PRINCIPLES OF MECHANISM.

INTRODUCTION.

1. EVERY machine is constructed for the purpose of performing certain mechanical operations, each of which supposes the existence of two other things beside the machine in question, namely, a moving power, and an object subjected to the operation, which may be termed the work to be done.

Machines, in fact, are interposed between the power and the work, for the purpose of adapting the one to the other.

2. As an example of a machine whose construction is familiar to all, the grinding machine so commonly seen in our streets may be cited, in which the grindstone is made to revolve by the application of the foot to a treadle.

Here the *moving power* is derived from muscular action. The *operation* is carried on by pressing the edge of the cutting instrument, which is the subject of it, against the surface of the grindstone, which is caused to travel rapidly under it.

The arrangement and form of this surface, and its connexion with the foot in such a manner that the pressure of the latter shall communicate the required motion to the former, is the office and object of the machine.

Two portions of the machine are given, the one by the nature of the power, and the other by that of the work. The first is a treadle placed at a proper level to receive the pressure of the foot, by the action of which it may be made to perform, without unnatural exertion, about eighty or ninety vertical oscillations in a minute. The second part of the machine is the cylindrical grindstone, which is mounted on a horizontal axis at the upper part of the frame, and at a convenient height to allow the tool to be pressed upon its revolving surface. The surface should pass under the edge of the tool at the rate of about 500 feet in a minute, and therefore supposing the diameter of the grindstone to be eight inches, it must revolve at the rate of 250 turns in a minute. The remainder of the mechanism serves to connect the treadle and grindstone, and may consist of any contrivance that will compel the latter to revolve when the former is made to oscillate, and in the proportion of 250 revolutions to 80 oscillations, or about three to one.

3. It appears, then, that this machine consists of a series of connected pieces, beginning with the treadle whose construction, position, and motion, are determined by the nature of the moving power, and ending with the grindstone, which in like manner is peculiar and adapted to the work. But this is, in fact, the description of every machine. There is always one or more series of connected pieces, at one end of each of which is a part especially adapted to receive the action of the power, such as a water-wheel, a wind-mill-sail or a horse-lever, a handle or a treadle. At the other end of each series will be a set of parts determined in form, position, and motion, by the nature of the work they have to do, and which may be called the working pieces. Between them are placed trains of mechanism connecting them

so that when the first parts move according to the law assigned them by the action of the power, the second must necessarily move according to the law required by the nature of the work.

4. These three classes of mechanical organs are so far independent of each other, that any given set of working parts may be supplied with power from any source: thus a grindstone may be turned either by the foot or by the hand of an assistant, by water or by a horse. Again, a given water-wheel or other receiver of power may be employed to give motion to any required set of working parts for whatever purpose. Also between a given receiver of power and set of working parts the interposed mechanism may be varied in many ways. Moreover the principles upon which the construction and arrangement of these three classes are founded are different. The *receivers of power* derive their form from a combination of mechanical principles with the physical laws which govern the respective sources of power. The *working parts* from a combination of mechanical principles, with considerations derived from the processes or objects in view. But the principles of the *interposed mechanism* admit of being developed without reference to the powers employed or transmitted, or to the resistances or work to be done, or, in fact, to the objects for which machinery is constructed. By defining mechanism in the abstract to be a combination of parts for the purpose of connecting two or more pieces, so that when one moves according to a given law, the others must move according to certain other given laws, this branch of the subject may be reduced to geometrical principles alone: whereas by considering mechanism as usual, as a modifier of force, the subject becomes embarrassed by a condition foreign to the connexion of parts by which the modification is pro-

duced; and which condition and its consequences admit more conveniently of subsequent consideration and separate investigation.

5. The hour-hand of a clock, for example, is connected with the minute-hand by a mechanism which compels the former to perform one revolution while the latter completes twelve; or generally, the angular velocity of the first is always one twelfth of that of the second. The connexion is independent of the force which puts the minute-hand in motion, and also of the actual velocity of the minute-hand. If this be turned by hand quickly or slowly, uniformly or variably, back or forwards, the hour-hand will still follow these motions at an angular rate of one twelfth of the original. The constant relation of the angular velocities depends in this as in other similar cases only upon the proportion between the diameters or number of teeth of the wheel-work that connects the two hands—a purely geometrical relation, the comprehension of which is rather obscured than assisted by the introduction of statical principles, of which the connexion is independent, but which find their proper place, when it becomes necessary to investigate the proportion between the forces and resistances in any given case, and the strains thrown upon the different parts of the mechanism by their application, and thus to find the requisite strength of each part.

