273.

INTERFERENCE OF SOUND.

[*Royal Institution Proceedings*, XVII. pp. 1—7, 1902;
Nature, LXVI. pp. 42—44, 1902.]

For the purposes of laboratory or lecture experiments it is convenient to use a pitch so high that the sounds are nearly or altogether inaudible. The wave-lengths (1 to 3 cm.) are then tolerably small, and it becomes possible to imitate many interesting optical phenomena. The ear as the percipient is replaced by the high pressure sensitive flame, introduced for this purpose by Tyndall, with the advantage that the effects are visible to a large audience.

As a source of sound a "bird-call" is usually convenient. A stream of air from a circular hole in a thin plate impinges centrically upon a similar hole in a parallel plate held at a little distance. Bird-calls are very easily made. The first plate, of 1 or 2 cm. in diameter, is cemented, or soldered, to the end of a short supply-tube. The second plate may conveniently be made triangular, the turned down corners being soldered to the first plate. For calls of medium pitch the holes may be made in tin plate. They may be as small as ½ mm. in diameter, and the distance between them as little as 1 mm. In any case the edges of the holes should be sharp and clean. There is no difficulty in obtaining wave-lengths (complete) as low as 1 cm., and with care wave-lengths of ·6 cm. may be reached, corresponding to about 50,000 vibrations per second. In experimenting upon minimum wave-lengths, the distance between the call and the flame should not exceed 50 cm., and the flame should be adjusted to the verge of flaring*. As most bird-calls are very dependent upon the precise pressure of the wind, a manometer in immediate connection is practically a necessity. The pressure, originally somewhat in excess, may be controlled by a screw pinch-cock operating on a rubber connecting tube.

* *Theory of Sound*, 2nd ed. § 371.

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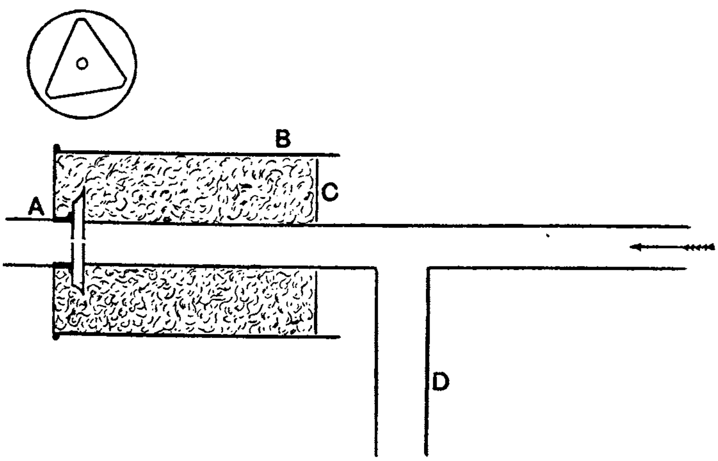
In the experiments with conical horns or trumpets, it is important that no sound should issue except through these channels. The horns end in short lengths of brass tubing which fit tightly to a short length of tubing (A) soldered air-tight on the face of the front plate of the bird-call. So far there is no difficulty; but if the space between the plates be boxed in air-tight, the action of the call is interfered with. To meet this objection a tin-plate box is soldered air-tight to A, and is stuffed with cotton-wool kept in position by a *loosely* fitting lid at C. In this way very little sound can escape except through the tube A, and yet the call speaks much as usual. The manometer is connected at the side tube D. The wind is best supplied from a gas-holder.

With the steadily maintained sound of the bird-call there is no difficulty in measuring accurately the wave-lengths by the method of nodes and loops.

Fig. 1.



A glass plate behind the flame, and mounted so as to be capable of sliding backwards and forwards, serves as reflecting wall. At the plate, and at any distance from it measured by an *even* number of quarter wave-lengths there are nodes, where the flame does not respond. At intermediate distances, equal to *odd* multiples of the quarter wave-length, the effect upon the flame is a maximum. For the present purpose it is best to use nodes, so adjusting the sensitiveness of the flame that it only just recovers its height at the minimum. The movement of the screen required to pass over ten intervals from minimum to minimum may be measured, and gives at once the length of five complete progressive waves. For the bird-call used in the experiments of this lecture the wave-length is 2 cm. very nearly.

