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Principles of Mechanism

Designed for the Use of Students in the Universities, and for Engineering Students Generally

ROBERT WILLIS

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PRINCIPLES

OF

MECHANISM,

DESIGNED FOR THE USE OF STUDENTS
IN THE UNIVERSITIES,

AND FOR

ENGINEERING STUDENTS GENERALLY.

BY

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PREFACE.

In the present work I have employed the term Mechanism as applying to combinations of machinery solely when considered as governing the relations of motion. Machinery as a modifier of force, has in the science of Mechanics occupied the attention of nearly every mathematician of eminence who has arisen in the world; but, by some strange chance, very few have attempted to give a scientific form to the attractive and valuable results of mechanism; for it cannot be said that the few and simple machines which form the examples in books of mechanics, are to be regarded as even forming a foundation for the principles upon which is to be based a science that will enable us either to reduce the movements and actions of a complex engine to system, or to give answers to the questions that naturally arise upon considering such engines;—for example, are the means by which the results are obtained the best that might have been employed? or what are the various methods that might have been substituted for them? Yet there appears no reason why the construction of a machine for a given purpose should not, like any usual problem, be so reduced to the dominion of the mathematician, as to enable him to obtain, by direct

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and certain methods, all the forms and arrangements that are applicable to the desired purpose, from which he may select at pleasure. At present, questions of this kind can only be solved by that species of intuition which long familiarity with a subject usually confers upon experienced persons, but which they are totally unable to communicate to others.

When the mind of a mechanician is occupied with the contrivance of a machine, he must wait until, in the midst of his meditations, some happy combination presents itself to his mind which may answer his purpose. Yet upon analysing the mental operations by which the nascent contrivance is gradually made to assume form and consistency, it will generally be observed, that the motions of the machine are the principal subject of contemplation, rather than the forces applied to it, or the work it has to do. For every machine will be found to consist of a train of pieces connected together in various ways, so that if one be made to move they all receive a motion, the relation of which to that of the first is governed by the nature of the connexion. The work which the machine has to do will require that the pieces appropriated to this work shall move with respect to each other in some given manner, and the forces applied to the machine to set it in motion must also move the piece which receives them in some other manner. Thus the question of contriving a machine by which a given kind of power may be made to perform given work, is reduced to a problem of mere motion,—to a question of connecting the pieces which receive the power and those which do the work; so that when the first move according to the law required by the economy of the power, the last shall necessarily receive the motion which will enable them to do the work. There are, of course, many essential considerations of force and arrangement which must be entered into before the machine can be completed, but they admit of being abstracted in the first instance; and it is only by so doing that we can hope to create a science of mechanism. Yet this view seems to have presented itself but lately, with due clearness, to the minds of writers on this subject; and it may be interesting to trace the history of its rise and progress.

Apart from the writings on the science of Mechanics, the history of which is well known, a number of books have been produced from time to time, having for their subject *Machinery*. At first, however, the leading principle of classification in these is derived from the purpose for which each machine is designed, and accordingly these books are either confined to machines destined for one particular kind of work, as in the early treatises of Valturius (1472) and Agricola (1550) on warlike and mining machinery respectively; or else they are collections of machines classed and described with reference to the objects for which they are constructed; divided, for example, into machines for raising water, for grinding flour, sawing timber, and so on. The earliest of these collections are the treatises of Besson (1569), Ramelli (1580), Strada (1618), Zonca (1621), Branca (1629), Bockler (1662); and the list might be continued without interruption to the present day*. The voluminous "Theatrum Machinarum" (1724) of Leupold, although it falls under the same description, yet in its first volume contains the first attempt to consider the parts of machinery separated from their work, and referred to the modifications of motion. And although these parts are made to follow the usual mechanical powers, and are mixed up with considerations of force, yet we find chapters on the *crank*, on *cams*, on machines for *converting a circular motion into a rectilinear*, or a *back and forwards motion*, and for converting a *back and forwards motion into a continued circular motion*; and so on. This must, in fact, be considered as the first attempt to produce a systematic treatise on Mechanism. The next step appears to have been made in 1794, by Monge, who, in planning the organization of the Ecole Polytechnique, proposed to devote two months of the first year of study to the *elements of machines*. "By these *elements* are to be understood the means by which the directions of motion are changed; those by which progressive motion in a right line, rotative motion, and reciprocating motion, are made each to produce the others. The most complicated machines being merely the result of a combination of some of these elements, it is necessary that a complete enumeration of them should be drawn up†." This enumeration formed the subject of part of his lectures, and was the basis of the two similar systems of Hachette, and of Lanz and Betancourt. The latter was finally

* This list might be preceded by Vitruvius, Book x., the works of Hero and other Greek mechanists, &c. Vide Veterum Mathematicorum Opera. Par. 1693.

