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Principles of Geometry

Volume 5: Analytical Principles of the Theory of Curves

H.F. Baker

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PRINCIPLES OF GEOMETRY

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VOLUME V

ANALYTICAL PRINCIPLES OF THE THEORY OF CURVES

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PREFACE

THE present volume is an account of the analytic principles of the theory of curves, of the rational functions belonging thereto and of the integrals of these functions, with a brief account of the methods, by loops and by Riemann surfaces, for dealing with the periods of these integrals. But the theory of correspondence, and some necessary references to involutions in a plane, find themselves in the succeeding volume, which is mainly devoted to the theory of surfaces.

It is perhaps desirable to explain the origin of these volumes. In the last fifty years a remarkable advance has been made in the theory of surfaces, and of algebraic loci in general; the English reader may find a description of the nature of this in a Presidential Address to the London Mathematical Society given in November 1912 (*Proceedings*, Vol. XII). But attempts, since the War, to expound these new results have continually shewn the necessity for a precise appreciation of the ideas out of which this advance has developed; in mathematics it is not sufficient to know the enunciation of a result; it is necessary to understand the proof. These two volumes have grown up in the attempt to meet this need. The further need of a volume explaining the applications of topological theory, especially to the periods of the integrals belonging to the higher loci, may, I hope, appeal to another. The volumes are necessarily very incomplete in their inclusion of detail, as the specialist in any branch will easily find; their object is to lay the foundations for a more detailed study.

The pursuit of the analytical principles has a fascination in itself; but since, for reasons of space, these volumes are so largely devoted to this, I may be allowed to add another remark. The study of the fundamental notions of geometry is not itself geometry; this is more an Art than a Science, and requires the constant play of an agile imagination, and a delight in exploring the relations of geometrical figures; only so do the exact ideas find their value. As when, upon a landscape of rugged hill and ruffled water, there breaks the morning sun, scattering the clouds, and anon bathing the whole in a glory of contrasting colour. If these volumes should help to increase the number of those to whom the comparison does not seem an exaggeration, they will have been worth the making.

To the University Press very special acknowledgments are due, for the care, and speed, with which the volumes have been printed.

H. F. B.

29 *August*, 1933.

TABLE OF CONTENTS

CHAPTER I. INTRODUCTORY ACCOUNT OF RATIONAL AND ELLIPTIC CURVES

CHAPTER II. THE ELIMINATION OF THE MULTIPLE POINTS OF A PLANE CURVE

CHAPTER III. THE BRANCHES OF AN ALGEBRAIC CURVE; THE ORDER OF A RATIONAL FUNCTION; ABEL'S THEOREM

CHAPTER IV. THE GENUS OF A CURVE. FUNDAMENTALS OF THE THEORY OF LINEAR SERIES

CHAPTER V. THE PERIODS OF ALGEBRAIC INTEGRALS. LOOPS IN A PLANE. RIEMANN SURFACES

CHAPTER VI. THE VARIOUS KINDS OF ALGEBRAIC INTEGRALS. RELATIONS AMONG PERIODS

CHAPTER VII. THE MODULAR EXPRESSION OF RATIONAL FUNCTIONS AND INTEGRALS

CHAPTER VIII. ENUMERATIVE PROPERTIES OF CURVES

Contents