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Principles of Geometry

Volume 6: Introduction to the Theory of Algebraic Surfaces and Higher Loci

H.F. Baker

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PRINCIPLES OF GEOMETRY

BY

H. F. BAKER, Sc.D., LL.D., F.R.S.

LOWNDEAN PROFESSOR, AND FELLOW OF ST JOHN'S COLLEGE,
IN THE UNIVERSITY

VOLUME VI

INTRODUCTION TO THE THEORY OF ALGEBRAIC SURFACES AND HIGHER LOCI

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PREFACE

THE origin and final purpose of this volume, and the preceding, have been stated in the preface to the latter. It may be useful to describe in outline the contents of the present volume. The first chapter deals with the theory of correspondence, mainly of points on one or two curves, with inclusion of the treatment by transcendental methods, and the connection with the theory of defective integrals. The second chapter attempts an exposition of Schubert's remarkable ideas, which are as interesting logically as geometrically, and of the extension of the theory of correspondence to aggregates of any dimension. The third chapter is in part a reminder of theorems which belong to plane geometry, and in part a sketch of general theorems for rational surfaces. In the fourth chapter the elementary preliminary properties of surfaces in ordinary space, and in space of four dimensions, are dealt with. Chapter V is that which is concerned with the most interesting and the most novel ideas of the volume. For this reason it is written in a tentative introductory manner, and will best have served its purpose if it leaves the reader convinced of the importance of the theory involved, and with a desire to follow it further. The next chapter develops in detail the theory of the intersections of manifolds in space of four dimensions. The last chapter collects together various particular theorems, and some easy applications of foregoing theory. Only want of space has led to the exclusion from the volume of many other results which are of interest.

I should like to give expression to my sense of how much this, and preceding volumes, owe to those who have been students with me during their composition; my experience has been of a remarkable and unremitting keenness in the prosecution of the matters treated. Without this encouragement and co-operation, I might not have persevered in the formulation of the ideas, especially in these last two volumes. But besides this personal

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reference, I would add that it is clear that the purely geometrical and descriptive aspects of the subject are felt by many of our students to offer a discipline which is a welcome complement to others which are open to them.

And, I would repeat, readers of these volumes are under much obligation to the staff of the University Press for the trouble and attention with which the printing has been executed.

H. F. B.

5 *October* 1933

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