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978-1-108-05031-9 - The Elementary Part of a Treatise on the Dynamics of a System of Rigid Bodies

Edward John Routh

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The Elementary Part of a Treatise on the Dynamics of a System of Rigid Bodies

As senior wrangler in 1854, Edward John Routh (1831–1907) was the man who beat James Clerk Maxwell in the Cambridge mathematics tripos. He went on to become a highly successful coach in mathematics at Cambridge, producing a total of twenty-seven senior wranglers during his career – an unrivalled achievement. In addition to his considerable teaching commitments, Routh was also a very able and productive researcher who contributed to the foundations of control theory and to the modern treatment of mechanics. First published in one volume in 1860, this textbook helped disseminate Routh's investigations into stability. This revised fifth edition was published in two volumes between 1891 and 1892. The first part establishes the principles of dynamics, providing formulae and examples throughout. While the growth of modern physics and mathematics may have forced out the problem-based mechanics of Routh's textbooks from the undergraduate syllabus, the utility and importance of his work is undiminished.

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CAMBRIDGE UNIVERSITY PRESS

Cambridge, New York, Melbourne, Madrid, Cape Town,
Singapore, São Paulo, Delhi, Mexico City

Published in the United States of America by Cambridge University Press, New York

www.cambridge.org

Information on this title: www.cambridge.org/9781108050319

© in this compilation Cambridge University Press 2013

This edition first published 1891

This digitally printed version 2013

ISBN 978-1-108-05031-9 Paperback

This book reproduces the text of the original edition. The content and language reflect the beliefs, practices and terminology of their time, and have not been updated.

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SUBJECT.

With numerous Examples.

BY

EDWARD JOHN ROUTH, Sc.D., LL.D., F.R.S., &c.

HON. FELLOW OF PETERHOUSE, CAMBRIDGE ;

FELLOW OF THE SENATE OF THE UNIVERSITY OF LONDON.

FIFTH EDITION, REVISED AND ENLARGED.

London:
MACMILLAN AND CO.
AND NEW YORK.

1891

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Cambridge University Press

978-1-108-05031-9 - The Elementary Part of a Treatise on the Dynamics of a System of Rigid Bodies

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First Edition, 1860. Second Edition, 1868.
Third Edition, 1877. Fourth Edition, 1882. Fifth Edition, 1891.

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Frontmatter

[More information](#)

PREFACE.

IN this edition many improvements have been made. Though some new matter has been added, most of the changes are in the form of additional explanations and generalizations of theorems already given. In some cases also the proofs have been simplified. The numbering of the articles is the same as in the last edition, except in some cases where the additions were such as to require a rearrangement.

When the last edition was printed, the book was divided into two parts in order to render it less bulky. This division has been retained. All the elements of the subject, together with some methods intended for the more advanced student, are placed in the first volume. In the second part the higher applications are given. In order that the plan of the book may be understood, a short summary of the subjects treated of in the second volume has been added to the table of contents.

As in the former editions, each chapter has been made as far as possible complete in itself, so that all that relates to any one part of the subject may be found in the same place. This arrangement is convenient for those who are already acquainted with dynamics, as it enables them to direct their attention to those parts in which they may feel most interested. It also enables the student to select his own order of reading. The student who is just beginning dynamics may not wish to be delayed by a long chapter of preliminary analysis before he enters on the real subject of the book. He may therefore begin with D'Alembert's Principle and read only those parts of chapter I. to which reference is made. Others may wish to pass on as

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[More information](#)

soon as possible to the great principles of Angular Momentum and Vis Viva. Though a different order may be found advisable for some readers, I have ventured to indicate a list of Articles to which those who are beginning dynamics should first turn their attention.

It will be observed that a chapter has been devoted to the discussion of Motion in Two Dimensions. This course has been adopted because it seemed expedient to separate the difficulties of dynamics from those of solid geometry.

