

CAMBRIDGE LIBRARY COLLECTION

Books of enduring scholarly value

Mathematical Sciences

From its pre-historic roots in simple counting to the algorithms powering modern desktop computers, from the genius of Archimedes to the genius of Einstein, advances in mathematical understanding and numerical techniques have been directly responsible for creating the modern world as we know it. This series will provide a library of the most influential publications and writers on mathematics in its broadest sense. As such, it will show not only the deep roots from which modern science and technology have grown, but also the astonishing breadth of application of mathematical techniques in the humanities and social sciences, and in everyday life.

Dynamics

Sir Horace Lamb (1849–1934) the British mathematician, wrote a number of influential works in classical physics. A pupil of Stokes and Clerk Maxwell, he taught for ten years as the first professor of mathematics at the University of Adelaide before returning to Britain to take up the post of professor of physics at the Victoria University of Manchester (where he had first studied mathematics at Owens College). As a teacher and writer his stated aim was clarity: ‘somehow to make these dry bones live.’ His Dynamics was first published in 1914 and the second edition, offered here, in 1923: it remained in print until the 1960s. It was intended as a sequel to his Statics (also reissued in this series), and like its predecessor is a textbook with examples.

Cambridge University Press
978-1-108-00533-3 - Dynamics
Horace Lamb
Frontmatter
[More information](#)

Cambridge University Press has long been a pioneer in the reissuing of out-of-print titles from its own backlist, producing digital reprints of books that are still sought after by scholars and students but could not be reprinted economically using traditional technology. The Cambridge Library Collection extends this activity to a wider range of books which are still of importance to researchers and professionals, either for the source material they contain, or as landmarks in the history of their academic discipline.

Drawing from the world-renowned collections in the Cambridge University Library, and guided by the advice of experts in each subject area, Cambridge University Press is using state-of-the-art scanning machines in its own Printing House to capture the content of each book selected for inclusion. The files are processed to give a consistently clear, crisp image, and the books finished to the high quality standard for which the Press is recognised around the world. The latest print-on-demand technology ensures that the books will remain available indefinitely, and that orders for single or multiple copies can quickly be supplied.

The Cambridge Library Collection will bring back to life books of enduring scholarly value across a wide range of disciplines in the humanities and social sciences and in science and technology.

Cambridge University Press
978-1-108-00533-3 - Dynamics
Horace Lamb
Frontmatter
[More information](#)

Dynamics

HORACE LAMB



CAMBRIDGE
UNIVERSITY PRESS

Cambridge University Press
978-1-108-00533-3 - Dynamics
Horace Lamb
Frontmatter
[More information](#)

CAMBRIDGE UNIVERSITY PRESS

Cambridge New York Melbourne Madrid Cape Town Singapore São Paulo Delhi

Published in the United States of America by Cambridge University Press, New York

www.cambridge.org

Information on this title: www.cambridge.org/9781108005333

© in this compilation Cambridge University Press 2009

This edition first published 1961
This digitally printed version 2009

ISBN 978-1-108-00533-3

This book reproduces the text of the original edition. The content and language reflect the beliefs, practices and terminology of their time, and have not been updated.

Cambridge University Press
978-1-108-00533-3 - Dynamics
Horace Lamb
Frontmatter
[More information](#)

DYNAMICS

DYNAMICS

By

HORACE LAMB

CAMBRIDGE
AT THE UNIVERSITY PRESS
1961

Cambridge University Press
978-1-108-00533-3 - Dynamics
Horace Lamb
Frontmatter
[More information](#)

PUBLISHED BY
THE SYNDICS OF THE CAMBRIDGE UNIVERSITY PRESS

Bentley House, 200 Euston Road, London, N.W.1
American Branch: 32 East 57th Street, New York 22, N.Y.
West African Office: P.O. Box 33, Ibadan, Nigeria

<i>First edition</i>	1914
<i>Reprinted</i>	1920
<i>Second edition</i>	1923
<i>Reprinted</i>	1926
	1929
	1942
	1945
	1946
	1947
	1951
	1960
	1961

*Printed in Great Britain at the University Press, Cambridge
(Brooke Crutchley, University Printer)*

PREFACE

THIS book is a sequel to a treatise on Statics published a little more than a year ago, and has a similar scope. To avoid repetitions, numerous references to the former volume are made.