When the sound whose wave-length is required is not maintained, the application of the method is, of course, more difficult. Nevertheless, results of considerable accuracy may be arrived at. A steel bar, about 22 cm. long, was so mounted as to be struck longitudinally every two or three seconds by a small hammer. Although in every position the flame shows some uneasiness at the stroke of the hammer, the distinction of loops and nodes is perfectly evident, and the measurement of wave-length can be effected with an accuracy of about 1 per cent. In the actual experiment the wave-length was nearly 3 cm.

The formation of stationary waves with nodes and loops by perpendicular reflection illustrates interference to a certain extent, but for the full development of the phenomenon the interfering sounds should be travelling in the same, or nearly the same, direction. The next example illustrates the theory of Huyghens' zones. Between the bird-call and the flame is placed a glass screen perforated with a circular hole. The size of the hole, the distances, and the wave-length are so related to one another that the aperture just includes the first and second zones. The operation of the sounds passing these zones is antagonistic, and the flame shows no response until a part of the aperture is blocked off. The part blocked off may be either the central circle or the annular region defined as the second zone. In either case the flame flares, affording complete proof of interference of the parts of the sound transmitted by the aperture*.

From a practical point of view the passage of sound through apertures in walls is not of importance, but similar considerations apply to its issue from the mouths of horns, at least when the diameter of the mouth exceeds the half wave-length. The various parts of the sound are approximately in the same phase when they leave the aperture, but the effect upon an observer depends upon the phases of the sounds, not as they leave, but as they arrive. If one part has further to go than another, a phase discrepancy sets in. To a point in the axis of the horn, supposed to be directed horizontally, the distances to be travelled are the same, so that here the full effect is produced, but in oblique directions it is otherwise. When the obliquity is such that the nearest and furthest parts of the mouth differ in distance by rather more than one complete wave-length, the sound may disappear altogether through antagonism of equal and opposite effects. In practice the attainment of a complete silence would be interfered with by reflections, and in many cases by a composite character of sound, viz. by the simultaneous occurrence of more than one wave-length.

In the fog signals established on our coasts the sound of powerful sirens issues from conical horns of circular cross-section. The influence of obliquity is usually very marked. When the sound is observed from a sufficient distance at sea, a deviation of even 20° from the axial line entails a considerable

* [1901. See Vol. III. p. 31.]

loss, to be further increased as the deviation rises to 40° or 60°. The difficulty thence arising is met, in the practice of the Trinity House, by the use of two distinct sirens and horns, the axes of the latter being inclined to one another at 120°. In this way an arc of 180° or more can be efficiently guarded, but a more equable distribution of the sound from a single horn remains a desideratum.

Guided by the considerations already explained, I ventured to recommend to the Trinity House the construction of horns of novel design, in which an attempt should be made to spread the sound out horizontally over the sea, and to prevent so much of it from being lost in an upward direction. The solution of the problem is found in a departure from the usual circular section, and the substitution of an elliptical or elongated section, of which the short diameter, placed horizontally, does not exceed the half wave-length; while the long diameter, placed vertically, may amount to two wave-lengths or more. Obliquity in the *horizontal plane* does not now entail much difference of phase, but when the horizontal plane is departed from such differences enter rapidly.

Horns upon this principle were constructed under the supervision of Mr Matthews, and were tried in the course of the recent experiments off St Catherine's. The results were considered promising, but want of time and the numerous obstacles which beset large scale operations prevented an exhaustive examination.