6. The term *mechanism*, then, must be understood to be in this work confined to those mechanical combinations which govern the relations of motion only, and which therefore admit of being entirely separated from the consideration of force. This, of course, excludes not only those mechanical organs which have been already alluded to, as receivers of power and working parts, but also those which

are employed to govern the motions of machinery; such as the escapements of clocks, and contrivances by which machinery is made self-acting and self-regulating; all of which are derived from combinations of pure mechanism with statical or dynamical principles, but from which they do not admit of separation. The exposition of such contrivances will naturally and easily follow from the principles of the present work, but are excluded from it by its plan, which is, to reduce the various combinations of *Pure Mechanism* to system, and to investigate them upon geometrical principles alone.

7. Neither is it my purpose to enter into minute details of the actual construction of machinery, of the different forms which each combination may assume, or of the infinitely varied methods of framing and putting them together; for, in the first place, the choice of these forms in every particular case is mainly determined by the strains to which the machinery is to be exposed; and, in the next place, this branch of the subject is sufficiently important and extensive to admit of separation from the others, under the name of *Constructive Mechanism*. Although some details of this kind are unavoidable in the present work, I have carefully avoided them when possible, and for this purpose have excluded from the drawings all unnecessary and extraneous framing or connexions that tend to individualize the combinations, and thus to oppose the very object which I have proposed to myself, namely, to introduce such a degree of generalization and system, as would give to *Pure Mechanism* a claim for admission into the ranks of the Sciences.

8. I must here recapitulate the ordinary definitions and measures of motion and velocity, for the purpose of

introducing certain modifications which they require to adapt them to our present purposes.

A body is absolutely at *rest* when it remains in the same position in space, and at rest relatively to another body when it continues in the same relative position to that body, as it is usually said to be at rest when it remains in the same relative position to the earth. Thus, too, a body which remains in the same place in a boat or a carriage, is at rest with respect to that boat or carriage, although these may be in motion; and so a wheel or other portion of a machine may be carried into different positions relatively to the fixed frame, and yet remain at rest with respect to the arm or carriage upon which it is mounted.

A body is in *motion* when it occupies successively different positions in space; motion being relative as well as rest. Two bodies moving with respect to a third will be at rest with respect to each other, if they retain in their motions the same relative positions; or a body absolutely at rest may be said to move with respect to another moving body, if the latter be assumed as the standard to which the motion is to be referred.

9. Motion is essentially continuous; that is to say, a point cannot pass from one position to another without going through a series of intermediate positions. Thus the motion of a point describes in space a line necessarily continuous, which line is termed its *path*. The path of a solid body must be understood as the line described by some principal point in that body, such as the center of a sphere.

The path of a body being assigned, there are only two *directions* in which it can move. Direction of motion being relative, may be indicated by naming some fixed point which the body is approaching or retiring from: as, for example, the points of the compass, the zenith or nadir, or by personal or other relations, such as right and left, larboard and starboard, windward and leeward, upwards and downwards, &c.; otherwise its direction of motion may be defined by comparing it with that of the sun or of the hands of a watch; the latter is an exceedingly convenient standard for rotative motion. By supposing the path of the sun projected upon the plane of motion, it may be employed as a standard for rotative direction in every case but that of motion in a plane perpendicular to its orbit.

The path and direction being assigned, the body may move in its given path and direction quickly or slowly, with a greater or less *velocity*; and this velocity is estimated by comparing the space passed over with the time occupied in describing it.

10. When a body describes equal portions of its path in equal successive times, the motion is said to be uniform, and the velocity measured by the space (that is, the length of path) described in the unit of time. The units usually employed are feet and seconds. Thus a body is said to move at the rate of 3 feet per second.

Since the same space is described in every unit of time, the entire space described is proportional to the time employed in describing it, and the measure of velocity is obtained by dividing the number of feet passed over by that of the seconds employed.

If V be the velocity, S the space in feet, T the time in seconds, $V = \frac{S}{T}$. The direction is indicated analytically by the sign of the velocity for a given path; if the velocity in one direction be assumed positive, that in the opposite direction will be negative.

11. The motion of a revolving body may be measured by the linear velocity of a point whose radial distance is equal to the unit of space. This is termed the angular velocity of the body, which is said to revolve uniformly when its angular velocity is uniform.

