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James Clerk Maxwell
Excerpt
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PART III.

MAGNETISM.

CHAPTER I.

ELEMENTARY THEORY OF MAGNETISM.

371.] CERTAIN bodies, as, for instance, the iron ore called load-stone, the earth itself, and pieces of steel which have been subjected to certain treatment, are found to possess the following properties, and are called Magnets.

If, near any part of the earth's surface except the Magnetic Poles, a magnet be suspended so as to turn freely about a vertical axis, it will in general tend to set itself in a certain azimuth, and if disturbed from this position it will oscillate about it. An unmagnetized body has no such tendency, but is in equilibrium in all azimuths alike.

372.] It is found that the force which acts on the body tends to cause a certain line in the body, called the Axis of the Magnet, to become parallel to a certain line in space, called the Direction of the Magnetic Force.

Let us suppose the magnet suspended so as to be free to turn in all directions about a fixed point. To eliminate the action of its weight we may suppose this point to be its centre of gravity. Let it come to a position of equilibrium. Mark two points on the magnet, and note their positions in space. Then let the magnet be placed in a new position of equilibrium, and note the positions in space of the two marked points on the magnet.

Since the axis of the magnet coincides with the direction of magnetic force in both positions, we have to find that line in the magnet which occupies the same position in space before and after the motion. It appears, from the theory of the motion of bodies of invariable form, that such a line always exists, and that a motion equivalent to the actual motion might have taken place by simple rotation round this line.

To find the line, join the first and last positions of each of the marked points, and draw planes bisecting these lines at right angles. The intersection of these planes will be the line required, which indicates the direction of the axis of the magnet and the direction of the magnetic force in space.

The method just described is not convenient for the practical determination of these directions. We shall return to this subject when we treat of Magnetic Measurements.

The direction of the magnetic force is found to be different at different parts of the earth's surface. If the end of the axis of the magnet which points in a northerly direction be marked, it has been found that the direction in which it sets itself in general deviates from the true meridian to a considerable extent, and that the marked end points on the whole downwards in the northern hemisphere and upwards in the southern.

The azimuth of the direction of the magnetic force, measured from the true north in a westerly direction, is called the Variation, or the Magnetic Declination. The angle between the direction of the magnetic force and the horizontal plane is called the Magnetic Dip. These two angles determine the direction of the magnetic force, and, when the magnetic intensity is also known, the magnetic force is completely determined. The determination of the values of these three elements at different parts of the earth's surface, the discussion of the manner in which they vary according to the place and time of observation, and the investigation of the causes of the magnetic force and its variations, constitute the science of Terrestrial Magnetism.

373.] Let us now suppose that the axes of several magnets have been determined, and the end of each which points north marked. Then, if one of these be freely suspended and another brought near it, it is found that two marked ends repel each other, that a marked and an unmarked end attract each other, and that two unmarked ends repel each other.

If the magnets are in the form of long rods or wires, uniformly and longitudinally magnetized, see below, Art. 384, it is found that the greatest manifestation of force occurs when the end of one magnet is held near the end of the other, and that the phenomena can be accounted for by supposing that like ends of the magnets repel each other, that unlike ends attract each other, and that the intermediate parts of the magnets have no sensible mutual action.

The ends of a long thin magnet are commonly called its Poles. In the case of an indefinitely thin magnet, uniformly magnetized throughout its length, the extremities act as centres of force, and the rest of the magnet appears devoid of magnetic action. In all actual magnets the magnetization deviates from uniformity, so that no single points can be taken as the poles. Coulomb, however, by using long thin rods magnetized with care, succeeded in establishing the law of force between two magnetic poles *.

The repulsion between two magnetic poles is in the straight line joining them, and is numerically equal to the product of the strengths of the poles divided by the square of the distance between them.

374.] This law, of course, assumes that the strength of each pole is measured in terms of a certain unit, the magnitude of which may be deduced from the terms of the law.

The unit-pole is a pole which points north, and is such that, when placed at unit distance from another unit-pole, it repels it with unit of force, the unit of force being defined as in Art. 6. A pole which points south is reckoned negative.

If m_1 and m_2 are the strengths of two magnetic poles, l the distance between them, and f the force of repulsion, all expressed numerically, then
$$f = \frac{m_1 m_2}{l^2}.$$

But if $[m]$, $[L]$ and $[F]$ be the concrete units of magnetic pole, length and force, then
$$f[F] = \left[\frac{m}{L}\right]^2 \frac{m_1 m_2}{l^2},$$
whence it follows that
$$[m^2] = [L^2 F] = \left[L^2 \frac{ML}{T^2}\right],$$
$$\text{or} \qquad [m] = [L^{\frac{3}{2}} T^{-1} M^{\frac{1}{2}}].$$
The dimensions of the unit pole are therefore $\frac{3}{2}$ as regards length, (-1) as regards time, and $\frac{1}{2}$ as regards mass. These dimensions are the same as those of the electrostatic unit of electricity, which is specified in exactly the same way in Arts. 41, 42.

