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Principles of Geometry

VOLUME 3:
SOLID GEOMETRY

H.F. BAKER



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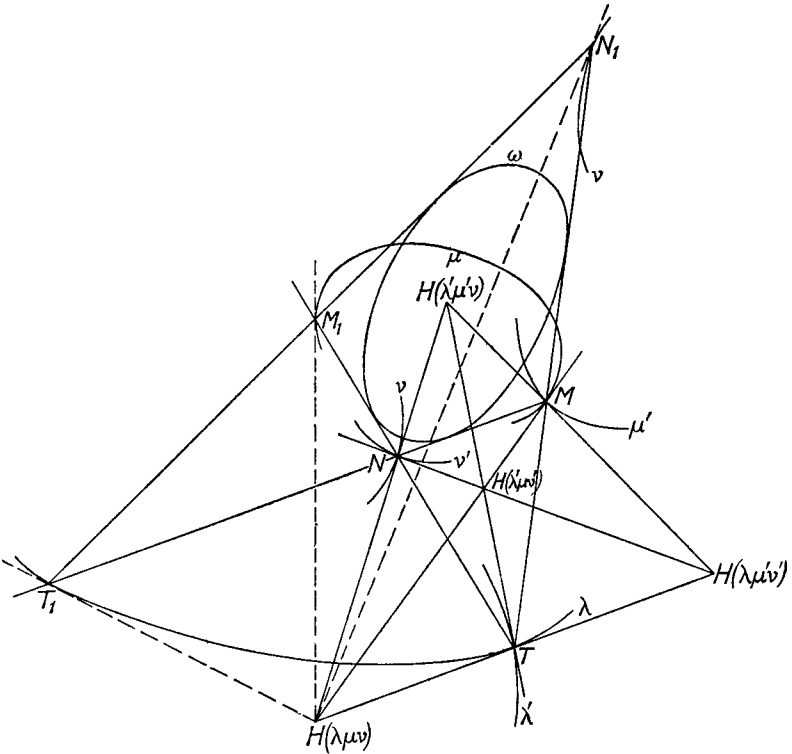
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PONCELET'S PORISM AND CONFOCAL QUADRICS
 (see p. 116)

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PRINCIPLES OF GEOMETRY

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VOLUME III

SOLID GEOMETRY

QUADRICS, CUBIC CURVES IN SPACE, CUBIC SURFACES

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PREFACE

THE present volume is devoted to geometry in three dimensions. The discussion of the logical standpoint, to which sufficient space has been given in the preceding volume, is left aside; and, from a desire to limit the size of the volume, many things are omitted which might well have been included. What is given may, however, be regarded as essential to any student who professes to have received a mathematical education. The aptitude for geometrical construction in space, important as it is in the applications of mathematics to physics and engineering, receives, in our educational system at present, less training than it deserves. It is the writer's hope that this volume may help to emphasize this; and may convey to readers something of the fascination and freedom which belongs to the reduction of intricate geometrical relations to the properties of a constructed figure. Only by such methods, moreover, can progress be made beyond the first principles of the subject.

Up to the end of Chapter III, this volume was in type when death severed an association to which the writer owed more help than he can well express. In business, James Bennet Peace was clear and honest; in friendship, constant and self-regardless; many beside the writer deplore his loss. To him, and to the co-operation of the other members of the Staff of the University Press, great acknowledgment is due.

H. F. BAKER.

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“TEODORO REYE,, che avevo cominciato ad ammirare fin da studente, leggendo la sua classica *Geometrie der Lage*; e col quale poi non avevo tardato ad entrare in relazione scientifica, ed anche personale, sì da poter apprezzare, oltre al valore del matematico, la grande bontà d'animo dell'uomo: vero gentiluomo!

Nato a Cuxhaven il 20 giugno 1838,, era passato verso il 1864 ad insegnare nel Politecnico di Zurigo.Aveva esordito nella scienza con lavori di *Fisica matematica* e di *Meteorologia*. Ma, poichè a Zurigo il corso del Culmann, fondatore della *Statica grafica*, si basava sulle teorie della *Geometria di posizione*, e il classico trattato di Staudt era troppo difficile per gli studenti; Reye fu condotto ad insegnare quelle teorie e ad esporle in un nuovo trattato, che uscì in due parti nel 1866 e nel 1868.

.....
 Artista non meno che scienziato, Reye ha molto contribuito a quella grandiosa e pure snella costruzione scientifica che è la *Geometria di posizione*, introducendo o svolgendo idee semplici e geniali; studiando, com'è carattere di essa, svariate figure in tal maniera da illuminarne di vivida luce le proprietà più profonde, e i legami che le uniscono. Non solo ci ha fatto conoscere nuovi veri; ma ci ha procurato squisiti godimenti estetici, quali solo può dare il bello. Onore e gratitudine a Lui!”

Corrado Segre, *Rendiconti...dei Lincei*, 2 Aprile 1922.

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