CONTENTS.

PRELIMINARY.

ON THE MEASUREMENT OF QUANTITIES.

Art.		Page
1.	The expression of a quantity consists of two factors, the numerical value, and the name of the concrete unit	1
2.	Dimensions of derived units	1
3–5.	The three fundamental units—Length, Time and Mass	2, 3
6.	Derived units	5
7.	Physical continuity and discontinuity	6
8.	Discontinuity of a function of more than one variable	7
9.	Periodic and multiple functions	8
10.	Relation of physical quantities to directions in space	8
11.	Meaning of the words Scalar and Vector	9
12.	Division of physical vectors into two classes, Forces and Fluxes	10
13.	Relation between corresponding vectors of the two classes	11
14.	Line-integration appropriate to forces, surface-integration to fluxes	12
15.	Longitudinal and rotational vectors	12
16.	Line-integrals and potentials	13
17.	Hamilton's expression for the relation between a force and its potential	15
18.	Cyclic regions and geometry of position	16
19.	The potential in an acyclic region is single valued	17
20.	System of values of the potential in a cyclic region	18
21.	Surface-integrals	19
22.	Surfaces, tubes, and lines of flow	21
23.	Right-handed and left-handed relations in space	24
24.	Transformation of a line-integral into a surface-integral	25
25.	Effect of Hamilton's operation ∇ on a vector function	27
26.	Nature of the operation ∇^2	29

PART I.

ELECTROSTATICS.

CHAPTER I.

DESCRIPTION OF PHENOMENA.

Art.		Page
27.	Electrification by friction. Electrification is of two kinds, to which the names of Vitreous and Resinous, or Positive and Negative, have been given	30
28.	Electrification by induction	31
29.	Electrification by conduction. Conductors and insulators	32
30.	In electrification by friction the quantity of the positive electrification is equal to that of the negative electrification	33
31.	To charge a vessel with a quantity of electricity equal and opposite to that of an excited body	33
32.	To discharge a conductor completely into a metallic vessel	34
33.	Test of electrification by gold-leaf electroscope	34
34.	Electrification, considered as a measurable quantity, may be called Electricity	35
35.	Electricity may be treated as a physical quantity	36
36.	Theory of Two fluids	37
37.	Theory of One fluid	39
38.	Measurement of the force between electrified bodies	40
39.	Relation between this force and the quantities of electricity	41
40.	Variation of the force with the distance	42
41, 42.	Definition of the electrostatic unit of electricity. — Its dimensions	42
43.	Proof of the law of electric force	43
44.	Electric field	44
45.	Electric potential	45
46.	Equipotential surfaces. Example of their use in reasoning about electricity	45
47.	Lines of force	47
48.	Electric tension	47
49.	Electromotive force	47
50.	Capacity of a conductor	48
51.	Properties of bodies.—Resistance	48

Art. Page
52. Specific Inductive capacity of a dielectric 50
53. 'Absorption' of electricity 50
54. Impossibility of an absolute charge 51
55. Disruptive discharge.—Glow 52
56. Brush 54
57. Spark 55
58. Electrical phenomena of Tourmaline 56
59. Plan of the treatise, and sketch of its results 57
60. Electric polarization and displacement 59
61. The motion of electricity analogous to that of an incompressible fluid 62
62. Peculiarities of the theory of this treatise 62

CHAPTER II.

ELEMENTARY MATHEMATICAL THEORY OF ELECTRICITY.

63. Definition of electricity as a mathematical quantity 66
64. Volume-density, surface-density, and line-density 67
65. Definition of the electrostatic unit of electricity 68
66. Law of force between electrified bodies 69
67. Resultant force between two bodies 69
68. Resultant force at a point 69
69. Line-integral of electric force; electromotive force 71
70. Electric potential 72
71. Resultant force in terms of the potential 72
72. The potential of all points of a conductor is the same 73
73. Potential due to an electrified system 74
74. Proof of the law of the inverse square 74
75. Surface-integral of electric induction 77
76. Introduction through a closed surface due to a single centre of force 77
77. Poisson's extension of Laplace's equation 79
78. Conditions to be fulfilled at an electrified surface 80
79. Resultant force on an electrified surface 82
80. The electrification of a conductor is entirely on the surface .. 83
81. A distribution of electricity on lines or points is physically impossible 84
82. Lines of electric induction 84
83. Specific inductive capacity 86

CHAPTER III.

SYSTEMS OF CONDUCTORS.