* His experiments on magnetism with the Torsion Balance are contained in the *Memoirs of the Academy of Paris*, 1780-9, and in Biot's *Traité de Physique*, tom. iii.

375.] The accuracy of this law may be considered to have been established by the experiments of Coulomb with the Torsion Balance, and confirmed by the experiments of Gauss and Weber, and of all observers in magnetic observatories, who are every day making measurements of magnetic quantities, and who obtain results which would be inconsistent with each other if the law of force had been erroneously assumed. It derives additional support from its consistency with the laws of electromagnetic phenomena.

376.] The quantity which we have hitherto called the strength of a pole may also be called a quantity of 'Magnetism,' provided we attribute no properties to 'Magnetism' except those observed in the poles of magnets.

Since the expression of the law of force between given quantities of 'Magnetism' has exactly the same mathematical form as the law of force between quantities of 'Electricity' of equal numerical value, much of the mathematical treatment of magnetism must be similar to that of electricity. There are, however, other properties of magnets which must be borne in mind, and which may throw some light on the electrical properties of bodies.

Relation between the Poles of a Magnet.

377.] The quantity of magnetism at one pole of a magnet is always equal and opposite to that at the other, or more generally thus :—

In every Magnet the total quantity of Magnetism (reckoned algebraically) *is zero.*

Hence in a field of force which is uniform and parallel throughout the space occupied by the magnet, the force acting on the marked end of the magnet is exactly equal, opposite and parallel to that on the unmarked end, so that the resultant of the forces is a statical couple, tending to place the axis of the magnet in a determinate direction, but not to move the magnet as a whole in any direction.

This may be easily proved by putting the magnet into a small vessel and floating it in water. The vessel will turn in a certain direction, so as to bring the axis of the magnet as near as possible to the direction of the earth's magnetic force, but there will be no motion of the vessel as a whole in any direction ; so that there can be no excess of the force towards the north over that towards the south, or the reverse. It may also be shewn from the fact that magnetizing a piece of steel does not alter its weight. It does alter the apparent position of its centre of gravity, causing it in these latitudes to shift along the axis towards the north. The centre of inertia, as determined by the phenomena of rotation, remains unaltered.

378.] If the middle of a long thin magnet be examined, it is found to possess no magnetic properties, but if the magnet be broken at that point, each of the pieces is found to have a magnetic pole at the place of fracture, and this new pole is exactly equal and opposite to the other pole belonging to that piece. It is impossible, either by magnetization, or by breaking magnets, or by any other means, to procure a magnet whose poles are unequal.

If we break the long thin magnet into a number of short pieces we shall obtain a series of short magnets, each of which has poles of nearly the same strength as those of the original long magnet. This multiplication of poles is not necessarily a creation of energy, for we must remember that after breaking the magnet we have to do work to separate the parts, in consequence of their attraction for one another.

379.] Let us now put all the pieces of the magnet together as at first. At each point of junction there will be two poles exactly equal and of opposite kinds, placed in contact, so that their united action on any other pole will be null. The magnet, thus rebuilt, has therefore the same properties as at first, namely two poles, one at each end, equal and opposite to each other, and the part between these poles exhibits no magnetic action.

Since, in this case, we know the long magnet to be made up of little short magnets, and since the phenomena are the same as in the case of the unbroken magnet, we may regard the magnet, even before being broken, as made up of small particles, each of which has two equal and opposite poles. If we suppose all magnets to be made up of such particles, it is evident that since the algebraical quantity of magnetism in each particle is zero, the quantity in the whole magnet will also be zero, or in other words, its poles will be of equal strength but of opposite kind.

Theory of Magnetic 'Matter.'

380.] Since the form of the law of magnetic action is identical with that of electric action, the same reasons which can be given for attributing electric phenomena to the action of one 'fluid' or two 'fluids' can also be used in favour of the existence of a magnetic matter, or of two kinds of magnetic matter, fluid or otherwise. In fact, a theory of magnetic matter, if used in a purely mathematical sense, cannot fail to explain the phenomena, provided new laws are freely introduced to account for the actual facts.

One of these new laws must be that the magnetic fluids cannot pass from one molecule or particle of the magnet to another, but that the process of magnetization consists in separating to a certain extent the two fluids within each particle, and causing the one fluid to be more concentrated at one end, and the other fluid to be more concentrated at the other end of the particle. This is the theory of Poisson.

