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Edward John Routh

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- Chap. 1. Theory of moments of inertia and the ellipsoids of inertia.
- Chap. 2. D'Alembert's Principle and other fundamental theorems.
- Chap. 3. Theory of motion about a fixed axis with applications to the pendulum, the numerical value of  $g$ , the watch balance, the ballistic pendulum, the anemometer.
- Chap. 4. General principles of motion in two dimensions. Special consideration of stress, friction, impulses and relative motion.
- Chap. 5. Geometry of motion in three dimensions, with Euler's equations.
- Chap. 6. On Momentum, with the discussion of sudden changes of motion.
- Chap. 7. On Vis Viva and Work, with some general theorems by Carnot, Bertrand, Thomson and Gauss.
- Chap. 8. Lagrange's equations. Theory of reciprocation, the Hamiltonian transformation and the Modified function.
- Chap. 9. Small oscillations. Several methods described. Lagrange's method, the energy test of stability and the Cavendish experiment.
- Chap. 10. Some special problems. Oscillations of rolling bodies, and Lagrange's rule with regard to large tautochronous motions.