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The Advanced Part of a Treatise on the Dynamics of a System of Rigid Bodies

As senior wrangler in 1854, Edward John Routh (1831–1907) was the man who beat James Clerk Maxwell in the Cambridge mathematics tripos. He went on to become a highly successful coach in mathematics at Cambridge, producing a total of twenty-seven senior wranglers during his career – an unrivalled achievement. In addition to his considerable teaching commitments, Routh was also a very able and productive researcher who contributed to the foundations of control theory and to the modern treatment of mechanics. First published in one volume in 1860, this textbook helped disseminate Routh's investigations into stability. This revised fifth edition was published in two volumes between 1891 and 1892. The second part develops the extensive coverage of dynamics, providing formulae and examples throughout. While the growth of modern physics and mathematics may have forced out the problem-based mechanics of Routh's textbooks from the undergraduate syllabus, the utility and importance of his work is undiminished.

The Advanced Part of a Treatise on the Dynamics of a System of Rigid Bodies

EDWARD JOHN ROUTH

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THE ADVANCED PART

OF A TREATISE ON THE

DYNAMICS OF A SYSTEM OF RIGID BODIES

BEING PART II. OF A TREATISE ON THE WHOLE SUBJECT.

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DYNAMICS OF A SYSTEM OF RIGID BODIES.

BEING PART II. OF A TREATISE ON THE WHOLE SUBJECT.

With numerous Examples.

BY

EDWARD JOHN ROUTH, Sc.D., LL.D., F.R.S., &c.

HON. FELLOW OF PETERHOUSE, CAMBRIDGE;
FELLOW OF THE SENATE OF THE UNIVERSITY OF LONDON.

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PREFACE.

In this edition many additions and improvements have been made, particularly in the last half of the book. Many parts have been re-written in the hope of making the explanations clearer and briefer. A few sections have been omitted to make room for more important matter. New subjects, not discussed in the former editions, have been introduced in order to make the treatise as complete as possible. Though more than a year has elapsed since the publication of the first volume of this edition, I have not found the time at my disposal during the interval too long for these changes.

Following the same plan as in Vol. I., the several chapters have been made as independent as possible. The object in view is that the reader should be able to select his own order of study. Historical notices and references have been given throughout the book. I have endeavoured to join to every theorem or problem the name of the writer who, as far as I know, was the first to enunciate or solve it.

Numerous examples have been given throughout the book. Some of these are intended to be merely simple exercises, but many are important as illustrating and completing the theories given in the text. Sometimes when the principles of a theory

had been explained numerous applications seemed to arise. Instead of loading the text with these it appeared preferable to put them into the form of examples and to give such hints as would make the solution easy. Everywhere the results have been given, and care has been taken to secure their accuracy; but amongst so many theorems, it cannot be expected that no errors have escaped detection.

I wish to express my thanks to Mr J. M. Dodds of Peterhouse for his kind assistance in correcting so many of the proof sheets.

EDWARD J. ROUTH.

PETERHOUSE,
February, 1892.

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In order that the plan of the book may be understood the following short summary is given of the subjects treated of in Part I.

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