† Vide Essai sur la Composition des Machines, par MM. Lanz and Betancourt, Par. 1808. p. 1.

adopted for the Ecole Polytechnique, and printed in 1808, under the title of "An Essay on the Composition of Machines." It was subsequently translated into English. Postponing for the moment the discussion of the system, we may observe, that Monge, in the above programme, distinctly proposes to study machines by treating them merely as contrivances for changing one kind of motion into another, apart from any considerations of force. We shall see presently, however, that this plan did not extend beyond the mere enumeration and description of the elements, without containing a provision for the calculation of the laws of the motion, or changes of motion produced. Ampère, however, appears to have contemplated the formation of a system that would also include these latter objects; for in his Essay on the Philosophy of the Sciences, published in 1834, we find it distinctly asserted, "that there exist certain considerations which if sufficiently developed would constitute a complete science, but which have been hitherto neglected, or have formed only the subject of memoirs or special essays. This science, (which he terms *Kinematics*,) ought to include all that can be said with respect to *motion* in its different kinds, independently of the forces by which it is produced. It should treat in the first place of spaces passed over, and of times employed in different motions, and of the determination of velocities according to the different relations which may exist between those spaces and times.

"It ought then to develope the different instruments by the help of which one motion may be converted into another, so that, calling these instruments by the usual name of *machines*, this science will define a machine to be, not as usual, *an instrument by means of which we may change the direction and intensity of a given force;* but, *an instrument by means of which we may change the direction and velocity of a given motion.* The definition is thus freed from the consideration of the forces which act on the machine; a consideration which merely distracts the attention of those who endeavour to unravel the mechanism.

"To understand, for example, the wheel-work by means of which the minute-hand of a watch makes twelve turns while the hour-hand makes but one, why need we trouble ourselves with the force that sets the watch in motion? The effect of the wheel-work, so far as it governs the relative velocity of the hands, is the same, by whatever cause the motion may be produced, as, for example, when the minute-hand is turned by the finger.

"After these general considerations relating to motion and velocity, this new science might pass on to the determination of the ratios that exist between the velocities of the different points of a machine, or generally of any system of material points, in all the movements of which the machine or system is susceptible; in a word, to the determination, independently of the forces applied to the material points, of what are called *virtual velocities;* a determination which is infinitely more comprehensible when thus separated from considerations of Force*."

* Vide Ampère, Essai sur la Philosophie des Sciences, 1835, p. 50.

It is much to be regretted that this distinguished writer did not attempt to follow up this clear and able view of the subject, by actually developing the science in question.

A similar separation of the principles of motion and force formed the basis of the Lectures on Mechanism which I delivered for the first time to the University of Cambridge, in 1837; and the same views were subsequently sanctioned by the high authority of Professor Whewell, who, in his Philosophy of the Inductive Sciences, has assigned a chapter to the Doctrine of Motion*, in which, under the title of Pure Mechanism, he has defined this science nearly in the above words of Ampère, whom he quotes.

To make the plan of the following pages more intelligible, it will be necessary in the first place to take a short review of the system of MM. Lanz and Betancourt, which, as we have seen, is founded upon the views of Monge. Their system is thus detailed at the opening of their work:

"The motions of the parts of machines are either (1) *Rectilinear*, (2) *circular*, (3) or *curvilinear*; and each of these may be *continuous* in direction or *alternate*, that is, *back and forward*. These six motions admit of being combined two and two in twenty-one different ways, each motion being supposed to be also combined with itself. The object of every simple machine being to counterchange or communicate these motions, the following system will include them all.

* Whewell, Philosophy of the Inductive Sciences, 1840, p. 144.