In uniform angular velocity the angles described by a given radius, are manifestly proportional to the times; and since the linear velocity of every point is the arc described in the unit of time, which arc is proportional to the radius, so the linear velocity of every point is proportional to its radius. If A be the angular velocity, R the radius of the point in feet, the linear velocity $V = RA$.

The motion of a uniformly revolving body may also be conveniently measured by the number of rotations performed in a given time. In uniform rotation the angles described are proportional to the times, and any given point describes its own circle with uniform linear velocity. Let T be the time of performing k revolutions, where k may be a whole number or a fraction. Then, since 2π is the circumference whose radius is unity; $2\pi k$ will be the space described in k revolutions by the point whose radius is unity, but A is the space described by the same point in the unit of time;

$$\therefore\ A : 2\pi k :: 1 : T; \quad \therefore\ T = \frac{2\pi k}{A}\ (1); \quad k = \frac{TA}{2\pi}\ (2);$$

Hence the number of turns in a given time varies as the angular velocity.

Let R be the radius of a wheel and V its perimetral velocity;

$$\therefore\ V = RA. \quad \text{And } k = \frac{TV}{2\pi R}\ (3);$$

whence the number of turns in a given time varies directly as the perimetral velocity, and inversely as the radius or diameter of the wheel*.

Let the time in which a wheel performs one complete revolution be termed its Period ($= P$); $\therefore P = \frac{2\pi}{A}$ {putting $k = 1$ in (1)}; and the period varies inversely as the angular velocity.

Also from (2) $k = \frac{T}{P}$; whence the period varies inversely as the number of turns in a given time. When the rotations of two wheels are to be compared, the number of turns they respectively make in a given time may be termed their synchronal rotations.

12. When the velocity is not uniform, these expressions can no longer be applied, because the velocity is different at different times. In this case, then, the velocity at every instant is measured by the space that would be described in the succeeding unit of time, were the velocity with which that unit is commenced continued uniformly throughout it.

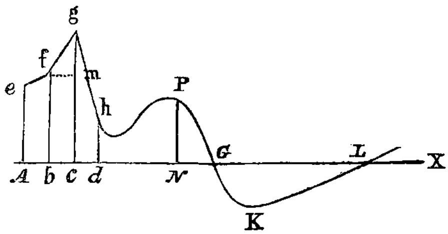
If the velocity of a body increase, it is said to be accelerated, and if the velocity diminish, to be retarded.

13. Varied motion admits of convenient graphical representation, by which its characteristic points and general laws are rendered much more easy of comprehension than they are by the use of formulæ alone.

* In practice linear velocity is commonly referred to seconds, and angular velocity to minutes; thus a millwright will define the velocity of a given wheel by either saying that it performs twenty revolutions in a minute, or that its circumference moves at the rate of three feet per second. In the expression (3) if k and T be expressed in minutes, and V is to be expressed in seconds, we must put $60V$ for V;

$$\therefore k = \frac{60\,TV}{2\pi R} = \frac{10\,TV}{R} \text{ very nearly.}$$

Thus to represent the motion of a body of which the velocities at certain given intervals of time are known, take an indefinite straight line *AX*, and from *A* set off abscissæ *Ab*, *Ac*, *Ad*...... proportional to the given intervals of time as measured from the beginning of the motion.



Upon *A*, *b*, *c*, *d*...... erect ordinates *Ae*, *bf*, *cg*, *dh*, respectively proportional to the velocity of the body at the beginning of the motion and after each interval of time. By joining the extremities of these ordinates, a polygon *efgh*...... is obtained, which if the intervals of time be taken with their differences sufficiently small, will become a curve as *hPGKL*, of which the abscissa *AN* at any given point *P*, will represent the time elapsed from the beginning, and the ordinate *NP* the corresponding velocity of the body.

If the motion of the body cease, its velocity becomes zero, and the curve meets the axis, as at *G* and *L*. If the body change its *direction* in its path, this is indicated by the change of sign in the velocity; for either direction being assumed positive, the other will be negative; and so in this curvilinear representation, the ordinates representing the velocity for one direction being set off upwards from the line, as from *e* to *G*, those of the opposite direction will be set off downwards as from *G* to *L*.

14. By another method a curve is constructed of which the abscissæ shall represent the time as before, but the ordinates the space described by the body. Thus, if the