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The Algebra of Invariants

JOHN HILTON GRACE
ALFRED YOUNG

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THE ALGEBRA OF INVARIANTS

BY

J. H. GRACE, M.A.

FELLOW OF PETERHOUSE

AND

A. YOUNG, M.A.

LECTURER IN MATHEMATICS AT SELWYN COLLEGE,
LATE SCHOLAR OF CLARE COLLEGE

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PREFACE.

THE object of this book is to provide an English introduction to the symbolical method in the theory of Invariants. It was started as an attempt to meet the need expressed by Elliott in the preface to *The Algebra of Quantics*—'a whole book which shall present to the English reader in his own language a worthy exposition of the method of the great German masters remains a desideratum.' Since then the need has been partly met by the article 'Algebra' by MacMahon in the Supplement to the *Encyclopædia Britannica*. The subject has been treated from the commencement in order that readers unacquainted with Elliott's treatise or any presentation of the elements may be able to understand the argument. Such readers should bear in mind that this treatise is only concerned with one part of a very extensive subject. The modern theory of Partitions will be found in the first part of the article by MacMahon mentioned above.

The first six chapters—a great portion of which, we hope, will be found easy reading—may be said to lead step by step to Gordan's wonderful proof of the finiteness of the system for a single binary form. The sixth chapter is, in fact, devoted to an exposition of Gordan's third proof, but here, as throughout the book, we have allowed ourselves a free hand in dealing with the memoirs and treatises quoted. For example, we have made much use of Jordan's great memoirs on Invariants in proving Gordan's theorem: in a later chapter on Types of Covariants the development of Jordan's method has led us to some results which we believe

to be important as well as novel, notably to an exact formula for the maximum order of an irreducible covariant of a system of binary forms.

The remainder of the book is mainly of geometrical interest: much space is devoted to Apolarity and Rational Curves, and the treatment of ternary forms is from the geometrical rather than the analytical point of view. The only complete system of ternary forms given is that for two Quadratics: it may be felt that more should have been said on this subject, but we think that with the methods known up to the present the treatment of ternary forms is too tedious for a text-book.

The number of references to Mathematical Journals etc. will perhaps be found unusually small: for this there is no need to apologise since the admirable *Bericht über den gegenwärtigen Stand der Invariantentheorie** of Meyer gives references up to the last few years and in a more complete fashion than is desirable in a book which makes no pretensions to being exhaustive.

We wish to thank Dr H. F. Baker for help given to us in our early reading and Professor Forsyth for encouragement while writing. For reading of proof-sheets we are indebted to Mr J. E. Wright, B.A., of Trinity College, Mr P. W. Wood, B.A., of Emmanuel College, and in a still greater degree to the late Mr A. P. Thompson, B.A., of Pembroke College, whose enthusiasm for Mathematics and research was most helpful and whose early death is deplored alike by his teachers and his fellow-workers. Our thanks are also due to the officials of the University Press for great help received during the course of printing.

J. H. GRACE.
A. YOUNG.

* *Jahresbericht der Deutschen Mathematiker Vereinigung*, Vol. I., 1892. French translation by Fehr; Gauthier-Villars, Paris, 1897. Italian translation by Vivanti; Pellerano, Naples, 1899. Article, *Invariantentheorie* in the *Encyclopädie der mathematischen Wissenschaften*.

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