Art. Page
84. On the superposition of electrified systems 88
85. Energy of an electrified system 88
86. General theory of a system of conductors. Coefficients of potential 89
87. Coefficients of induction. Capacity of a conductor. Dimensions of these coefficients 90
88. Reciprocal property of the coefficients 91
89. A theorem due to Green 92
90. Relative magnitude of the coefficients of potential 92
91. And of induction 93
92. The resultant mechanical force on a conductor expressed in terms of the charges of the different conductors of the system and the variation of the coefficients of potential 94
93. The same in terms of the potentials, and the variation of the coefficients of induction 94
94. Comparison of electrified systems 96

CHAPTER IV.

GENERAL THEOREMS.

95. Two opposite methods of treating electrical questions 98
96. Characteristics of the potential function 99
97. Conditions under which the volume-integral
$$\iiint \left(u\frac{dV}{dx} + v\frac{dV}{dy} + w\frac{dV}{dz}\right) dx\,dy\,dz$$
vanishes 100
98. Thomson's theorem of the unique minimum of
$$\iiint \frac{1}{K}(a^2 + b^2 + c^2)\,dx\,dy\,dz \quad .. \quad .. \quad .. \quad .. \quad 103$$
99. Application of the theorem to the determination of the distribution of electricity 107
100. Green's theorem and its physical interpretation 108
101. Green's functions 113
102. Method of finding limiting values of electrical coefficients .. 115

CHAPTER V.

MECHANICAL ACTION BETWEEN ELECTRIFIED BODIES.

Art. Page
103. Comparison of the force between different electrified systems .. 119
104. Mechanical action on an element of an electrified surface .. 121
105. Comparison between theories of direct action and theories of stress 122
106. The kind of stress required to account for the phenomenon .. 123
107. The hypothesis of stress considered as a step in electrical science 126
108. The hypothesis of stress shewn to account for the equilibrium of the medium and for the forces acting between electrified bodies 128
109. Statements of Faraday relative to the longitudinal tension and lateral pressure of the lines of force 131
110. Objections to stress in a fluid considered 131
111. Statement of the theory of electric polarization 132

CHAPTER VI.

POINTS AND LINES OF EQUILIBRIUM.

112. Conditions of a point of equilibrium 135
113. Number of points of equilibrium 136
114. At a point or line of equilibrium there is a conical point or a line of self-intersection of the equipotential surface 137
115. Angles at which an equipotential surface intersects itself .. 138
116. The equilibrium of an electrified body cannot be stable 139

CHAPTER VII.

FORMS OF EQUIPOTENTIAL SURFACES AND LINES OF FLOW.

117. Practical importance of a knowledge of these forms in simple cases 142
118. Two electrified points, ratio 4 : 1. (Fig. I) 143
119. Two electrified points, ratio 4 : −1. (Fig. II) 144
120. An electrified point in a uniform field of force. (Fig. III) .. 145
121. Three electrified points. Two spherical equipotential surfaces. (Fig. IV) 145
122. Faraday's use of the conception of lines of force 146
123. Method employed in drawing the diagrams 147

CHAPTER VIII.

SIMPLE CASES OF ELECTRIFICATION.

Art. Page
124. Two parallel planes 150
125. Two concentric spherical surfaces 152
126. Two coaxal cylindric surfaces 154
127. Longitudinal force on a cylinder, the ends of which are surrounded by cylinders at different potentials 155

CHAPTER IX.

SPHERICAL HARMONICS.

128. Singular points at which the potential becomes infinite 157
129. Singular points of different orders defined by their axes 158
130. Expression for the potential due to a singular point referred to its axes 160
131. This expression is perfectly definite and represents the most general type of the harmonic of i degrees 162
132. The zonal, tesseral, and sectorial types 163
133. Solid harmonics of positive degree. Their relation to those of negative degree 165
134. Application to the theory of electrified spherical surfaces .. 166
135. The external action of an electrified spherical surface compared with that of an imaginary singular point at its centre 167
136. Proof that if Y_i and Y_j are two surface harmonics of different degrees, the surface-integral $\iint Y_i\, Y_j\, dS = 0$, the integration being extended over the spherical surface 169
137. Value of $\iint Y_i\, Y_j\, dS$ where Y_i and Y_j are surface harmonics of the same degree but of different types 169
138. On conjugate harmonics 170
139. If Y_j is the zonal harmonic and Y_i any other type of the same degree
$$\iint Y_i\, Y_j\, dS = \frac{4\pi a^2}{2i+1}\, Y_{i(j)}$$
where $Y_{i(j)}$ is the value of Y_i at the pole of Y_j 171
140. Development of a function in terms of spherical surface harmonics 172
141. Surface-integral of the square of a symmetrical harmonic .. 173

Art. Page
142. Different methods of treating spherical harmonics 174
143. On the diagrams of spherical harmonics. (Figs. V, VI, VII, VIII, IX) 175
144. If the potential is constant throughout any finite portion of space it is so throughout the whole region continuous with it within which Laplace's equation is satisfied 176
145. To analyse a spherical harmonic into a system of conjugate harmonics by means of a finite number of measurements at selected points of the sphere 177
146. Application to spherical and nearly spherical conductors .. 178

CHAPTER X.

