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James Clerk Maxwell

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A TREATISE

ON

ELECTRICITY AND MAGNETISM

*MAXWELL*

VOL. II.

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Clarendon Press Series

A TREATISE  
ON  
ELECTRICITY AND MAGNETISM

BY  
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VOL. II

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