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The Elementary Part of a Treatise on the Dynamics of a System of Rigid Bodies

As senior wrangler in 1854, Edward John Routh (1831–1907) was the man who beat James Clerk Maxwell in the Cambridge mathematics tripos. He went on to become a highly successful coach in mathematics at Cambridge, producing a total of twenty-seven senior wranglers during his career – an unrivalled achievement. In addition to his considerable teaching commitments, Routh was also a very able and productive researcher who contributed to the foundations of control theory and to the modern treatment of mechanics. First published in one volume in 1860, this textbook helped disseminate Routh's investigations into stability. This revised fifth edition was published in two volumes between 1891 and 1892. The first part establishes the principles of dynamics, providing formulae and examples throughout. While the growth of modern physics and mathematics may have forced out the problem-based mechanics of Routh's textbooks from the undergraduate syllabus, the utility and importance of his work is undiminished.

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Edward John Routh

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OF A TREATISE ON THE

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BEING PART I. OF A TREATISE ON THE WHOLE SUBJECT.

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With numerous Examples.

BY

EDWARD JOHN ROUTH, Sc.D., LL.D., F.R.S., &c.

HON. FELLOW OF PETERHOUSE, CAMBRIDGE;
FELLOW OF THE SENATE OF THE UNIVERSITY OF LONDON.

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PREFACE.

In this edition many improvements have been made. Though some new matter has been added, most of the changes are in the form of additional explanations and generalizations of theorems already given. In some cases also the proofs have been simplified. The numbering of the articles is the same as in the last edition, except in some cases where the additions were such as to require a rearrangement.

When the last edition was printed, the book was divided into two parts in order to render it less bulky. This division has been retained. All the elements of the subject, together with some methods intended for the more advanced student, are placed in the first volume. In the second part the higher applications are given. In order that the plan of the book may be understood, a short summary of the subjects treated of in the second volume has been added to the table of contents.

As in the former editions, each chapter has been made as far as possible complete in itself, so that all that relates to any one part of the subject may be found in the same place. This arrangement is convenient for those who are already acquainted with dynamics, as it enables them to direct their attention to those parts in which they may feel most interested. It also enables the student to select his own order of reading. The student who is just beginning dynamics may not wish to be delayed by a long chapter of preliminary analysis before he enters on the real subject of the book. He may therefore begin with D'Alembert's Principle and read only those parts of chapter I. to which reference is made. Others may wish to pass on as

soon as possible to the great principles of Angular Momentum and Vis Viva. Though a different order may be found advisable for some readers, I have ventured to indicate a list of Articles to which those who are beginning dynamics should first turn their attention.

It will be observed that a chapter has been devoted to the discussion of Motion in Two Dimensions. This course has been adopted because it seemed expedient to separate the difficulties of dynamics from those of solid geometry.

A slight historical notice of each result has been attempted whenever it could be briefly given. Such additions, if not carried too far, add greatly to the interest of the subject. But the success of the attempt is far from complete. In the earlier history there was the guidance of Montucla, and further on there was Prof. Cayley's Report to the British Association. With the help of these the task became comparatively easy; but in some other portions the number of memoirs which have been written is so vast that anything but a slight notice is impossible. A useful theorem is many times discovered, and probably each time with some variations. It is thus often difficult to ascertain who is the first author. It has therefore been found necessary to correct some of the references given in the former editions, and to add references where there were none before. It has not however been thought necessary to refer to the author's own additions to the subject except when they had already been printed elsewhere.

Throughout each chapter there will be found numerous examples, many being very easy, while others are intended for the more advanced student. In order to obtain as great a variety of problems as possible, a further collection has been added at the end of each chapter, taken from the Examination Papers which have been set in the University and in the Colleges. As these problems have been constructed by many different examiners, I hope that this selection will enable the student to acquire facility in solving all kinds of dynamical problems.

In constructing the examples my first care has been to follow closely the principle which each is intended to illustrate. But such instruments or applications of principles have been sought for as have been found useful in practice. Whenever some useful instrument has been found, which did not require so lengthy a description as to unfit it for an illustration, it has been preferred as an example to a merely curious and artificial construction.

In the former editions differential coefficients with regard to the time have been represented by accents in the chapters after the seventh. However unsuitable such a notation may be when several independent variables are used in the same investigation, it has some advantages in such a subject as dynamics, where the differentiations are nearly always taken with regard to the time. It was not used in the earlier chapters because it was thought that it would add to the initial difficulties of the subject those of an unaccustomed notation. But now that the representation of differential coefficients by dots has been used in several standard books both on elementary and on advanced mechanics, this reason has lost much of its force. Dots and accents have therefore been used throughout this edition whenever a shortened notation has appeared to be desirable. One objection to the use of this notation is that the meaning of the symbol may be changed by a slight error in the number of the dots or accents. As this might increase the difficulties of the subject to a beginner, the use of dots in the earlier chapters has been restricted chiefly to the working of examples, and care has been taken that the results should be clearly stated.

I cannot conclude without expressing how much I am indebted to Mr J. M. Dodds of Peterhouse for his assistance in correcting the proof sheets. I hope that the work, having had the advantage of his revision, will be found clear of serious errors.

EDWARD J. ROUTH.

PETERHOUSE,
December 8, 1890.

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VIS VIVA.

CHAPTER VIII.

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The following subjects will be treated of in the second volume.

Theory of moving axes, Clairaut's theorem, and motion relative to the earth.

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The student, to whom the subject is entirely new, is advised to read *first* the following articles: Chap. I. 1—25, 33—36, 47—52. Chap. II. 66—87. Chap. III. 88—93, 98—104, 110, 112—118. Chap. IV. 130—164, 168—175, 179—186, 199. Chap. V. 214—245, 248—256, 261—269. Chap. VI. 282—285, 288—295, 299—309. Chap. VII. 332—374. Chap. VIII. 395—409. Chap. IX. 432—463, 467—476. Chap. X. 483, 488—499.

ERRATUM.

Page 190, line 25, *for* $\left(\frac{gt^2}{2} - a\theta\right)$ *read* $\left(\frac{gt^2}{2} - a\theta\right)^2$.