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Sir Horace Lamb (1849–1934) the British mathematician, wrote a number of influential works in classical physics. A pupil of Stokes and Clerk Maxwell, he taught for ten years as the first professor of mathematics at the University of Adelaide before returning to Britain to take up the post of professor of physics at the Victoria University of Manchester (where he had first studied mathematics at Owens College). As a teacher and writer his stated aim was clarity: ‘somehow to make these dry bones live’. His Statics was first published in 1912, and the third edition, offered here, in 1928. It was intended as a textbook for students with some knowledge of mechanics, and deals mainly with two-dimensional problems: examples are provided at the end of each section.

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Statics

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Theory of Elasticity*

HORACE LAMB



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STATICS

INCLUDING HYDROSTATICS AND THE
ELEMENTS OF THE THEORY OF
ELASTICITY

BY

HORACE LAMB, M.A., LL.D., Sc.D., F.R.S.

HONORARY FELLOW OF TRINITY COLLEGE, CAMBRIDGE

LATELY PROFESSOR OF MATHEMATICS IN THE VICTORIA UNIVERSITY OF MANCHESTER

THIRD EDITION

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PREFACE TO THE THIRD EDITION

THIS book had its origin in a course of lectures given for a number of years in the Manchester University, but has received various additions and developments. It is intended for students who have already some knowledge of elementary Mechanics, and who have arrived at the stage at which they may usefully begin to apply the methods of the Calculus. It deals mainly with two-dimensional problems, but occasionally, where the extension to three dimensions is easy, theorems are stated and proved in their more general form.

The present volume differs from many academical manuals in the prominence given to geometrical methods, and in particular to those of Graphical Statics. These methods, especially in relation to the theory of frames, have imported a new interest into a subject which was in danger of becoming fossilized. I have not attempted, however, to enter into details which are best learned from technical treatises, or in engineering practice.

It seemed natural and convenient to treat of Hydrostatics, to a similar degree of development, and I have also, for reasons stated at the beginning of Chap. xv, included the rudiments of the theory of Elasticity. A number of important problems are here discussed by quite elementary methods.

The chapter on Mass-Systems has an interest in the present subject, but I have also had in view, here and elsewhere, the requirements of the companion volume on Dynamics.

I am indebted for some valuable suggestions to Prof. A. Föppl's excellent *Vorlesungen über technische Mechanik*. I have also derived some interesting references from the *Encyclopädie der mathematischen Wissenschaften*.

The examples for practice have been selected (or devised) with some care, and it is hoped that most of them will serve as genuine illustrations of statical principles rather than as exercises in

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PREFACE

Algebra or Trigonometry. Problems of a mainly mathematical character have been excluded, unless there appeared to be some special interest or elegance in the results.

The present edition has been carefully revised; a few sections have been re-written, and some rather serious oversights have been corrected. I shall be much obliged to any readers who will call my attention to such errors or omissions as have still escaped detection.

H. L.

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