A writer who undertakes to explain the elements of Dynamics has the choice, either to follow one or other of the traditional methods which, however effectual from a practical point of view, are open to criticism on logical grounds, or else to adopt a treatment so abstract that it is likely to bewilder rather than to assist the student who looks to learn something about the behaviour of actual bodies which he can see and handle. There is no doubt as to which is the proper course in a work like the present; and I have not hesitated to follow the method adopted by Maxwell, in his *Matter and Motion*, which forms, I think, the best elementary introduction to the 'absolute' system of Dynamics. Some account of the more abstract, if more logical, way of looking at dynamical questions is, however, given in its proper place, which is at the end, rather than at the beginning of the book.

There is some latitude of judgment as to the order in which the different parts of the subject should be taken. To many students it is more important that they should gain, as soon as possible, some power of dealing with the simpler questions of 'rigid' Dynamics, than that they should master the more intricate problems of 'central forces,' or of motion under various laws of resistance. This consideration has dictated the arrangement here adopted, but as the later chapters are largely independent of one another, they may be read in a different order without inconvenience.

Some pains have been taken in the matter of examples for practice. The standard collections, and the text-books of several generations, supply at first sight abundant material for appropriation, but they do not always reward the search for problems which are really exercises on dynamical theory, and not merely algebraical or trigonometrical puzzles in disguise. In the present treatise, preference has been given to examples which are simple rather than elaborate from the analytical point of view. Most of those which are in any degree original have been framed with this intention.

I am again greatly indebted to Prof. F. S. Carey, and Mr J. H. C. Searle, for their kindness in reading the proofs, and for various useful suggestions. The latter has moreover verified most of the examples. Miss Mary Taylor has also given kind assistance in the later stages of the passage through the press.

H. L.

THE UNIVERSITY,
MANCHESTER.
January 1914.

In this issue a few pages have been re-written, and a number of errors have been corrected, chiefly in the examples. An additional set of miscellaneous exercises has also been inserted near the end. It will be understood that the 'Appendix,' which remains unaltered, is intended merely as a summary of the 'Newtonian' standpoint, from which an elementary text-book is necessarily written.

H. L.

CAMBRIDGE.
February, 1923.

CONTENTS

CHAPTER I

KINEMATICS OF RECTILINEAR MOTION.

ART.	PAGE
1. Velocity	1
2. Acceleration	3
3. Units and Dimensions	6
4. The Acceleration of Gravity	8
5. Differential Equations	9
<i>Examples I</i>	14

CHAPTER II

DYNAMICS OF RECTILINEAR MOTION.

6. Dynamical Principles. Gravitational Units	17
7. The Absolute System of Dynamics	20
8. Application to Gravity	21
9. General Equation of Motion. Impulse	23
10. Simple-Harmonic Motion	24
11. The Pendulum	29
12. Disturbed Simple-Harmonic Motion. Constant Disturbing Force	30
13. Periodic Disturbing Force	32
14. General Disturbing Force	36
15. Motion about Unstable Equilibrium	38
16. Motion under Variable Gravity	39
17. Work; Power	41
18. Equation of Energy	43
19. Dynamical Units and their Dimensions	46
<i>Examples II, III, IV</i>	49

CHAPTER III.

TWO-DIMENSIONAL KINEMATICS.

ART.		PAGE
20.	Velocity	55
21.	Hodograph. Acceleration	58
22.	Relative Motion	61
23.	Epicyclic Motion	62
24.	Superposition of Simple-Harmonic Vibrations	64
	<i>Examples V</i>	66

CHAPTER IV.

DYNAMICS OF A PARTICLE IN TWO DIMENSIONS.

CARTESIAN COORDINATES.

25.	Dynamical Principle	69
26.	Cartesian Equations	71
27.	Motion of a Projectile	72
28.	Elliptic-Harmonic Motion	76
29.	Spherical Pendulum. Blackburn's Pendulum	79
30.	Equation of Energy	81
31.	Properties of a Conservative Field of Force	84
32.	Oscillations about Equilibrium. Stability	85
33.	Rotating Axes	88
	<i>Examples VI, VII, VIII</i>	92

CHAPTER V.

TANGENTIAL AND NORMAL ACCELERATIONS.

CONSTRAINED MOTION.

34.	Tangential and Normal Accelerations	98
35.	Dynamical Equations	101
36.	Motion on a Smooth Curve	103
37.	The Circular Pendulum	105
38.	The Cycloidal Pendulum	110
39.	Oscillations on a Smooth Curve; Finite Amplitude	113
	<i>Examples IX. X</i>	116

CONTENTS

ix

CHAPTER VI.