A slight historical notice of each result has been attempted whenever it could be briefly given. Such additions, if not carried too far, add greatly to the interest of the subject. But the success of the attempt is far from complete. In the earlier history there was the guidance of Montucla, and further on there was Prof. Cayley's Report to the British Association. With the help of these the task became comparatively easy; but in some other portions the number of memoirs which have been written is so vast that anything but a slight notice is impossible. A useful theorem is many times discovered, and probably each time with some variations. It is thus often difficult to ascertain who is the first author. It has therefore been found necessary to correct some of the references given in the former editions, and to add references where there were none before. It has not however been thought necessary to refer to the author's own additions to the subject except when they had already been printed elsewhere.

Throughout each chapter there will be found numerous examples, many being very easy, while others are intended for the more advanced student. In order to obtain as great a variety of problems as possible, a further collection has been added at the end of each chapter, taken from the Examination Papers which have been set in the University and in the Colleges. As these problems have been constructed by many different examiners, I hope that this selection will enable the student to acquire facility in solving all kinds of dynamical problems.

Cambridge University Press

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Frontmatter

[More information](#)

PREFACE.

vii

In constructing the examples my first care has been to follow closely the principle which each is intended to illustrate. But such instruments or applications of principles have been sought for as have been found useful in practice. Whenever some useful instrument has been found, which did not require so lengthy a description as to unfit it for an illustration, it has been preferred as an example to a merely curious and artificial construction.

In the former editions differential coefficients with regard to the time have been represented by accents in the chapters after the seventh. However unsuitable such a notation may be when several independent variables are used in the same investigation, it has some advantages in such a subject as dynamics, where the differentiations are nearly always taken with regard to the time. It was not used in the earlier chapters because it was thought that it would add to the initial difficulties of the subject those of an unaccustomed notation. But now that the representation of differential coefficients by dots has been used in several standard books both on elementary and on advanced mechanics, this reason has lost much of its force. Dots and accents have therefore been used throughout this edition whenever a shortened notation has appeared to be desirable. One objection to the use of this notation is that the meaning of the symbol may be changed by a slight error in the number of the dots or accents. As this might increase the difficulties of the subject to a beginner, the use of dots in the earlier chapters has been restricted chiefly to the working of examples, and care has been taken that the results should be clearly stated.

I cannot conclude without expressing how much I am indebted to Mr J. M. Dodds of Peterhouse for his assistance in correcting the proof sheets. I hope that the work, having had the advantage of his revision, will be found clear of serious errors.

EDWARD J. ROUTH.

PETERHOUSE,

December 8, 1890.

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Edward John Routh

Frontmatter

[More information](#)

CONTENTS.

CHAPTER I.

ON MOMENTS OF INERTIA.

ARTS.		PAGES
1— 2.	On finding Moments of Inertia by integration	1
3— 9.	Definitions, elementary propositions and reference table	2—9
10—11.	Method of Differentiation	9—10
12—14.	Theorem of Parallel Axes	10—13
15—17.	Theorem of the Six Constants of a Body	13—16
18.	Method of Transformation of Axes	16—17
19—32.	Ellipsoids of Inertia, Invariants, &c.	17—23
33—39.	Equipomental Bodies, Triangle, Tetrahedron, &c.	24—28
40—44.	Theory of Projections	29—31
45.	Moments with higher powers	31—32
46.	Theory of Inversion	32—33
47.	Centre of Pressure, &c.	33—35
48—51.	Principal Axes	35—37
52—55.	Foci of Inertia	38—40
56—59.	Arrangement of Principal Axes	40—43
60—61.	Condition that a Line should be a Principal Axis	43—45
62—65.	Locus of Equal Moments, Equipomental Surface, &c.	45—49

CHAPTER II.

D'ALEMBERT'S PRINCIPLE, &C.

66—78.	D'Alembert's Principle and the Equations of Motion	50—61
79—82.	Independence of Translation and Rotation	61—63
83.	General method of using D'Alembert's Principle	63—65
84—87.	Impulsive Forces	65—69

CHAPTER III.

MOTION ABOUT A FIXED AXIS.