CONFOCAL SURFACES OF THE SECOND DEGREE.

147. The lines of intersection of two systems and their intercepts by the third system 181
148. The characteristic equation of V in terms of ellipsoidal co-ordinates 182
149. Expression of α, β, γ in terms of elliptic functions 183
150. Particular solutions of electrical distribution on the confocal surfaces and their limiting forms 184
151. Continuous transformation into a figure of revolution about the axis of z 187
152. Transformation into a figure of revolution about the axis of x 188
153. Transformation into a system of cones and spheres 189
154. Confocal paraboloids 189

CHAPTER XI.

THEORY OF ELECTRIC IMAGES.

155. Thomson's method of electric images 191
156. When two points are oppositely and unequally electrified, the surface for which the potential is zero is a sphere 192
157. Electric images 193
158. Distribution of electricity on the surface of the sphere 195
159. Image of any given distribution of electricity 196
160. Resultant force between an electrified point and sphere 197
161. Images in an infinite plane conducting surface 198
162. Electric inversion 199
163. Geometrical theorems about inversion 201
164. Application of the method to the problem of Art. 158 202

Art. Page

165. Finite systems of successive images 203
166. Case of two spherical surfaces intersecting at an angle $\frac{\pi}{n}$ .. 204
167. Enumeration of the cases in which the number of images is finite 206
168. Case of two spheres intersecting orthogonally 207
169. Case of three spheres intersecting orthogonally 210
170. Case of four spheres intersecting orthogonally 211
171. Infinite series of images. Case of two concentric spheres .. 212
172. Any two spheres not intersecting each other 213
173. Calculation of the coefficients of capacity and induction 216
174. Calculation of the charges of the spheres, and of the force between them 217
175. Distribution of electricity on two spheres in contact. Proof sphere 219
176. Thomson's investigation of an electrified spherical bowl 221
177. Distribution on an ellipsoid, and on a circular disk at potential V 221
178. Induction on an uninsulated disk or bowl by an electrified point in the continuation of the plane or spherical surface .. 222
179. The rest of the sphere supposed uniformly electrified 223
180. The bowl maintained at potential V and uninfluenced 223
181. Induction on the bowl due to a point placed anywhere 224

CHAPTER XII.

CONJUGATE FUNCTIONS IN TWO DIMENSIONS.

182. Cases in which the quantities are functions of x and y only .. 226
183. Conjugate functions 227
184. Conjugate functions may be added or subtracted 228
185. Conjugate functions of conjugate functions are themselves conjugate 229
186. Transformation of Poisson's equation 231
187. Additional theorems on conjugate functions 232
188. Inversion in two dimensions 232
189. Electric images in two dimensions 233
190. Neumann's transformation of this case 234
191. Distribution of electricity near the edge of a conductor formed by two plane surfaces 236
192. Ellipses and hyperbolas. (Fig. X) 237
193. Transformation of this case. (Fig. XI) 238

Art. Page
194. Application to two cases of the flow of electricity in a conducting sheet 239
195. Application to two cases of electrical induction 239
196. Capacity of a condenser consisting of a circular disk between two infinite planes 240
197. Case of a series of equidistant planes cut off by a plane at right angles to them 242
198. Case of a furrowed surface 243
199. Case of a single straight groove 243
200. Modification of the results when the groove is circular 244
201. Application to Sir W. Thomson's guard-ring 245
202. Case of two parallel plates cut off by a perpendicular plane. (Fig. XII) 246
203. Case of a grating of parallel wires. (Fig. XIII) 248
204. Case of a single electrified wire transformed into that of the grating 248
205. The grating used as a shield to protect a body from electrical influence 249
206. Method of approximation applied to the case of the grating .. 251

CHAPTER XIII.

ELECTROSTATIC INSTRUMENTS.