Continuous Rectilinear*, changed into	rectilinear	continuous†	1
		alternate†	2
	circular...	continuous†	3
		alternate†	4
	curvilinear	continuous†	5
		alternate†	6
Continuous Circular*, into	rectilinear	alternate†	7
	circular...	continuous†	8
		alternate†	9
	curvilinear	continuous†	10
		alternate†	11
Continuous Curvilinear*, into	rectilinear	alternate†	12
	circular...	alternate†	13
	curvilinear	continuous†	14
		alternate†	15
Alternate Rectilinear*, into	rectilinear	alternate†	16
	circular...	alternate†	17
	curvilinear	alternate†	18
Alternate Circular*, into............	circular...	alternate†	19
	curvilinear	alternate†	20
Alternate Curvilinear*, into.........	curvilinear	alternate†	21"

Of many of these combinations, however, no direct solution is given. Thus for (2) we are told to convert rectilinear motion into circular by one of the combinations in (3), and then to convert this into alternate rectilinear by one of those in (7). In this way also classes 5, 6, 11, 12, 13, 15, 16, 18, and 21, are disposed of; so that there remain only twelve, under which our authors proceed to arrange the elementary combinations into which, according to them, mechanism may be resolved.

This celebrated system, which has been pretty generally received, must however be considered as a merely popular arrangement, notwithstanding the apparently scientific sim-

* With velocity either uniform or varying according to a given law.

† With a velocity of the same nature as that which produces it, preserving a constant proportion to it or varying according to a given law. In the same or in different planes.

plicity of the scheme. In the first place, it is not confined to pure combinations of mechanism, but is embarrassed by the intrusion of several dynamical and even hydraulic contrivances. Thus, a water-wheel and a windmill-sail are considered to be a means of converting continuous rectilinear motion into continuous circular; and a ferry-boat attached to one end of a long rope, of which the other is fixed to the bank, is admitted into Class 4, as a means of converting continuous rectilinear motion into alternate circular. Fly-wheels, pendulums with their escapements, parallel motions, are all placed in one class or other of this scheme. No attempt is made to subject the motions to calculation, or to reduce these laws to general formulæ, for which indeed the system is totally unfitted.

The plan of the great work of Borgnis, published in 1818, is much more comprehensive and complete, really embracing the whole subject of machinery, instead of being confined by its plan to elementary combinations for the modification of motion. Borgnis, in the volume on the *Composition of Machines*, divides mechanical organs into six orders, each of which have subordinate classes. His orders are*; (1) Receivers of power; (2) Communicators; (3) Modifiers; (4) Frame-work, fixed and moveable: (5) Regulators; (6) Working parts.

For the mere purposes of descriptive mechanism this system is much better adapted than that of MM. Lanz

* In the original, (1) Récepteurs, (2) Communicateurs, (3) Modificateurs, (4) Supports, (5) Régulateurs, (6) Opérateurs.

and Betancourt, but still does not provide for the investigation of the laws of the modifications of motion, which is an especial object of the proposed science of Kinematics. Many essays, however, have been from time to time written concerning various detached portions of this science. The teeth of wheels is the most remarkable of these, from having occupied the attention of so many of the best mathematicians. But in fact, the description of all the mechanical curves, as epicycloids and conchoids, may be held to belong to this science, which would thus be made to include a great mass of matter that has hitherto been classed with geometry. The calculation of trains of wheel-work is also a branch of it, to which the first contribution was made by Huyghens, who employed continued fractions, for the purpose of obtaining approximate numbers for the trains of his Planetarium*.

The following pages must not however be considered as an attempt to carry out the able and comprehensive views of Ampère, being confined to machinery alone, and not passing from it to the more abstract generalities of motion, which he seems to have contemplated.

My object has been to form a system that would embrace all the elementary combinations of mechanism, and at the same time admit of a mathematical investigation of the laws by which their modifications of motion are governed. I have confined myself to the Elements of Pure

* Vide also Young's Nat. Philosophy, vol. II. p. 55. Arts. 365, 366, the substance of which will be found in this work. Arts. 34 and 237.

Mechanism, that is, to those contrivances by which motion is communicated purely by the connexion of parts, without requiring the essential intermixture of dynamical effects.

I have taken a different course from the one hitherto followed, in respect that instead of considering a machine to be an instrument by means of which we may change the direction and velocity of a *given motion*, I have treated it as an instrument by means of which we may produce any *relations of motion* between two pieces.

For Monge and his followers began by dividing motion into rectilinear and rotative, continuous and reciprocating, and so based their system upon the *actual motion* of the parts; and Ampère defines his machine in the words quoted above as modifying a *given motion*. But a little consideration will shew that any given element of machinery can only govern the *relations of velocity and direction* of the pieces it serves to connect; and that this connexion and the law of its action are for the most part independent of the *actual velocities*. By establishing a system upon the relations of motion instead of upon the actual motions, it will be found that many of the redundancies and difficulties that have hitherto obscured the subject are got rid of.

