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Scientific Papers

Volume 6: 1911–1919

Baron John William Strutt Rayleigh

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SCIENTIFIC PAPERS

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VOL. VI.

1911—1919

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PREFACE

THIS volume completes the collection of my Father's published papers. The two last papers (Nos. 445 and 446) were left ready for the press, but were not sent to any channel of publication until after the Author's death.

Mr W. F. Sedgwick, late Scholar of Trinity College, Cambridge, who had done valuable service in sending corrections of my Father's writings during his lifetime, kindly consented to examine the proofs of the later papers of this volume [No. 399 onwards] which had not been printed off at the time of the Author's death. He has done this very thoroughly, checking the numerical calculations other than those embodied in tables, and supplying footnotes to elucidate doubtful or obscure points in the text. These notes are enclosed in square brackets [ ] and signed W. F. S. It has not been thought necessary to notice minor corrections.

RAYLEIGH.

Sept. 1920.

CONTENTS

* [1914. It would have been in better accordance with usage to have said "of Relative Index differing little from Unity."]

* [1917. It would be more correct to say $P_n(\cos\theta)$, where $\cos\theta$ lies between ± 1.]

ERRATA

(INCLUDING THE ERRATA NOTED IN VOLUME V. PAGE XIII.)

VOLUME I.

Page viii, line 4.
,, 64, line 8. } *For* end lies *read* ends lie.

,, 86, last line. *For* 1882 *read* 1881.

,, 89, line 10. *Insert* comma *after* maximum.

,, 144, line 6 from bottom. *For* D *read* D_1.

,, 324, equation (8). *Insert* negative sign *before the single integral.*
,, ,, line 2 from bottom. *For* (1) *read* (5).
,, 325, line 10. *For* $-nVH$ *read* $+n\rho VH$.
} And *Theory of Sound*, Vol. I. (1894), p. 477, equation (8) and last line, and p. 478, line 12.

,, 442, line 9. *After* $\frac{\rho'-\rho}{\rho'}$ *insert* y.

,, 443, line 9. *For* (7) *read* (8).

,, 443, line 10. *For* η *read* ξ.

,, 446, line 10. *For* ϕ *read* ϕ'.

,, 448, line 5. *For* v *read* c.

,, 459, line 17. *For* 256, 257 *read* 456, 457.

,, 492, line 7 from bottom. *For* $r\sqrt{2n}$ *read* $r/\sqrt{2n}$.

,, 494, lines 10 and 12. *For* $-\frac{2mr^2}{n^2-4m^2}\cos 2\theta$ *read* $+\frac{2mr^2}{n^2-4m^2}\cos 2\theta$.

,, 523, line 9. *For* n/λ *read* n/k.

,, 524. In the second term of equations (32) and following *for* ΔK^{-1} *read* $\Delta\mu^{-1}$.

,, 525, line 11. *For* f *read* f_1.

,, 526, line 13. *For* $f:g$ *read* $f_1:g_1$.

,, 528, line 3 from bottom. *For* e^{int} *read* $e^{i(nt-kr_0)}$.

,, 538, line 11 from bottom. This passage is incorrect (*see* Vol. VI. Art. 355, p. 41).

,, 556. In line 8 *after* (15) *add* with $s\phi-\frac{1}{2}\pi$ for $s\phi$; in line 9 *for* δA_s *read* $\delta A_s'$; and *for* line 10 *substitute* $+\delta A_s'$ *as* $\{\cos\frac{1}{2}s\pi+\cos(\frac{1}{2}s\pi+s\pi)\}F$.
Throughout lines 12—25 *for* A_s, A_1, A_2, ... A_6, δA_s *read* A_s', A_1', A_2', ... A_6', $\delta A_s'$; *for* $\sin\frac{1}{2}s\pi$ *read* $-\cos\frac{1}{2}s\pi$; and *reverse* the signs of the expressions for A_2', A_4', A_6'. Similarly, in *Theory of Sound*, Vol. I. (1894), p. 427, *substitute* $s\phi+\frac{1}{2}\pi$ *for* $s\phi$ in (32) (*see* p. 424), and in lines 11—26 *for* A_s', A_s, δA_s *read* A_s, A_s', $\delta A_s'$, and *for* sin *read* +cos. Also in (43) and (47) *for* s^2-s *read* s^3-s. [In both cases the work done corresponding to δA_s vanishes whether s be odd or even.]