207. The frictional electrical machine 254
208. The electrophorus of Volta 255
209. Production of electrification by mechanical work.—Nicholson's Revolving Doubler 256
210. Principle of Varley's and Thomson's electrical machines 256
211. Thomson's water-dropping machine 259
212. Holtz's electrical machine 260
213. Theory of regenerators applied to electrical machines 260
214. On electrometers and electroscopes. Indicating instruments and null methods. Difference between registration and measurement 262
215. Coulomb's Torsion Balance for measuring charges 263
216. Electrometers for measuring potentials. Snow Harris's and Thomson's 266
217. Principle of the guard-ring. Thomson's Absolute Electrometer 267
218. Heterostatic method 269
219. Self-acting electrometers.—Thomson's Quadrant Electrometer 271
220. Measurement of the electric potential of a small body 274
221. Measurement of the potential at a point in the air 275

Art. Page
222. Measurement of the potential of a conductor without touching it 276
223. Measurement of the superficial density of electrification. The proof plane 277
224. A hemisphere used as a test 278
225. A circular disk 279
226. On electric accumulators. The Leyden jar 281
227. Accumulators of measurable capacity 282
228. The guard-ring accumulator 283
229. Comparison of the capacities of accumulators 285

PART II.

ELECTROKINEMATICS.

CHAPTER I.

THE ELECTRIC CURRENT.

230. Current produced when conductors are discharged 288
231. Transference of electrification 288
232. Description of the voltaic battery 289
233. Electromotive force 290
234. Production of a steady current 290
235. Properties of the current 291
236. Electrolytic action 291
237. Explanation of terms connected with electrolysis 292
238. Different modes of passage of the current 292
239. Magnetic action of the current 293
240. The Galvanometer 294

CHAPTER II.

CONDUCTION AND RESISTANCE.

241. Ohm's Law 295
242. Generation of heat by the current. Joule's Law 296
243. Analogy between the conduction of electricity and that of heat 297
244. Differences between the two classes of phenomena 297
245. Faraday's doctrine of the impossibility of an absolute charge .. 298

CHAPTER III.

ELECTROMOTIVE FORCE BETWEEN BODIES IN CONTACT.

Art. Page
246. Volta's law of the contact force between different metals at the same temperature 299
247. Effect of electrolytes 300
248. Thomson's voltaic current in which gravity performs the part of chemical action 300
249. Peltier's phenomenon. Deduction of the thermoelectric electromotive force at a junction 300
250. Seebeck's discovery of thermoelectric currents 302
251. Magnus's law of a circuit of one metal 302
252. Cumming's discovery of thermoelectric inversions 304
253. Thomson's deductions from these facts, and discovery of the reversible thermal effects of electric currents in copper and in iron 304
254. Tait's law of the electromotive force of a thermoelectric pair .. 305

CHAPTER IV.

ELECTROLYSIS.

255. Faraday's law of electrochemical equivalents 307
256. Clausius's theory of molecular agitation 309
257. Electrolytic polarization 309
258. Test of an electrolyte by polarization 310
259. Difficulties in the theory of electrolysis 310
260. Molecular charges 311
261. Secondary actions observed at the electrodes 313
262. Conservation of energy in electrolysis 315
263. Measurement of chemical affinity as an electromotive force .. 316

CHAPTER V.

ELECTROLYTIC POLARIZATION.

264. Difficulties of applying Ohm's law to electrolytes 318
265. Ohm's law nevertheless applicable 318
266. The effect of polarization distinguished from that of resistance 318
267. Polarization due to the presence of the ions at the electrodes. The ions not in a free state 319
268. Relation between the electromotive force of polarization and the state of the ions at the electrodes 320

Art. Page
269. Dissipation of the ions and loss of polarization 321
270. Limit of polarization 321
271. Ritter's secondary pile compared with the Leyden jar 322
272. Constant voltaic elements.—Daniell's cell 325

CHAPTER VI.

MATHEMATICAL THEORY OF THE DISTRIBUTION OF ELECTRIC CURRENTS.

273. Linear conductors 329
274. Ohm's Law 329
275. Linear conductors in series 329
276. Linear conductors in multiple arc 330
277. Resistance of conductors of uniform section 331
278. Dimensions of the quantities involved in Ohm's law 332
279. Specific resistance and conductivity in electromagnetic measure 333
280. Linear systems of conductors in general 333
281. Reciprocal property of any two conductors of the system .. 335
282. Conjugate conductors 336
283. Heat generated in the system 336
284. The heat is a minimum when the current is distributed according to Ohm's law 337

CHAPTER VII.

CONDUCTION IN THREE DIMENSIONS.

