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978-1-108-00534-0 - An Elementary Course of Infinitesimal Calculus

Horace Lamb

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An Elementary Course of Infinitesimal Calculus

Sir Horace Lamb (1849–1934) the British mathematician, wrote a number of influential works in classical physics. A pupil of Stokes and Clerk Maxwell, he taught for ten years as the first professor of mathematics at the University of Adelaide before returning to Britain to take up the post of professor of physics at the Victoria University of Manchester (where he had first studied mathematics at Owens College). As a teacher and writer his stated aim was clarity: ‘somehow to make these dry bones live’. The first edition of this work was published in 1897, the third revised edition in 1919, and a further corrected version just before his death. This edition, reissued here, remained in print until the 1950s. As with Lamb’s other textbooks, each section is followed by examples.

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BY
SIR HORACE LAMB

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PREFACE

THIS book was described in the original preface as an attempt to teach those portions of the Calculus which are of primary importance in the applications of the subject. The general arrangement of the work was, at the time, somewhat unusual, but appears to have been found convenient.

The present edition has been revised throughout, and a number of changes have been made. Apart from minor alterations and rearrangements, there are one or two points which call for remark.

A special chapter is devoted to the exponential and allied functions, the exponential function being now defined as the standard solution of the equation

$$\frac{dy}{dx} = y.$$

It is to this property, entirely, that the function owes its importance in Mathematics, and it seems therefore most natural to take this as the starting-point. No theory of the exponential series which has any pretensions to be rigorous can be said to be altogether elementary, but it is claimed that the method here followed is, from the standpoint of the Calculus, no more difficult than any other, whilst there can be no question as to its being the most appropriate.

Another considerable change is in the treatment of infinite series, their differentiation and integration. In previous editions these questions were discussed in a general manner, by the light of the theory of uniform convergence. There was perhaps some justification for including this theory, at a time when it was hardly accessible in any English manual, but it was out of

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perspective with the rest of the book, and is now omitted. It is replaced by a discussion restricted to *power-series* only, which are the only type which the student is likely to be concerned with until he reaches a more advanced stage.

Finally, some sections on mass-centres, quadratic moments, and the like, have been condensed or omitted. They have in the meantime been transferred, for the most part, to other books by the author.

HORACE LAMB.

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