Thus, to follow up the example given by Ampère of the hands of a watch, it is clear that the connexion governs the relation of their angular velocities, which at every instant is in the proportion of twelve to one; and also provides that they shall both revolve the same way, whether that be to the right or to the left. If then the one be made to revolve through a small angle back and forwards, the other will also revolve back and forwards through an angle of one twelfth of that described by the first. Now in the usual system this identical contrivance, which in its ordinary employment belongs to the class of conversion from continuous circular into continuous circular, is thus also thrown into the class of alternate circular changed into alternate circular. In the system which I propose, this contrivance at once finds its place as a combination in which the velocity ratio and directional relation are constant.

I have also dismissed, or given a subordinate place, to the distinction between circular and rectilinear motion, and have introduced a new distinction between those motions which are capable of being from the nature of the contrivance continued indefinitely in either direction, and those of which the extent is limited by the nature of the contrivance.

The first ground of my classification is the effect of the combination upon the Velocity Ratio of the pieces, and upon the relation of their directions of motion, or Directional Relation; from which considerations I have divided all the Elementary Combinations into three classes.

The second ground of the classification, and the one by means of which the calculation of the law of communication of the velocities and directions is effected, is the *mode in which the motion is transmitted;* a part of the subject which appears wholly neglected by the writers already referred to. These modes I have divided into Rolling and Sliding Contact, Link-work, Wrapping Connexion, and Reduplication. The relative motions produced by each of these methods will be found to be governed by a different geometrical principle, and every possible mode of communication may be placed under one or other of these divisions. Many combinations, however, derive their principle of action from a mixture of two or more of these methods of communication. In this case their place in the system is always determined by that method which has the greatest influence; besides which, each combination is reduced to its equivalent simple form, and its position determined by that alone; for the object of the system is to reduce the motions to calculation; and for this purpose the equivalent simple form of every combination must be employed.

For example, the action of combinations in which rows of teeth are used depends partly upon rolling contact and partly upon sliding contact; for the action of the individual teeth is of the latter kind, but the total action of them is equivalent to the rolling contact of their pitch-lines, and the pitch-lines only need be considered in calculating the motion. Accordingly, all combinations in which rows of teeth are employed will be found under the head of Rolling Contact. Again, when cam-plates or curves are used a friction roller is often employed for these plates to act against. At first sight this would appear to convert the action of the combination into rolling contact. But besides that this contrivance merely transfers the sliding action to the axis of the roller, and that our definition of rolling contact supposes the two axes of motion of the rolling curves to be fixed in position, the calculation of the motion of all such combinations is effected by supposing the roller reduced to a point, and the curve thus obtained upon the principles of pure sliding contact, is afterwards adapted to the roller by tracing a second curve within it at a normal distance equal to the radius of the roller. All combinations of this kind are therefore placed under Sliding Contact, notwithstanding the employment of friction-rollers. Either of these considerations, the Velocity Ratio and Directional Relation, or the modes of communication, might have been made the primary ground of the classification; and some advantages might result from adopting the second for this purpose. I was induced to select the first, because it enabled me to separate from the others all that most important class of combinations in which the Velocity Ratio and Directional Relation remain constant, and which are also the foundation of most of those contained in the subsequent classes. The Synoptical Table, which immediately follows this Preface, will shew the general arrangement of the Elementary Combinations under the proposed system.

In the Second Part of the work I have assembled a number of contrivances which appear to me to be connected by a general principle which has not hitherto been defined; these I have ventured to term Aggregate Motions. One portion only of these contrivances has usually been treated as a separate class under the name of Differential Motions.

The Third Part contains several problems relating to the calculation and arrangement of mechanism in which it is necessary to have the power of altering the relations of motion at pleasure. This part, for the reasons which will be found in its first chapter, is not to be considered as a complete essay on this branch of the subject.

I have, in the course of the work, endeavoured in every case to acknowledge the sources from whence I have derived any portion of its contents by references at the foot of the page. But so little of the subject has been hitherto treated mathematically, that I must hold myself answerable for the greatest portion of it. The teeth of wheels, which occupies the first part of Chapter III. is the only branch of mechanism in which the original papers have been already wrought into a system, and published in a collected form. This was first done by Camus, and has been subsequently effected by Buchanan in his Essays, and by Hachette.

