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Henry Frederick Baker (1866–1956) was a renowned British mathematician specialising in algebraic geometry. He was elected a Fellow of the Royal Society in 1898 and appointed the Lowndean Professor of Astronomy and Geometry in the University of Cambridge in 1914. First published between 1922 and 1925, the six-volume *Principles of Geometry* was a synthesis of Baker's lecture series on geometry and was the first British work on geometry to use axiomatic methods without the use of co-ordinates. The first four volumes describe the projective geometry of space of between two and five dimensions, with the last two volumes reflecting Baker's later research interests in the birational theory of surfaces. The work as a whole provides a detailed insight into the geometry which was developing at the time of publication. This, the fourth volume, describes the principal configurations of space of four and five dimensions.

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HIGHER GEOMETRY

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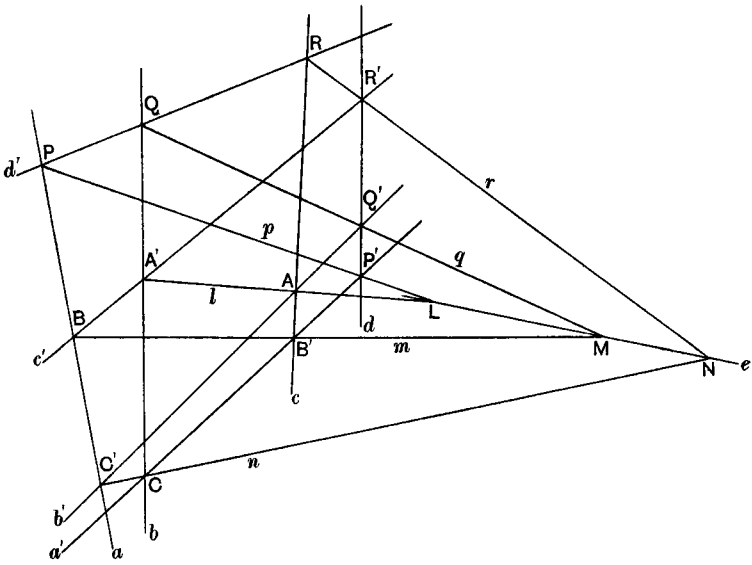
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THE FIGURE OF FIFTEEN LINES AND FIFTEEN POINTS, IN SPACE OF FOUR DIMENSIONS
 (See Ch. V.)

PRINCIPLES OF GEOMETRY

BY

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VOLUME IV

HIGHER GEOMETRY

BEING ILLUSTRATIONS OF THE UTILITY OF THE
CONSIDERATION OF HIGHER SPACE, ESPECIALLY
OF FOUR AND FIVE DIMENSIONS

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PREFACE

THE present volume, the first written and the most revised, of the book, for which indeed, mostly, the earlier volumes were undertaken, still bears many marks of the difficulty of compressing the matter into brief compass. But the writer hopes that it may seem to the reader as remarkable as it does to him, that it should be possible to comprehend under one point of view, and that so simple, the introduction to nearly all the surfaces ordinarily studied in the geometry of three dimensions, as well as the usual line geometry. Chapters v, vi, vii seek to make clear that this is so. To these the earlier chapters are auxiliary. But Chapters ii and iv have been introduced as much for their own interest as for their illustrative value; the results obtained in these two chapters are not required in the subsequent pages. It is hoped that the Table of Contents, and the Index, may make it easy to use the volume. It will of course be understood that the volume is throughout intended to be introductory and illustrative; hardly anywhere is it complete.

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H. F. B.

1 June 1925.

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