On a laboratory scale there is no difficulty in illustrating the action of the elliptical horns. They may be made of thin sheet brass. In one case the total length is 20 cm., while the dimensions of the mouth are 5 cm. for the long diameter and $1\frac{1}{4}$ cm. for the shorter diameter. The horn is fitted at its narrow end to A (Fig. 1), and can rotate about the common horizontal axis. When this axis is pointed directly at the flame, flaring ensues; and the rotation of the horn has no visible effect. If now, while the long diameter of the section remains vertical, the axis be slewed round in the horizontal plane until the obliquity reaches 50° or 60°, there is no important falling off in the response of the flame. But if at obliquities exceeding 20° or 30° the horn is rotated through a right angle, so as to bring the long diameter horizontal, the flame recovers as if the horn had ceased sounding. The fact that there is really no falling off may be verified with the aid of a reflector, by which the sound proceeding at first in the direction of the axis may be sent towards the flame.

When the obliquity is 60° or 70° it is of great interest to observe how moderate a departure from the vertical adjustment of the longer diameter causes a cessation of effect. The influence of maladjustment is shown even more strikingly in the case of a larger horn. According to theory and observation a serious falling off commences when the tilt is such that the difference of distances from the flame of the two extremities of the long diameter reaches the half wave-length—in this case 1 cm. It is thus abundantly proved that the sound issuing from the properly adjusted elliptical cone is confined to a comparatively narrow belt round the horizontal plane and that in this plane it covers efficiently an arc of 150° or 160°.

Another experiment, very easily executed with the apparatus already described, illustrates what are known in Optics as Lloyd's bands. These bands are formed by the interference of the direct vibration with its very oblique reflection. If the bird-call is pointed toward the flame, flaring ensues. It is only necessary to hold a long board horizontally under the direct line to obtain a reflection. The effect depends upon the precise height at which the board is held. In some positions the direct and reflected vibrations co-operate at the flame and the flaring is more pronounced than when the board is away. In other positions the waves are antagonistic and the flame recovers as if no sound were reaching it at all. This experiment was made many years ago by Tyndall who instituted it in order to explain the very puzzling phenomenon of the "silent area." In listening to fog-signals from the sea it is not unfrequently found that the signal is lost at a distance of a mile or two and recovered at a greater distance in the same direction. During the recent experiments the Committee of the Elder Brethren of the Trinity House had several opportunities of making this observation. That the surface of the sea must act in the manner supposed by Tyndall cannot be doubted, but there are two difficulties in the way of accepting the simple explanation as complete. According to it the interference should always be the same, which is certainly not the case. Usually there is no silent area. Again, although according to the analogy of Lloyd's bands there might be a dark or silent place at a particular height above the water, say on the bridge of the *Irene*, the effect should be limited to the neighbourhood of the particular height. At a height above the water twice as great, or near the water level itself, the sound should be heard again. In the latter case there were some difficulties, arising from disturbing noises, in making a satisfactory trial; but as a matter of fact, neither by an observer up the mast nor by one near the water level, was a sound lost on the bridge ever recovered.

The interference bands of Fresnel's experiment may be imitated by a bifurcation of the sound issuing from *A* (Fig. 1). For this purpose a sort of T-tube is fitted, the free ends being provided with small elliptical cones, similar to that already described, whose axes are parallel and distant from another by about 40 cm. The whole is constructed with regard to symmetry, so that sounds of equal intensity and of the same phase issue from the two cones whose long diameters are vertical. If the distances of the burner from the mouths of the cones be precisely equal, the sounds arrive in the same phase and the flame flares vigorously. If, as by the hand held between, one of the sounds is cut off, the flaring is reduced, showing that with this adjustment the two sounds are more powerful than one. By an almost imperceptible slewing round of the apparatus on its base-board the adjustment above spoken of is upset and the flame is induced to recover its tall equilibrium condition. The sounds now reach the flame in opposition of phase and practically neutralise one another. That this is so is proved in a moment. If the hand be introduced between either orifice and the flame, flaring ensues, the sound not intercepted being free to produce its proper effect.

The analogy with Fresnel's bands would be most complete if we kept the sources of sound at rest and caused the burner to move transversely so as to occupy in succession places of maximum and minimum effect. It is more convenient with our apparatus and comes to the same thing, if we keep the burner fixed and move the sources transversely, sliding the base-board without rotation. In this way we may verify the formula, connecting the width of a band with the wave-length and the other geometrical data of the experiment.