ARTS.		PAGES
88—91.	The Fundamental Theorem	70—72
92—93.	The Pendulum and the Centre of Oscillation	72—76
94—96.	Effects of a change of temperature and of the buoyancy of the air	76—79
97.	Moments of Inertia found by experiment	79—80
98—105.	Length of the seconds pendulum with correction for resistance of the air	80—84
106—107.	Construction of a Pendulum	84—85
108.	The Pendulum as a Standard of Length	86—87
109.	Oscillation of a watch balance	87—89
110—113.	Pressures on the fixed Axis. Bodies symmetrical and not symmetrical	90—95
114.	Analysis of results	95—97
115—116.	Dynamical and Geometrical Similarity	97—98
117—119.	Permanent Axes of Rotation, Initial Axes	98—100
120.	The Centre of Percussion	100—102
121—125.	The Ballistic Pendulum	102—107
126—129.	The Anemometer	107—109

CHAPTER IV.

MOTION IN TWO DIMENSIONS.

130—133.	General methods of forming the Equations of Motion	110—113
134.	Angular Momentum	113—114
135—137.	Method of Solution by Differentiation	114—117
138—143.	Vis Viva, Force Function and Work	117—122
144—148.	Examples of Solution	122—131
149.	Characteristics of a Body	131—133
150—152.	Stress at any point of a rod	133—137
153—157.	Laws of Friction	137—139
158—160.	Discontinuity of Friction, and Indeterminate Motion	140—141
161—163.	A Sphere on an imperfectly rough plane	141—143
164.	Friction Couples	143
165—166.	Friction of a carriage and other examples	144—146
167.	Rigidity of cords	146—147
168—169.	Impulsive Forces, General Principles	147—148
170—175.	Examples of sudden changes of motion, reel, sphere, disc, column, &c. Earthquakes	148—152
176—178.	Impact of Compound Inelastic Bodies, &c.	152—156
179—180.	Impact of Smooth Elastic Bodies	156—158
181—198.	The general problem of the Impact of two Bodies, smooth or rough, elastic or inelastic. The representative point	158—173

CONTENTS.

xi

ARTS.		PAGES
199—202.	Initial Motions	173—178
203—213.	Relative Motion and Moving Axes	178—186
	Examples	187—191

CHAPTER V.

MOTION IN THREE DIMENSIONS.

214—228.	Translation and Rotation. Base Point, Central Axis	192—198
229—234.	Composition of Rotations, &c.	198—202
235—237.	Analogy to Statics	202—204
238—239.	The Velocity of any point	204—206
240—247.	Composition of Screws, &c.	206—212
248—259.	Euler's Equations	212—219
260.	The Centrifugal Forces of a Body	219—221
261—267.	Angular Momentum with Fixed or Moving Axes	221—227
268—270.	Examples of Top and Sphere	228—232
271—281.	Finite Rotations. Theorems of Rodrigues and Sylvester. Screws, &c.	233—238

CHAPTER VI.

ON MOMENTUM.

282—287.	The Fundamental Theorem, with examples	239—244
288—298.	Sudden fixtures and changes	244—250
299—300.	Gradual changes	251—254
301—305.	The Invariable Plane	254—259
306—314.	Impulsive forces in three dimensions	259—263
315—331.	The general problem of the Impact of two Bodies in three dimensions, the bodies being smooth or rough, elastic or inelastic. The representative point	264—274
	Examples	274—276

CHAPTER VII.

VIS VIVA.

332—341.	Force, Function and Work	277—282
342.	Work done by Gravity and Units of Work	282
343.	Work of an Elastic String	282—283
344.	Work of Collecting a Body	283—286
345.	Work of a Gaseous Pressure	286
346.	Work of an Impulse	286—288
347.	Work of a Membrane	288

ARTS.	PAGES
348—349. Work of Bending a Rod	288—289
350—362. Principle of Vis Viva, Potential and Kinetic Energy	289—295
363—364. Expressions for Vis Viva of a Body	295—298
365—366. Examples on Vis Viva	298—300
367—372. Principle of Similitude. Models	300—304
373—374. Theory of Dimensions	304
375—376. Clausius' theory of stationary motion. The Virial	304—306
377—381. Carnot's theorems	307—308
382—386. The equation of Virtual Work applied to Impulses	308—310
387—388. Thomson's theorem. Bertrand's theorem	310—311
389. Imperfectly elastic and rough bodies	311—313
390—394. Gauss' principle of Least Constraint	314—317
Examples	317—320

CHAPTER VIII.

