAGE x.

(*Corrections which are important are marked with an asterisk.*)

	Page	1,	Line 7,	*for* volume	*read* weight.
			,, 10,	,, gramme-million	,, gramme-six.
*	,,	6,	,, 5,	,, 1·4971499	,, ·4971499.
*			,, 6,	,, ·6679358	,, 1·1447299.
*	Art.	123,	,, 2,	,, $2\sqrt{5}$	,, $2\sqrt{15}$.
*	,,	259,	,, 2,	,, $\alpha^2+\beta^2$	,, $(\alpha^2+\beta^2)^n$.
*	,,	276,	,, 6,	,, $3n^2+n-1$	,, $3n^2+3n-1$.
	,,	291,	,, 1,	,, δ	,, γ.
	,,	292,	,, 3 & 4,	,, a	,, α.
*	,,	322,	,, 9,	,, 45 and 13	,, 35 and 10.
*	,,	361,	,, 7,	,, 3528	,, 8584.
*	,,	459,	,, 3 & 9,	,, $-$	,, -16.
	,,	470,	,, 1,	,, x	,, x .
*	,,	489,	,, 9,		,, $-\frac{4q}{3}$.
*	,,	555,	,, 14,	,, a number of rows	,, two columns.
	,,	593,	,, 11 & 12,	,, R and R	,, R_1 and R_2.
	,,	604,	,, 2,	,, one-sixtieth	,, one-ninetieth.
	,,	713,	,, 2,	,, II.	,, III.
*	,,	897,	,, 5,	,, $\cos\frac{1}{2}c$	,, $\sin\frac{1}{2}c$.
	,,	922 ii.,		,, b^2+2c^2	,, $2b^2+c^2$.
	,,	949,	,, last,	,, D	,, C.
*	,,	1076,	,, 2,	*dele* "The projections . . . are parallel."	
	,,	1158,	,, last,	,, 1201	,, 1217.
	,,	1178,	,, 4,	,, PS	,, PS'.
	,,	1241,	,, 1,	,, parallel	,, conjugate.
	,,	1413,	,, 3,	,, $-du\,dv$	,, $+du\,dv$.
	,,	1491,	,, 3,	,, $-$	,, $=$
	,,	1849,	,, 1,	,, $+$	,, $-$
	,,	1903,	*footnote*,	,, $\int(x)$	,, $f(x)$.
	,,	1925,		*supply* dx.	
	,,	1954-6,		*supply* x.	
	,,	2030-2,		erroneous, because l in (1427) is necessarily an integer.	
	,,	2035,	,, 1,	,, ax	,, a.
*	,,	2140,	,, 1,	$\sqrt{}$ applies to the whole denominator.	
	,,	2136,	,, last,	,, 2294	,, 2293.
	,,	2354,		,, $-$	,, $+$
*	,,	2392,		,, x^p	,, $x\ -1$.
*	,,	2465,		,, $\frac{p}{2}$	,, $\frac{\pi}{2}$.
	,,	3237,	,, 1,	,, $(n-1)x$	,, $(n-1)$.
	,,	3751,		*supply* u_x.	
*	,,	4678,		*dele* 2 in the second term.	
*	,,	4680,		*supply* the factor 4 on the left.	
*	,,	4692,	,, 5,	*dele* 2 in the second term.	
*	,,	4903,	,, 3,	*supply* the factor 4 on the left.	
	,,	5154,	,, 4,	,, 3155	,, 5155.
	,,	5330,	,, 2,	,, m	,, b.

and refer to Fig. 129, Art. 5332 on the cardioid is wanting.

TABLE OF CONTENTS.

PART I.

SECTION I.—MATHEMATICAL TABLES.

SECTION II.—ALGEBRA.

Cambridge University Press
978-1-108-05067-8 - A Synopsis of Elementary Results in Pure and Applied Mathematics: Containing Propositions, Formulae, and Methods of Analysis, with Abridged Demonstrations: Volume 1
George Shoobridge Carr
Frontmatter
More information

SECTION III.—THEORY OF EQUATIONS.

SECTION IV.—PLANE TRIGONOMETRY.

SECTION V.—SPHERICAL TRIGONOMETRY.

SECTION VI.—ELEMENTARY GEOMETRY.

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George Shoobridge Carr
Frontmatter
More information

SECTION VII.—GEOMETRICAL CONICS.

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George Shoobridge Carr
Frontmatter
More information

INDEX TO PROPOSITIONS OF EUCLID

REFERRED TO IN THIS WORK.

The references to Euclid are made in Roman and Arabic numerals; *e.g.* (VI. 19).

BOOK I.