The phase discrepancy necessary for interference may be introduced, without disturbing the equality of distances, by inserting in the path of one of the sounds a layer of gas having different acoustical properties from air. In the lecture carbonic acid was employed. This gas is about half as heavy again as air, so that the velocity of sound is less in the proportion of 1 : 1·25. If l be the thickness of the layer, the *retardation* is ·25 l; and if this be equal to the half wave-length, the interposition of the layer causes a transition from complete agreement to complete opposition of phase. Two cells of tin plate were employed, fitted with tubes above and below, and closed with films of collodion. The films most convenient for this purpose are those formed upon water by the evaporation of a few drops of a solution of celluloid in pear-oil. These cells were placed one in the path of each sound, and the distances of the cones adjusted to maximum flaring. The insertion of carbonic acid into *one* cell quieted the flame, which flared again when the second cell was charged so as to restore symmetry. Similar effects were produced as the gas was allowed to run out at the lower tubes, so as to be replaced by air entering above*.

Many vibrating bodies give rise to sounds which are powerful in some directions but fail in others—a phenomenon that may be regarded as due to interference. The case of tuning forks (unmounted) is well known. In the lecture a small and thick wine-glass was vibrated, after the manner of a bell, with the aid of a violin bow. When any one of the four vibrating segments was presented to the flame, flaring ensued; but the response failed

* In a still atmosphere the hot gases arising from lighted candles may be substituted for the layers of CO_2.

when the glass was so held at the same distance that its *axis* pointed to the flame. In this position the effects of adjacent segments neutralise one another and the aggregate is zero. Another example, which, strangely enough, does not appear to have been noticed, is afforded by the familiar open organ pipe. The vibrations issuing from the two ends are in the same phase as they start, so that if the two ends are equally distant from the percipient, the effects conspire. If, however, the pipe be pointed towards the percipient, there is a great falling off, inasmuch as the length of the pipe approximates to the half wave-length of the sound. The experiment may be made in the lecture-room with the sensitive flame and one of the highest pipes of an organ, but it succeeds better and is more striking when carried out in the open air with a pipe of lower pitch, simply listened to with the unaided ear of the observer. Within doors reflections complicate all experiments of this kind.

[1910. Some further discussion of interfering sources will be found in *Phil. Mag.* Sept. 1903, Art. 290 below.]

274.

SOME GENERAL THEOREMS CONCERNING FORCED VIBRATIONS AND RESONANCE.

[*Philosophical Magazine*, III. pp. 97—117, 1902.]

THE general equation for the small vibrations of a system whose configuration is defined by the generalized coordinates $\psi_1, \psi_2, \ldots$ may be written*

$$\frac{d}{dt}\frac{dT}{d\dot{\psi}} + \frac{dF}{d\dot{\psi}} + \frac{dV}{d\psi} = \Psi, \quad \ldots\ldots\ldots\ldots(1)$$

where T, F, V, denoting respectively the kinetic energy, the dissipation function, and the potential energy, have the forms

$$\left.\begin{aligned} T &= \tfrac{1}{2}a_{11}\dot{\psi}_1^2 + \tfrac{1}{2}a_{22}\dot{\psi}_2^2 + \ldots + a_{12}\dot{\psi}_1\dot{\psi}_2 + \ldots \\ F &= \tfrac{1}{2}b_{11}\dot{\psi}_1^2 + \tfrac{1}{2}b_{22}\dot{\psi}_2^2 + \ldots + b_{12}\dot{\psi}_1\dot{\psi}_2 + \ldots \\ V &= \tfrac{1}{2}c_{11}\psi_1^2 + \tfrac{1}{2}c_{22}\psi_2^2 + \ldots + c_{12}\psi_1\psi_2 + \ldots \end{aligned}\right\}, \quad \ldots\ldots\ldots(2)$$

in which the coefficients a, b, c are constants.