VOLUME II.

,, 197, line 19. *For* nature *read* value.

,, 240, line 22. *For* dp/dx *read* dp/dy.

,, 241, line 2. *For* du/dx *read* du/dy.

,, 244, line 4. *For* k/n *read* n/k.

,, 323, lines 7 and 16 from bottom. *For* Thomson *read* C. Thompson.

,, 345, line 8 from bottom. *For* as pressures *read* at pressures.

,, 386, lines 12, 15, and 19. *For* cos CBD *read* cos CBB'.

,, 389, line 6. *For* minor *read* mirror.

,, 414, line 5. *For* favourable *read* favourably.

,, 551, first footnote. *For* 1866 *read* 1886.

VOLUME III.

Page 11, footnote. *For* has *read* have.
,, 92, line 4. *For* Vol. I. *read* Vol. II.
,, 129, equation (12). *For* $e^{u(i-x)}dx$ *read* $e^{u(i-x)}du$.
,, 162, line 19, and p. 224, second footnote. *For* Jellet *read* Jellett.
,, 179, line 15. *For* Provostaye *read* De la Provostaye.
,, 224, equation (20). *For* 2χ *read* χ.
,, ,, second footnote. *For* p. 179 *read* p. 343. } And *Theory of Sound*, Vol. I. (1894), p. 412, equation (12), and p. 423 (footnote).
,, 231, line 5 of first footnote. *For* 171 *read* 172.
,, 273, lines 15 and 20. *For* $\{\phi(x)\}^2$ *read* $\int_{-\infty}^{+\infty}\{\phi(x)\}^2dx$.
,, 314, line 1. *For* (38) *read* (39).
,, 326. In the lower part of the Table, under Ampton *for* $c\flat+4$ *read* $e\flat+4$, and under Terling (3) *for* $b\flat+6$ *read* $b+6$ (and in *Theory of Sound*, Vol. I. (1894), p. 393).
,, 522, equation (31). *Insert as factor of last term* $1/R$.
,, 548, second footnote. *For* 1863 *read* 1868.
,, 569, second footnote. *For* alcohol *read* water.
,, 580, line 3. Prof. Orr remarks that a is a function of r.

VOLUME IV.

,, 14, lines 6 and 8. *For* 38 *read* 42.
,, 267, lines 6, 10, and 20, and p. 269, line 1. *For* van t' Hoff *read* van 't Hoff. Also in Index, p. 604 (the entry should be under Hoff).
,, 277, equation (12). *For* dz *read* dx.
,, 299, first footnote. *For* 1887 *read* 1877.
,, 369, footnote. *For* 1890 *read* 1896.
,, 400, equation (14). A formula equivalent to this was given by Lorenz in 1890.
,, 418. In table opposite 6 *for* ·354 *read* ·324.
,, 453, line 8 from bottom. *For* $\frac{2}{n-1}$ *read* $-\frac{2}{n-1}$.
,, 556, line 8 from bottom. *For* reflected *read* rotated.
,, 570, line 7 (Section III). *For* 176 *read* 179.
,, 576, line 7 from bottom.
,, 586, line 20. } *For* end lies *read* ends lie.
,, 582, last line. *For* 557 *read* 555.
,, 603. *Transfer* the entry under Provostaye *to* De la Provostaye.
,, 604. *Transfer the entry* II 553 *from* W. Weber *to* H. F. Weber.

VOLUME V.