I. 4.—Triangles are equal and similar if two sides and the included angle of each are equal each to each.

I. 5.—The angles at the base of an isosceles triangle are equal.

I. 6.—The converse of 5.

I. 8.—Triangles are equal and similar if the three sides of each are equal each to each.

I. 16.—The exterior angle of a triangle is greater than the interior and opposite.

I. 20.—Two sides of a triangle are greater than the third.

I. 26.—Triangles are equal and similar if two angles and one corresponding side of each are equal each to each.

I. 27.—Two straight lines are parallel if they make equal alternate angles with a third line.

I. 29.—The converse of 27.

I. 32.—The exterior angle of a triangle is equal to the two interior and opposite; and the three angles of a triangle are equal to two right angles.

Cor. 1.—The interior angles of a polygon of n sides $= (n-2)\pi$.

Cor. 2.—The exterior angles $= 2\pi$.

I. 35 to 38.—Parallelograms or triangles upon the same or equal bases and between the same parallels are equal.

I. 43.—The complements of the parallelograms about the diameter of a parallelogram are equal.

I. 47.—The square on the hypotenuse of a right-angled triangle is equal to the squares on the other sides.

I. 48.—The converse of 47.

BOOK II.

II. 4.—If a, b are the two parts of a right line, $(a+b)^2 = a^2+2ab+b^2$.

If a right line be bisected, and also divided, internally or externally, into two unequal segments, then—

II. 5 and 6.—The rectangle of the unequal segments is equal to the difference of the squares on half the line, and on the line between the points of section; or $(a+b)(a-b) = a^2-b^2$.

II. 9 and 10.—The squares on the same unequal segments are together double the squares on the other parts; or

$$(a+b)^2+(a-b)^2 = 2a^2+2b^2.$$

II. 11.—To divide a right line into two parts so that the rectangle of the whole line and one part may be equal to the square on the other part.

II. 12 and 13.—The square on the base of a triangle is equal to the sum of the squares on the two sides *plus* or *minus* (as the vertical angle is *obtuse* or *acute*), twice the rectangle under either of those sides, and the projection of the other upon it; or $a^2 = b^2+c^2-2bc\cos A$ (702).

BOOK III.

III. 3.—If a diameter of a circle bisects a chord, it is perpendicular to it: and conversely.

III. 20.—The angle at the centre of a circle is twice the angle at the circumference on the same arc.

III. 21.—Angles in the same segment of a circle are equal.

III. 22.—The opposite angles of a quadrilateral inscribed in a circle are together equal to two right angles.

III. 31.—The angle in a semicircle is a right angle.

III. 32.—The angle between a tangent and a chord from the point of contact is equal to the angle in the alternate segment.

III. 33 and 34.—To *describe* or to *cut off* a segment of a circle which shall contain a given angle.

III. 35 and 36.—The rectangle of the segments of any chord of a circle drawn through an *internal* or *external* point is equal to the square of the semi-chord perpendicular to the diameter through the internal point, or to the square of the tangent from the external point.

III. 37.—The converse of 36. If the rectangle be equal to the square, the line which meets the circle touches it.

BOOK IV.

IV. 2.—To inscribe a triangle of given form in a circle.
IV. 3.—To describe the same about a circle.
IV. 4.—To inscribe a circle in a triangle.
IV. 5.—To describe a circle about a triangle.
IV. 10.—To construct two-fifths of a right angle.
IV. 11.—To construct a regular pentagon.

BOOK VI.

VI. 1.—Triangles and parallelograms of the same altitude are proportional to their bases.
VI. 2.—A right line parallel to the side of a triangle cuts the other sides proportionally; and conversely.
VI. 3 and A.—The bisector of the *interior* or *exterior* vertical angle of a triangle divides the base into segments proportional to the sides.
VI. 4.—Equiangular triangles have their sides proportional homologously.
VI. 5.—The converse of 4.
VI. 6.—Two triangles are equiangular if they have two angles equal, and the sides about them proportional.
VI. 7.—Two triangles are equiangular if they have two angles equal and the sides about two *other* angles proportional, provided that the third angles are both greater than, both less than, or both equal to a right angle.
VI. 8.—A right-angled triangle is divided by the perpendicular from the right angle upon the hypotenuse into triangles similar to itself.
VI. 14 and 15.—Equal *parallelograms*, or *triangles* which have two angles equal, have the sides about those angles reciprocally proportional; and conversely, if the sides are in this proportion, the figures are equal.
VI. 19.—Similar triangles are in the duplicate ratio of their homologous sides.
VI. 20.—Likewise similar polygons.
VI. 23.—Equiangular parallelograms are in the ratio compounded of the ratios of their sides.
VI. B.—The rectangle of the sides of a triangle is equal to the square of the bisector of the vertical angle *plus* the rectangle of the segments of the base.

BOOK XI.