285. Notation 338
286. Composition and resolution of electric currents 338
287. Determination of the quantity which flows through any surface 339
288. Equation of a surface of flow 340
289. Relation between any three systems of surfaces of flow 340
290. Tubes of flow 340
291. Expression for the components of the flow in terms of surfaces of flow 341
292. Simplification of this expression by a proper choice of parameters 341
293. Unit tubes of flow used as a complete method of determining the current 341
294. Current-sheets and current-functions 342
295. Equation of 'continuity' 342
296. Quantity of electricity which flows through a given surface .. 344

CHAPTER VIII.

RESISTANCE AND CONDUCTIVITY IN THREE DIMENSIONS.

Art. Page

297. Equations of resistance 345
298. Equations of conduction 346
299. Rate of generation of heat 346
300. Conditions of stability 347
301. Equation of continuity in a homogeneous medium 348
302. Solution of the equation 348
303. Theory of the coefficient T. It probably does not exist .. 349
304. Generalized form of Thomson's theorem 350
305. Proof without symbols 351
306. Strutt's method applied to a wire of variable section.—Lower limit of the value of the resistance 353
307. Higher limit 356
308. Lower limit for the correction for the ends of the wire 358
309. Higher limit 358

CHAPTER IX.

CONDUCTION THROUGH HETEROGENEOUS MEDIA.

310. Surface-conditions 360
311. Spherical surface 362
312. Spherical shell 363
313. Spherical shell placed in a field of uniform flow 364
314. Medium in which small spheres are uniformly disseminated .. 365
315. Images in a plane surface 366
316. Method of inversion not applicable in three dimensions 367
317. Case of conduction through a stratum bounded by parallel planes 367
318. Infinite series of images. Application to magnetic induction .. 368
319. On stratified conductors. Coefficients of conductivity of a conductor consisting of alternate strata of two different substances 369
320. If neither of the substances has the rotatory property denoted by T the compound conductor is free from it 370
321. If the substances are isotropic the direction of greatest resistance is normal to the strata 371
322. Medium containing parallelepipeds of another medium 371
323. The rotatory property cannot be introduced by means of conducting channels 372
324. Construction of an artificial solid having given coefficients of longitudinal and transverse conductivity 373

CHAPTER X.

CONDUCTION IN DIELECTRICS.

Art. Page
325. In a strictly homogeneous medium there can be no internal charge 374
326. Theory of a condenser in which the dielectric is not a perfect insulator 375
327. No residual charge due to simple conduction 376
328. Theory of a composite accumulator 376
329. Residual charge and electrical absorption 378
330. Total discharge 380
331. Comparison with the conduction of heat 381
332. Theory of telegraph cables and comparison of the equations with those of the conduction of heat 381
333. Opinion of Ohm on this subject 384
334. Mechanical illustration of the properties of a dielectric 385

CHAPTER XI.

MEASUREMENT OF THE ELECTRIC RESISTANCE OF CONDUCTORS.

335. Advantage of using material standards of resistance in electrical measurements 388
336. Different standards which have been used and different systems which have been proposed 388
337. The electromagnetic system of units 389
338. Weber's unit, and the British Association unit or Ohm 389
339. Professed value of the Ohm 10,000,000 metres per second .. 389
340. Reproduction of standards 390
341. Forms of resistance coils 391
342. Coils of great resistance 392
343. Arrangement of coils in series 392
344. Arrangement in multiple arc 393
345. On the comparison of resistances. (1) Ohm's method 394
346. (2) By the differential galvanometer 394
347. (3) By Wheatstone's Bridge 398
348. Estimation of limits of error in the determination 399
349. Best arrangement of the conductors to be compared 400
350. On the use of Wheatstone's Bridge 402
351. Thomson's method for small resistances 404
352. Matthiessen and Hockin's method for small resistances 406

Art. Page
353. Comparison of great resistances by the electrometer 408
354. By accumulation in a condenser 409
355. Direct electrostatic method 409
356. Thomson's method for the resistance of a galvanometer 410
357. Mance's method of determining the resistance of a battery .. 411
358. Comparison of electromotive forces 413

CHAPTER XII.

ELECTRIC RESISTANCE OF SUBSTANCES.

359. Metals, electrolytes, and dielectrics 415
360. Resistance of metals 416
361. Resistance of mercury 417
362. Table of resistance of metals 418
363. Resistance of electrolytes 419
364. Experiments of Paalzow 419
365. Experiments of Kohlrausch and Nippoldt 420
366. Resistance of dielectrics 421
367. Gutta-percha 423
368. Glass 423
369. Gases 424
370. Experiments of Wiedemann and Rühlmann 425