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Henry Frederick Baker (1866–1956) was a renowned British mathematician specialising in algebraic geometry. He was elected a Fellow of the Royal Society in 1898 and appointed the Lowndean Professor of Astronomy and Geometry in the University of Cambridge in 1914. First published between 1922 and 1925, the six-volume *Principles of Geometry* was a synthesis of Baker's lecture series on geometry and was the first British work on geometry to use axiomatic methods without the use of co-ordinates. The first four volumes describe the projective geometry of space of between two and five dimensions, with the last two volumes reflecting Baker's later research interests in the birational theory of surfaces. The work as a whole provides a detailed insight into the geometry which was developing at the time of publication. This, the first volume, describes the foundations of projective geometry.

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VOLUME 1:  
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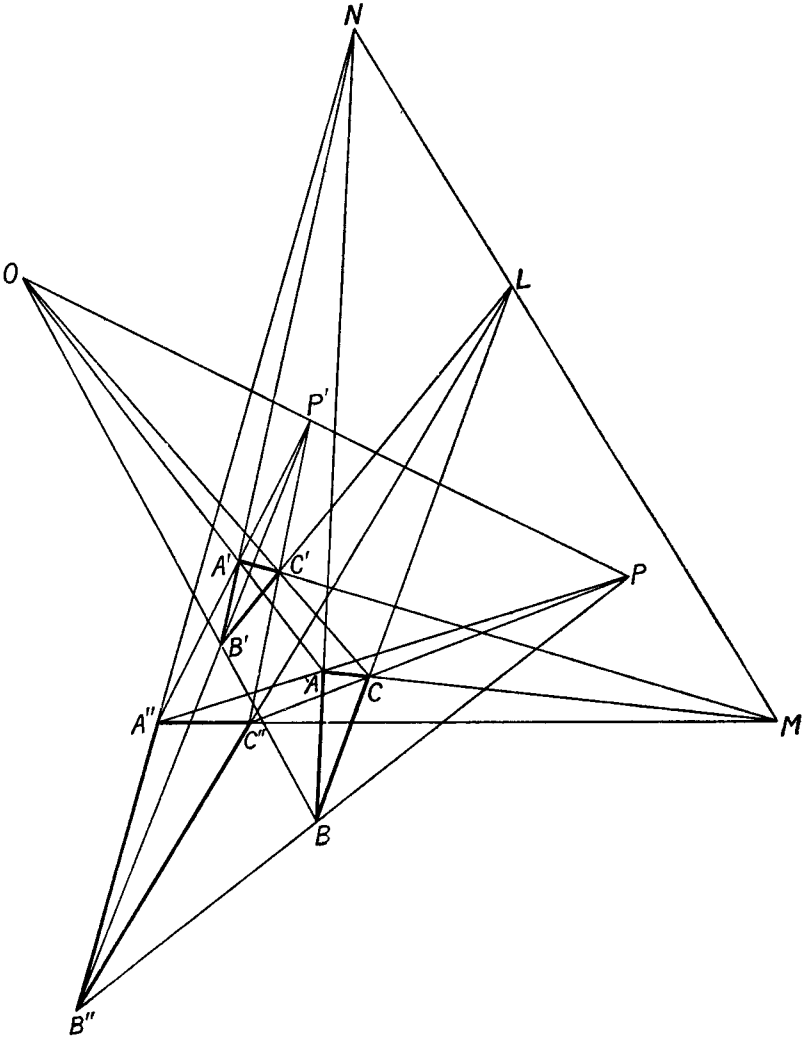
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# PRINCIPLES OF GEOMETRY

BY

H. F. BAKER, Sc.D., F.R.S.,

LOWNDEAN PROFESSOR OF ASTRONOMY AND GEOMETRY, AND FELLOW OF  
ST JOHN'S COLLEGE, IN THE UNIVERSITY OF CAMBRIDGE

VOLUME I  
FOUNDATIONS

*Amplissima est et pulcherrima  
scientia figurarum*

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## PREFACE

THE volumes of which this is the first have the purpose of introducing the reader to those parts of geometry which precede the theory of higher plane curves and of irrational surfaces. The present volume is devoted to the indispensable logical preliminaries. It assumes only those relations of position, for points, lines and planes, which, furnished with a pencil, a ruler, some rods and some string, a student may learn by drawing diagrams and making models. It seeks to set these relations in an ordered framework of deduction, gradually rendered comprehensive and precise enough to include all the subsequent theory; to this end it puts aside, at first, most of those intricate details which make up the burden of what is generally called elementary geometry. That such a plan can be carried through, thanks to the work of many generations of thinkers, is well enough known; and experience has shewn that many students, especially of the class who look forward to becoming Engineers or Physicists, to whom the geometry of the usual text-books is tiresome, find such a course stimulating and easy, when the matter is properly presented to them. The mathematician who has followed such a course will find that he has no cause to think he has learnt the wrong things. The fundamental theorems in this method of approaching the subject are indeed of Greek origin; only, these are here made to lead to general principles, giving a command of detail unknown to the Greeks. Subsequent volumes will deal, on the basis of the results obtained in this volume, with conics (and circles), with quadric surfaces and cubic curves in space, and with cubic surfaces and certain quartic surfaces. These volumes are ready to print; it is hoped that they may appear in no long time.

Speaking in more detail of the present volume, it rejects the consideration of distance, and of congruence, as fundamental ideas; these are, in effect, replaced by a theory of related ranges; the geometry usually described with the help of the notion of distance appears later, in a more general, but not more difficult, form. By what means it is possible, so to dispense with this notion, should be of interest to others than the student of geometry. An account is given, however, of the consequences of accepting as fundamental the continuity of the real points of the line. As it is necessary to provide for the consideration of the so-called imaginary elements, room has been given to the justification of these; the ideas to which they lead are indeed an essential part of the power which belongs to the point

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*Preface*

of view adopted. The references to the theory of space of more than three dimensions are also vital to the plan of the work ; it will be seen later with what simplicity this theory enables us to deal with curves and surfaces whose properties can otherwise be developed only at great length, with complicated analysis. The geometrical theory is accompanied by an algebraic symbolism, which serves to help to fix ideas, and for purposes of verification ; it is necessary also to include the proof that this symbolism is appropriate to the purpose. It is held, however, that the geometrical argument should be complete in itself, independently of the symbols ; a geometry should have such a comprehensive grasp of geometrical relations that all its results are clear by consideration of the geometrical entities alone.

The writer is under great obligations to Mr J. B. Peace, M.A., for his interest in the work, and to the Staff of the University Press for their ready cooperation, especially at this time of difficulty in the production of books.

H. F. B.

26 *September* 1921

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