MOTION OF A PAIR OF PARTICLES.

ART.		PAGE
40.	Conservation of Momentum	121
41.	Instantaneous Impulses. Impact	123
42.	Kinetic Energy	126
43.	Conservation of Energy	127
44.	Oscillations about Equilibrium	129
	<i>Examples XI</i>	134

CHAPTER VII.

DYNAMICS OF A SYSTEM OF PARTICLES.

45.	Linear and Angular Momentum	137
46.	Kinetic Energy	139
47.	Principle of Linear Momentum	141
48.	Principle of Angular Momentum	141
49.	Motion of a Chain	142
50.	Steady Motion of a Chain	144
51.	Impulsive Motion of a Chain	147
	<i>Examples XII</i>	148

CHAPTER VIII.

DYNAMICS OF RIGID BODIES.

ROTATION ABOUT A FIXED AXIS.

52.	Introduction	150
53.	D'Alembert's Principle	150
54.	Rotation about a Fixed Axis	155
55.	The Compound Pendulum	158
56.	Determination of g	160
57.	Torsional Oscillations	163
58.	Bifilar Suspension	164
59.	Reactions on a Fixed Axis	165
60.	Application to the Pendulum	167
	<i>Examples XIII, XIV</i>	169

CHAPTER IX.

DYNAMICS OF RIGID BODIES (CONTINUED). MOTION IN
TWO DIMENSIONS.

ART.		PAGE
61.	Comparison of Angular Momenta about Parallel Axes	173
62.	Rate of Change of Angular Momentum	175
63.	Application to Rigid Bodies	177
64.	Equation of Energy	181
65.	General Theory of a System with One Degree of Freedom	183
66.	Oscillations about Equilibrium. Stability	186
67.	Forced Oscillations of a Pendulum. Seismographs	192
68.	Oscillations of Multiple Systems	196
69.	Stresses in a Moving Body	198
70.	Initial Reactions	200
71.	Instantaneous Impulses	202
72.	The Ballistic Pendulum	207
73.	Effect of Impulses on Energy	209
	<i>Examples XV, XVI</i>	<i>210</i>

CHAPTER X.

LAW OF GRAVITATION.

74.	Statement of the Law	216
75.	Simple Astronomical Applications	217
76.	The Problem of Two Bodies	219
77.	Construction of Orbits	223
78.	Hodograph	227
79.	Formulae for Elliptic Motion	228
80.	Kepler's Three Laws	231
81.	Correction to Kepler's Third Law	234
82.	Perturbations	237
83.	The Constant of Gravitation	241
	<i>Examples XVII, XVIII, XIX</i>	<i>243</i>

CHAPTER XI.

CENTRAL FORCES.

84.	Determination of the Orbit	247
85.	The Inverse Problem	249
86.	Polar Coordinates	251
87.	Disturbed Circular Orbit	256
88.	Apses	259

CONTENTS

xi

ART.		PAGE
89.	Critical Orbits	261
90.	Differential Equation of Central Orbits	263
91.	Law of the Inverse Cube	266
	<i>Examples XX</i>	271

CHAPTER XII.

DISSIPATIVE FORCES.

92.	Resistance varying as the Velocity	274
93.	Constant Propelling Force, with Resistance	276
94.	Theory of Damped Oscillations	277
95.	Forced Oscillations	282
96.	The Spherical Pendulum	287
97.	Quadratic Law of Resistance	290
98.	Case of a Constant Propelling Force	291
99.	Effect of Resistance on Projectiles	294
100.	Effect of Resistance on Planetary Orbits	297
	<i>Examples XXI</i>	298

CHAPTER XIII.

SYSTEMS OF TWO DEGREES OF FREEDOM.

101.	Motion of a Particle on a Smooth Surface	301
102.	Motion on a Spherical Surface	303
103.	The Spherical Pendulum	305
104.	General Motion of a Particle. Lagrange's Equations	308
105.	Applications	313
106.	Mechanical Systems of Double Freedom. Lagrange's Equations	315
107.	Energy. Momentum. Impulse	320
108.	General Theorems	322
109.	Oscillations about Equilibrium.	324
110.	Normal Modes of Vibration	328
111.	Forced Oscillations	331
	<i>Examples XXII, XXIII</i>	334

APPENDIX.

	NOTE ON DYNAMICAL PRINCIPLES	345
--	--	-----

	INDEX	350
--	-----------------	-----