A particle of a magnetizable body is, on this theory, analogous to a small insulated conductor without charge, which on the two-fluid theory contains indefinitely large but exactly equal quantities of the two electricities. When an electromotive force acts on the conductor, it separates the electricities, causing them to become manifest at opposite sides of the conductor. In a similar manner, according to this theory, the magnetizing force causes the two kinds of magnetism, which were originally in a neutralized state, to be separated, and to appear at opposite sides of the magnetized particle.

In certain substances, such as soft iron and those magnetic substances which cannot be permanently magnetized, this magnetic condition, like the electrification of the conductor, disappears when the inducing force is removed. In other substances, such as hard steel, the magnetic condition is produced with difficulty, and, when produced, remains after the removal of the inducing force.

This is expressed by saying that in the latter case there is a Coercive Force, tending to prevent alteration in the magnetization, which must be overcome before the power of a magnet can be either increased or diminished. In the case of the electrified body this would correspond to a kind of electric resistance, which, unlike the resistance observed in metals, would be equivalent to complete insulation for electromotive forces below a certain value.

This theory of magnetism, like the corresponding theory of electricity, is evidently too large for the facts, and requires to be restricted by artificial conditions. For it not only gives no reason why one body may not differ from another on account of having more of both fluids, but it enables us to say what would be the properties of a body containing an excess of one magnetic fluid. It is true that a reason is given why such a body cannot exist, but this reason is only introduced as an after-thought to explain this particular fact. It does not grow out of the theory.

381.] We must therefore seek for a mode of expression which shall not be capable of expressing too much, and which shall leave room for the introduction of new ideas as these are developed from new facts. This, I think, we shall obtain if we begin by saying that the particles of a magnet are Polarized.

Meaning of the term 'Polarization.'

When a particle of a body possesses properties related to a certain line or direction in the body, and when the body, retaining these properties, is turned so that this direction is reversed, then if as regards other bodies these properties of the particle are reversed, the particle, in reference to these properties, is said to be polarized, and the properties are said to constitute a particular kind of polarization.

Thus we may say that the rotation of a body about an axis constitutes a kind of polarization, because if, while the rotation continues, the direction of the axis is turned end for end, the body will be rotating in the opposite direction as regards space.

A conducting particle through which there is a current of electricity may be said to be polarized, because if it were turned round, and if the current continued to flow in the same direction as regards the particle, its direction in space would be reversed.

In short, if any mathematical or physical quantity is of the nature of a vector, as defined in Art. 11, then any body or particle to which this directed quantity or vector belongs may be said to be Polarized*, because it has opposite properties in the two opposite directions or poles of the directed quantity.

The poles of the earth, for example, have reference to its rotation, and have accordingly different names.

* The word Polarization has been used in a sense not consistent with this in Optics, where a ray of light is said to be polarized when it has properties relating to its sides, which are identical on opposite sides of the ray. This kind of polarization refers to another kind of Directed Quantity, which may be called a Dipolar Quantity, in opposition to the former kind, which may be called Unipolar.

When a dipolar quantity is turned end for end it remains the same as before. Tensions and Pressures in solid bodies, Extensions, Compressions and Distortions and most of the optical, electrical, and magnetic properties of crystallized bodies are dipolar quantities.

The property produced by magnetism in transparent bodies of twisting the plane of polarization of the incident light, is, like magnetism itself, a unipolar property. The rotatory property referred to in Art. 303 is also unipolar.

Meaning of the term 'Magnetic Polarization.'

382.] In speaking of the state of the particles of a magnet as magnetic polarization, we imply that each of the smallest parts into which a magnet may be divided has certain properties related to a definite direction through the particle, called its Axis of Magnetization, and that the properties related to one end of this axis are opposite to the properties related to the other end.

The properties which we attribute to the particle are of the same kind as those which we observe in the complete magnet, and in assuming that the particles possess these properties, we only assert what we can prove by breaking the magnet up into small pieces, for each of these is found to be a magnet.

Properties of a Magnetized Particle.

383.] Let the element $dx\,dy\,dz$ be a particle of a magnet, and let us assume that its magnetic properties are those of a magnet the strength of whose positive pole is m, and whose length is ds. Then if P is any point in space distant r from the positive pole and r' from the negative pole, the magnetic potential at P will be $\frac{m}{r}$ due to the positive pole, and $-\frac{m}{r'}$ due to the negative pole, or

$$V = \frac{m}{rr'}(r'-r). \tag{1}$$

If ds, the distance between the poles, is very small, we may put

$$r'-r = ds\cos\epsilon, \tag{2}$$

where ϵ is the angle between the vector drawn from the magnet to P and the axis of the magnet, or

$$V = \frac{m\,ds}{r^2}\cos\epsilon. \tag{3}$$

Magnetic Moment.