These works being so well known I have not so constantly referred to my authorities in this chapter, but it will be found that I have incorporated into it extracts from the valuable paper of Professor Airy, as well as the entire contents of my own paper from the Transactions of the Society of Civil Engineers, and have added several original investigations relating to the proportions of the teeth, and their least numbers. Some of these questions have been discussed by Kæstner*, but not in a manner

* Commentat. Gott. 1781, 1782.

adapted to practice. Tredgold has also given some results*, but has unfortunately vitiated them by the coarseness of his approximations. It will be found that I have calculated all the results that are required in practice, and have arranged them in tables for reference.

On the whole, it will be seen that the present volume is limited to a very small portion of the important subject of machinery. The object of it is, as has been already stated, to systematize the subject, and to free it from the considerations of force, with which it has been usually mixed up.

To complete the plan, therefore, it will be necessary, in the next place, to apply these considerations of force to the combinations thus obtained, as well as to describe and investigate those parts of machinery in the action of which forces are essential; a task which I shall probably undertake at some future time.

In carrying out this branch of the subject, great assistance will be derived from the works of modern French writers—Navier, Poncelet, Morin, &c., who have with so much success and originality applied themselves to this purpose. Their works are now beginning to attract the attention of our own writers; and in the present year Professor Whewell has introduced many of their results into his Mechanics of Engineering, generalizing them with his usual ability; and in the same work he has flattered me by the

* Vide Buchanan's Essays.

adoption of my own views upon the classification of the modes in which motion is communicated from one piece to another of a machine, adding to them the investigation of the effects of force and resistance; which may be considered as carrying out a portion of the plan above alluded to, as necessary to complete this arrangement of the science of Machinery.

I am not without hopes, that in addition to its principal object of giving a scientific and systematic form to its subject, the results of the volume which I now venture to present to the world, may be found a useful addition to mathematical studies in general, by affording simple illustrations of the application and interpretation of formulæ, and by suggesting new subjects for problems, and for farther investigation.

CAMBRIDGE,
Sept. 28, 1841.

SYNOPTICAL TABLE

OF THE

ELEMENTARY COMBINATIONS OF PURE MECHANISM.

	DIRECTIONAL RELATION CONSTANT.		DIRECTIONAL RELATION CHANGING PERIODICALLY.
	Velocity-Ratio Constant.	*Velocity-Ratio Varying.*	*Velocity-Ratio Constant or Varying.*
	Class A.	Class B.	Class C.
Division A. By Rolling Contact.	Rolling cylinders, cones, and hyperboloids. General arrangement and form of toothed wheels. Pitch.	Rolling curves and rolling curve-wheels. Roëmer's & Huyghens' wheels, &c. Wheels with intermitted teeth. Rolling-curve levers.	Mangle-wheels. Mangle-racks. Escaping geerings.
Division B. By Sliding Contact.	Forms of the individual teeth of wheels. Cams. Screws. Endless screws or worms and their wheels.	Pin and slit lever. Cams. Unequal worm. Geneva stop and other intermittent motions.	Pin and slit lever. Cams in general. Swash plate. Double screw. Spiral and solid cams. Escapements.
Division C. By Wrapping Connectors.	Arrangement and material of bands. Form of their pullies. Guide pullies. Geering chains. Arrangements for limited motions.	Curvilinear pully. Fusees.	Curvilinear pully and lever.
Division D. By Link-work.	Cranks and link-work for equal rotations. Cranks for limited motions. Bell crank-work.	Link-work. Hooke's joints.	Cranks, excentrics, and other link-work. Ratchet wheels and clicks. Intermittent link-work.
Division E. By Reduplication.	Tackle of all kinds, with parallel cords and in trains.	Tackle with unparallel cords.	

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PART THE FIRST.

ON TRAINS OF MECHANISM IN GENERAL.

CLASS A. { DIRECTIONAL RELATION CONSTANT. VELOCITY RATIO CONSTANT. } DIVISION A. BY ROLLING CONTACT.

By pure Rolling Contact.

To apply these Solutions to practice.

CLASS A. DIVISION B. BY SLIDING CONTACT.

Axes Parallel.

LIST OF TECHNICAL AND NEW TERMS,

WHICH ARE DEFINED OR EXPLAINED IN THE FOLLOWING ARTICLES.