LAGRANGE'S EQUATIONS.

395—399. Typical Equation for Finite Forces	321—325
400. Indeterminate Multipliers	325—326
401—404. Lagrange's equations for Impulsive Forces	326—328
405—408. Examples on Lagrange's equations	328—331
409—413. The Reciprocal Function	331—334
414—417. Hamilton's equations	334—337
418—421. The modified Lagrangian Function. Its use in forming Lagrange's and Hamilton's equations	337—339
422—425. Co-ordinates which appear only as velocities	339—342
426—428. Non-conservative Forces	342—344
429—431. Geometrical equations which contain differential coefficients with regard to the time	344—347
Examples	347—348

CHAPTER IX.

SMALL OSCILLATIONS.

432—438. Oscillations with one degree of freedom	349—353
439—440. Moments about the Instantaneous Axis	353—354
441—444. Oscillations of Cylinders, with the use of the circle of stability 445. Oscillations of a body guided by two curves	355—358 358—359
446. Oscillation when the path of the Centre of Gravity is known	359—360
447. Oscillations deduced from Vis Viva	360—361
448. Moments about the Central Axis	361—362
449—452. Oscillations deduced from the ordinary equations of motion	362—366
453—462. Lagrange's method	366—377
463—466. Initial motions	377—379
467—469. The energy test of stability	379—381
470—476. The Cavendish Experiment	382—388
Examples	388—390

Cambridge University Press

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Frontmatter

[More information](#)

CONTENTS.

xiii

CHAPTER X.

ON SOME SPECIAL PROBLEMS.

ARTS.		PAGES
477.	Oscillations of a rocking body in three dimensions . . .	391
478—479.	Relative indicatrix	391—392
480—482.	Cylinder of stability and the time of oscillation	392—394
483—487.	Oscillations of rough cones rolling on each other to the first order of small quantities	394—398
488—492.	Lagrange's formula for large Tautochronous motions	398—401
493—494.	Large oscillations of a particle on a rough cycloid in a resisting medium	401—403
495—498.	Large Tautochronous motions on any rough curve, with applications to epicycloid, &c.	403—406
499.	Effect of a resisting medium on the time of oscillation	406
500—507.	Conditions of stability and times of oscillations of rough cylinders to any order of small quantities	406—410
503—510.	Conditions of stability and times of oscillation of rough cones to any order	410—412

The following subjects will be treated of in the second volume.

Theory of moving axes, Clairaut's theorem, and motion relative to the earth.

Theory of small oscillations with several degrees of freedom both about a position of equilibrium and about a state of steady motion.

Motion of a body about a fixed point under no forces.

Motion of a body under any forces.

Theory of free and forced oscillations.

Methods of Isolation and of Multipliers.

Applications of the calculus of finite differences.

Applications of the calculus of variations.

Precession and Nutation.

Motion of a string or chain.

Motion of a membrane.

The student, to whom the subject is entirely new, is advised to read *first* the following articles: Chap. I. 1—25, 33—36, 47—52. Chap. II. 66—87. Chap. III. 88—93, 98—104, 110, 112—118. Chap. IV. 130—164, 168—175, 179—186, 199. Chap. V. 214—245, 248—256, 261—269. Chap. VI. 282—285, 288—295, 299—309. Chap. VII. 332—374. Chap. VIII. 395—409. Chap. IX. 432—463, 467—476. Chap. X. 483, 488—499.

ERRATUM.

Page 190, line 25, for $\left(\frac{gt^2}{2} - a\theta\right)$ read $\left(\frac{gt^2}{2} - a\theta\right)^2$.