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Sir James Jeans (1877–1946) is regarded as one of the founders of British cosmology, and was the first to suggest (in 1928) the steady state theory, which assumes a continuous creation of matter in the universe. He made many major contributions over a wide area of mathematical physics, but was also well known as an accessible writer for the non-specialist. Problems of Cosmology and Stellar Dynamics, first published in 1919, is a theoretical prelude to Jeans's later and more mature work on the subject, Astronomy and Cosmogony. The impetus for publishing his theories on the behaviour of rotating masses, and on general dynamical theory, was the 1917 Adams Prize on the ‘rotating and gravitating fluid mass’. Jeans won the prize with the core text of this volume. Enlarging on that work, and utilising the burgeoning results of astronomy, as well as his bolder theoretical conjectures, this book became a solid foundation for substantial progress in cosmology.
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Problems of Cosmology and Stellar Dynamics

James Jeans
PROBLEMS OF COSMOGONY

AND

STEellar DYNAMICS
PROBLEMS OF COSMOGONY
AND
STEellar DYNAMICS

BY

J. H. JEANS, M.A., F.R.S.

FORMERLY STOKES LECTURER IN APPLIED MATHEMATICS IN THE UNIVERSITY OF CAMBRIDGE;
SOMETIME PROFESSOR OF APPLIED MATHEMATICS IN PRINCETON UNIVERSITY

BEING AN ESSAY TO WHICH THE ADAMS PRIZE
OF THE UNIVERSITY OF CAMBRIDGE FOR
THE YEAR 1917 WAS ADJUDGED

CAMBRIDGE
AT THE UNIVERSITY PRESS
1919
PREFACE

The present essay is primarily an attempt to follow up a line of research initiated by Laplace and Maclaurin, and extended in various directions by Roche, Lord Kelvin, Jacobi, Poincaré and Sir G. Darwin. Within two years of the close of his life, Darwin remarked that the way to further progress in cosmogony was blocked by our ignorance of the figures of equilibrium of rotating gaseous masses. He wrote as follows (Darwin and Modern Science, p. 563, and Tides, 3rd edition, p. 401):

"As we have seen, the study of the forms of equilibrium of rotating liquids is almost complete, and a good beginning has been made in the investigation of the equilibrium of gaseous stars, but much more remains to be discovered.

"As a beginning we should like to know how a moderate degree of compressibility would alter the results for liquid, and...to understand more as to the manner in which rotation affects the equilibrium and stability of rotating gas. The field for the mathematician is a wide one, and in proportion as the very arduous exploration of that field is attained, so will our knowledge of the processes of cosmical evolution increase...."

"Human life is too short to permit us to watch the leisurely procedure of cosmical evolution, but the celestial museum contains so many exhibits that it may become possible, by the aid of theory, to piece together bit by bit the processes through which stars pass in the course of their evolution."

Guided possibly by considerations such as these, the Adjudicators of the Adams Prize announced as the subject for the 1917 Essay:

The course of evolution of the configurations possible for a rotating and gravitating fluid mass, including the discussion of the stabilities of the various forms.

At this time I had for some years been engaged in an attack on this problem. The announcement offered an excuse not only for putting together my own results in essay form but also for welding them on to the earlier results obtained in the classical papers of Darwin, Poincaré and other workers at this problem. After the adjudication of the prize, the essay was enlarged by the addition of some further results which had been obtained in the interval, and the present volume is the result.
Preface

It is hoped that the book will be read in the spirit of the remarks of Darwin just quoted. The main object of the essay is to build a framework of absolute mathematical truth; the backbone of the structure is the theoretical investigation into the behaviour of rotating masses. Of this my own contribution forms only a small part; the book contains also an account of general dynamical theory, and of the researches of Darwin, Poincaré and others, in so far as they relate to the main problem in hand. This part of the book has been made as concise as possible, and I have ventured to hope that it will prove of value to those who are embarking on a study of the general problem of cosmic evolution.

I have tried not only to build a skeleton but also to clothe it. When a firm theoretical framework had been constructed, it seemed permissible and proper to try to fit the facts of observational astronomy into their places. If ever a complete mathematical theory is achieved, it will probably be an easy task to trace out the order of evolution of stellar objects, but at present our theoretical knowledge is so incomplete that a large element of speculation must necessarily enter into every attempt to connect up theory and observation. I have tried throughout to keep speculation within reasonable limits, and have applied as many checks and tests as I could to the various conjectural hypotheses brought forward. Many astronomers necessarily will disagree with a number of these conjectures; it is in this way that science advances. To any critic who may think the conjectures ought not to have been brought forward at all, I would reply in the words of Herschel:

"If we indulge a fanciful imagination and build worlds of our own,... these will vanish like the Cartesian vortices, that soon gave way when better theories were offered. On the other hand, if we add observation to observation, without attempting to draw not only certain conclusions but also conjectural views from them, we offend against the very end for which only observations ought to be made. I will endeavour to keep a proper medium; but if I should deviate from that I could not wish to fall into the latter error."

The more speculative chapters fall naturally together at the end of the book. Many readers may find these the most interesting, and I have tried to arrange the book so that they will prove intelligible to those readers who prefer to take mathematical investigations as read. In the present state of our knowledge any attempt to dictate final conclusions on the main problems of cosmogony could be nothing but pure dogmatism; I should not have ventured even to suggest a conclusion except that the various theoretical results obtained seemed to point with considerable unanimity in one particular direction. Consequently a definite scheme of cosmogony has been
Preface

suggested; not in the belief that it will prove to be true, but in the hope that it may in some degree help others ultimately to find the truth. This scheme will be found to contain nothing fundamentally new; it consists only of a patchwork of parts of existing theories. This is perhaps hardly surprising; so many cosmogonical conjectures have been made that it is unlikely that any really novel hypothesis remains to be put forward. In any case a theoretical investigation such as that of the present book is necessarily destructive rather than constructive; primarily it serves to test and eliminate existing theories rather than to indicate new possibilities.

It is a pleasure to thank many friends who have helped me in various ways. First I must thank the great number of astronomers who have allowed me to draw on their stores of astronomical knowledge. I have to express my obligation and cordial thanks to Professor Hale, Professor Ritchey and Mr F. G. Pease of Mount Wilson Observatory for permission to reproduce the very fine photographs which enrich my book. Finally it is a pleasure to express to the officials and staff of the Cambridge University Press my appreciation of their unfailing courtesy and the care they have bestowed on the printing of the work.

J. H. JEANS.

Dec. 18, 1918.
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