If we substitute in (1) the values of T, F, and V, and write D for d/dt, we obtain a system of equations which may be put into the form

$$\left.\begin{aligned} e_{11}\psi_1 + e_{12}\psi_2 + e_{13}\psi_3 + \ldots &= \Psi_1 \\ e_{21}\psi_1 + e_{22}\psi_2 + e_{23}\psi_3 + \ldots &= \Psi_2 \\ e_{31}\psi_1 + e_{32}\psi_2 + e_{33}\psi_3 + \ldots &= \Psi_3 \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \end{aligned}\right\}, \quad \ldots\ldots\ldots(3)$$

where e_{rs} denotes the quadratic operator

$$e_{rs} = a_{rs}D^2 + b_{rs}D + c_{rs}. \quad \ldots\ldots\ldots\ldots(4)$$

And it is to be remarked that since

$$a_{rs} = a_{sr}, \qquad b_{rs} = b_{sr}, \qquad c_{rs} = c_{sr},$$

it follows that

$$e_{rs} = e_{sr}. \ldots\ldots\ldots\ldots\ldots\ldots(5)$$

* See *Theory of Sound*, Vol. I. §§ 82, 84, 104.

If we multiply the first of equations (3) by $\dot{\psi}_1$, the second by $\dot{\psi}_2$, &c., and then add, we obtain

$$\frac{d(T+V)}{dt} + 2F = \Psi_1\dot{\psi}_1 + \Psi_2\dot{\psi}_2 + \dots \qquad (6)$$

In this the first term represents the rate at which energy is being stored in the system; $2F$ is the rate of dissipation; and the two together account for the work done upon the system in time dt by the external forces Ψ_1, Ψ_2, &c.

In considering forced vibrations of simple type we take

$$\Psi_1 = E_1 e^{ipt}, \qquad \Psi_2 = E_2 e^{ipt}, \qquad \text{\&c.}, \qquad (7)$$

and assume that ψ_1, ψ_2, &c. are also proportional to e^{ipt}. The coordinates are then determined by the system of algebraic equations resulting from the substitution in (4), (3) of ip for D. The most general motion possible under the assumed forces would require the inclusion of *free* vibrations, but (unless $F=0$) these die out as time progresses.

By the theory of determinants the solution of equations (3) may be expressed in the form

$$\left.\begin{array}{l} \nabla \cdot \psi_1 = \dfrac{d\nabla}{de_{11}}\Psi_1 + \dfrac{d\nabla}{de_{12}}\Psi_2 + \dots \\ \nabla \cdot \psi_2 = \dfrac{d\nabla}{de_{12}}\Psi_1 + \dfrac{d\nabla}{de_{22}}\Psi_2 + \dots \\ \dots\dots\dots\dots\dots\dots \end{array}\right\}, \qquad (8)$$

where ∇ denotes the determinant of the symbols e. If there be no dissipation, ∇, or as we may write it with fuller expressiveness $\nabla(ip)$, is an even function of ip vanishing when p corresponds to one of the natural frequencies of vibration. In such a case the coordinates ψ_1, &c. in general become infinite. When there is dissipation, $\nabla(ip)$ does not vanish for any (real) value of p. If we write

$$\nabla(ip) = \nabla_1(ip) + ip\,\nabla_2(ip), \qquad (9)$$

in which ∇_1, ∇_2 are *even* functions of ip, ∇_2 depends entirely upon the dissipation, while if the dissipation be small, ∇_1 is approximately the same as if there were none.

As it will be convenient to have a briefer notation than that of (8), we will write

$$\left.\begin{array}{l} \psi_1 = A_{11}e^{i\alpha_{11}}\Psi_1 + A_{12}e^{i\alpha_{12}}\Psi_2 + \dots \\ \psi_2 = A_{21}e^{i\alpha_{21}}\Psi_1 + A_{22}e^{i\alpha_{22}}\Psi_2 + \dots \\ \dots\dots\dots\dots\dots\dots \end{array}\right\}, \qquad (10)$$

in which A, α are real and are subject to the relations

$$A_{rs} = A_{sr}, \qquad \alpha_{rs} = \alpha_{sr}. \qquad (11)$$

In order to take account of the phases of the forces, we may suppose similarly that in (7)

$$E_1 = R_1 e^{i\theta_1}, \qquad E_2 = R_2 e^{i\theta_2}, \qquad \&c. \qquad \text{..............(12)}$$

Work Done.