384.] The product of the length of a uniformly and longitudinally magnetized bar magnet into the strength of its positive pole is called its Magnetic Moment.

Intensity of Magnetization.

The intensity of magnetization of a magnetic particle is the ratio of its magnetic moment to its volume. We shall denote it by I.

The magnetization at any point of a magnet may be defined by its intensity and its direction. Its direction may be defined by its direction-cosines λ, μ, ν.

Components of Magnetization.

The magnetization at a point of a magnet (being a vector or directed quantity) may be expressed in terms of its three components referred to the axes of coordinates. Calling these A, B, C,

$$A = I\lambda, \qquad B = I\mu, \qquad C = I\nu,$$

and the numerical value of I is given by the equation (4)

$$I^2 = A^2 + B^2 + C^2. \tag{5}$$

385.] If the portion of the magnet which we consider is the differential element of volume $dx\,dy\,dz$, and if I denotes the intensity of magnetization of this element, its magnetic moment is $I\,dx\,dy\,dz$. Substituting this for $m\,ds$ in equation (3), and remembering that

$$r\cos\epsilon = \lambda(\xi - x) + \mu(\eta - y) + \nu(\zeta - z), \tag{6}$$

where ξ, η, ζ are the coordinates of the extremity of the vector r drawn from the point (x, y, z), we find for the potential at the point (ξ, η, ζ) due to the magnetized element at (x, y, z),

$$\delta V = \{A(\xi - x) + B(\eta - y) + C(\zeta - z)\}\frac{1}{r^3}\,dx\,dy\,dz. \tag{7}$$

To obtain the potential at the point (ξ, η, ζ) due to a magnet of finite dimensions, we must find the integral of this expression for every element of volume included within the space occupied by the magnet, or

$$V = \iiint \{A(\xi - x) + B(\eta - y) + C(\zeta - z)\}\frac{1}{r^3}\,dx\,dy\,dz. \tag{8}$$

Integrating by parts, this becomes

$$V = \iint A\frac{1}{r}\,dy\,dz + \iint B\frac{1}{r}\,dz\,dx + \iint C\frac{1}{r}\,dx\,dy$$
$$-\iiint \frac{1}{r}\left(\frac{dA}{dx} + \frac{dB}{dy} + \frac{dC}{dz}\right)dx\,dy\,dz,$$

where the double integration in the first three terms refers to the surface of the magnet, and the triple integration in the fourth to the space within it.

If l, m, n denote the direction-cosines of the normal drawn outwards from the element of surface dS, we may write, as in Art. 21, the sum of the first three terms,

$$\iint (lA + mB + nC)\frac{1}{r}\,dS,$$

where the integration is to be extended over the whole surface of the magnet.

If we now introduce two new symbols σ and ρ, defined by the equations

$$\sigma = lA + mB + nC,$$

$$\rho = -\left(\frac{dA}{dx} + \frac{dB}{dy} + \frac{dC}{dz}\right),$$

the expression for the potential may be written

$$V = \iint \frac{\sigma}{r}\, dS + \iiint \frac{\rho}{r}\, dx\, dy\, dz.$$

386.] This expression is identical with that for the electric potential due to a body on the surface of which there is an electrification whose surface-density is σ, while throughout its substance there is a bodily electrification whose volume-density is ρ. Hence, if we assume σ and ρ to be the surface- and volume-densities of the distribution of an imaginary substance, which we have called 'magnetic matter,' the potential due to this imaginary distribution will be identical with that due to the actual magnetization of every element of the magnet.

The surface-density σ is the resolved part of the intensity of magnetization I in the direction of the normal to the surface drawn outwards, and the volume-density ρ is the 'convergence' (see Art. 25) of the magnetization at a given point in the magnet.

This method of representing the action of a magnet as due to a distribution of 'magnetic matter' is very convenient, but we must always remember that it is only an artificial method of representing the action of a system of polarized particles.

On the Action of one Magnetic Molecule on another.

387.] If, as in the chapter on Spherical Harmonics, Art. 129, we make

$$\frac{d}{dh} = l\frac{d}{dx} + m\frac{d}{dy} + n\frac{d}{dz}, \tag{1}$$

where l, m, n are the direction-cosines of the axis h, then the potential due to a magnetic molecule at the origin, whose axis is parallel to h_1, and whose magnetic moment is m_1, is

$$V_1 = -\frac{d}{dh_1}\frac{m_1}{r} = \frac{m_1}{r^2}\lambda_1, \tag{2}$$

where λ_1 is the cosine of the angle between h_1 and r.

Again, if a second magnetic molecule whose moment is m_2, and whose axis is parallel to h_2, is placed at the extremity of the radius vector r, the potential energy due to the action of the one magnet on the other is