,, 43, line 19. *For* (5) *read* (2).
,, 137, line 14. μ is here used in two senses, which must be distinguished.
,, 149, line 3. *For* P_0 *read* P_1.
,, 209, footnote. *For* XLX. *read* XIX.
,, 241, line 10 from bottom. *For* position *read* supposition.
,, 255, first footnote. *For* Matthews *read* Mathews.
,, 256, line 6. *For* 1889 *read* 1899.
,, 265, line 16 from bottom. *For* § 351 *read* § 251.
,, ,, ,, 15 ,, ,, *For* solution *read* relation.
,, 266, lines 5 and 6, and *Theory of Sound*, § 251. An equivalent result had at an earlier date been obtained by De Morgan (*see* Volume VI. p. 233).
,, 286, line 7. *For a read x.*

VOLUME V—*continued.*

Page 364, title, and p. ix, Art. 320. *After* Acoustical Notes *add* VII.
,, 409, first line of P.S. *For* anwer *read* answer.
,, 444, line 2 of footnote. *For* p. 441, line 9 *read* p. 442, line 9.
,, 496, equation (4). *Substitute* equation (19) on p. 253 of Volume VI. (*see* pp. 251—253), *reading* $\left(\frac{l'}{l}-\frac{l}{l'}\right)$ *for* $\left(\frac{l'}{l}+\frac{l}{l'}\right)$.
,, 549, equation (48). *For* e^{-ikr} *read* e^{-ikr_0}.
,, 619, line 3. *Omit* the second expression for $J_n(n)$.
,, ,, lines 11, 12, 19. *For* 2·1123 *read* 1·3447.
,, ,, line 12. *For* 1·1814 *read* 1·8558.
,, ,, line 19. *For* ·51342 *read* ·8065.

} *See* the first footnote on p. 211 of Volume VI.

VOLUME VI.

,, 4, first footnote. *After* equation (8) *add*:—*Scientific Papers*, Vol. V. p. 619. *See also* Errata last noted above.
,, 5, line 3. *For* $(2n+1)z^2=4n(n+1)(n+2)$ *read* $z^2=2n(n+2)$, so that z^2 is an integer.
,, 11, last footnote. *For* § 230 *read* § 250 (fourth edition).
,, 13, equation (17). *For* $\frac{4}{5}k^4a^4$ *read* $\frac{5}{4}k^4a^4$.
,, 14, footnote. *For* § 247 *read* § 251 (fourth edition).
,, 78, footnote. *Add*:—*Scientific Papers*, Vol. V. p. 400.
,, 87, footnote. *Add*:—Thomson and Tait's *Natural Philosophy*, Vol. I. p. 497.
,, 89, second footnote. *For* 328 *read* 329.
,, 90, second footnote. *Add*:—*Math. and Phys. Papers*, Vol. IV. p. 77.
,, 138, footnote. *For* 1868 *read* 1865, *and for* Vol. II. p. 128, *read* Vol. I. p. 526.
,, 148, footnote. *Add*:—*Scientific Papers*, Vol. IV. p. 407, and this Volume, p. 47.
,, 155, footnote. *For* Vol. IV. *read* Vol. III.
,, 222, second footnote. *For* Vol. II. *read* Vol. I. And in *Theory of Sound*, Vol. I. (1894), last line of § 207, *for* 4·4747 *read* 4·4774
,, 223, line 5 from bottom. *For* 0·5772156 *read* 0·5772157.
,, 225, line 1. *For* much greater *read* not much greater.
,, ,, line 6 from bottom. *For* 13·094 *read* 3·3274.
,, 253, equation (19). *For* $\left(\frac{l'}{l}+\frac{l}{l'}\right)$ *read* $\left(\frac{l'}{l}-\frac{l}{l'}\right)$.
,, 259, line 5. *For* $-\frac{2}{a}\frac{dy}{dz}$ *read* $\mp\frac{2}{a}\frac{dy}{dz}$.
,, 263, equation (24). *For* $\frac{\omega^2 a}{2T}$ *read* $\frac{\omega^2 a^3}{2T}$.
,, ,, ,, (25). *For* $\left(1-\frac{3r^2}{a^2}\right)$ *read* $\left(1+\frac{3}{4}\frac{r^2}{a^2}\right)$.
,, 282, footnote. *For* p. 77 *read* p. 71.
,, 303, line 17. *For* $\surd(bvc/\kappa)$ *read* $\surd(bv\kappa)$.
,, 307, line 8. *For* $\frac{d\phi}{dy}$ *read* $-\frac{d\phi}{dy}$.
,, 315, line 2. *Delete* 195.
,, 341, second footnote. *Add*:—[This Volume, p. 275].
,, 351, line 13 from bottom. *For* $Tg\rho$ *read* $T/g\rho$.