If we suppose that but *one* force, say Ψ_1, acts upon the system, the values of the coordinates are given by the first terms of the right-hand members of (10). The work done by the force in time dt depends upon that part of $d\psi_1/dt$ which is in the same phase with it, corresponding to the part of ψ_1 which is in quadrature with the force. Thus, taking the real parts only of the symbolic quantities, so that

$$\Psi_1 = R_1 \cos(pt + \theta_1), \qquad \psi_1 = A_{11} R_1 \cos(pt + \theta_1 + \alpha_{11}), \text{......(13)}$$

we have as the work done (on the average) in time t

$$-pA_{11}R_1^2 \int \cos(pt + \theta_1) \,.\, \sin(pt + \theta_1 + \alpha_{11})\, dt,$$

or

$$-\tfrac{1}{2} p R_1^2 A_{11} \sin \alpha_{11} \,.\, t. \qquad \text{......................(14)}$$

As was to be expected, this is independent of θ_1.

Another expression for the same quantity may be obtained by considering how this work is dissipated. From (6) we see that

$$\int \Psi_1 \dot{\psi}_1 dt = 2 \int F dt = b_{11} \int \dot{\psi}_1^2 dt + b_{22} \int \dot{\psi}_2^2 dt + \ldots + 2b_{12} \int \dot{\psi}_1 \dot{\psi}_2 dt + \ldots \text{...(15)}$$

Taking again the real parts in (10), we have

$$\int \dot{\psi}_1^2 dt = \tfrac{1}{2} p^2 R_1^2 A_{11}^2 \,.\, t, \text{.................................(16)}$$

$$\int \dot{\psi}_1 \dot{\psi}_2 dt = \tfrac{1}{2} p^2 R_1^2 A_{11} A_{12} \cos(\alpha_{11} - \alpha_{12}) \,.\, t, \text{..............(17)}$$

$$\int \dot{\psi}_2 \dot{\psi}_3 dt = \tfrac{1}{2} p^2 R_1^2 A_{12} A_{13} \cos(\alpha_{12} - \alpha_{13}) \,.\, t; \text{..............(18)}$$

so that by (15) the work dissipated in time t is

$$\tfrac{1}{2} p^2 R_1^2 t \{b_{11} A_{11}^2 + b_{22} A_{21}^2 + \ldots + 2b_{12} A_{11} A_{12} \cos(\alpha_{11} - \alpha_{12}) + \ldots\}. \quad \text{...(19)}$$

Equating the equivalent quantities in (14) and (19), we get

$$-p^{-1} A_{11} \sin \alpha_{11} = b_{11} A_{11}^2 + b_{22} A_{21}^2 + \ldots + 2b_{12} A_{11} A_{12} \cos(\alpha_{11} - \alpha_{12}) + \ldots \text{.(20)}$$

This assumes a specially simple form when F is a function of the squares only of $d\psi_1/dt$, &c., so that b_{12}, &c. vanish.

In (14) we have calculated the work done by a force Ψ_1 acting alone upon the system. If other forces act, the expression for ψ_1 will deviate from (13); but in any case we may write

$$\Psi_1 = R_1 e^{i\theta_1}, \qquad \psi_1 = r_1 e^{i\phi_1}, \text{....................(21)}$$

and the work done in unit of time by the real part of Ψ_1 on the real part of ψ_1 will be

$$-\tfrac{1}{2} p R r \sin(\phi_1 - \theta_1), \text{..........................(22)}$$

and depends upon the product of the moduli and the *difference* of phases.

If ψ_1 consist of two or more parts of the form (21), the work done is to be found by addition of the terms corresponding to the various parts.