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The Theory of Sets of Points

William Henry Young
Grace Chisholm Young

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THE THEORY
OF
SETS OF POINTS

by

W. H. YOUNG, M.A., Sc.D.
Sometime Fellow of Peterhouse, Cambridge,
Lecturer in Higher Analysis at the University of Liverpool,

and

GRACE CHISHOLM YOUNG, Phil.Doc. (Gött.),
Formerly Sir Francis Goldsmid Scholar of Girton College, Cambridge.

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PREFACE.

THE present volume is an attempt at a simple presentation of one of the most recent branches of mathematical science. It has involved an amount of labour which would seem to the average reader quite out of proportion to the size of the book; yet I can scarcely hope that the mode of presentation will appeal equally to all mathematicians. There are no definitely accepted landmarks in the didactic treatment of Georg Cantor's magnificent theory, which is the subject of the present volume. A few of the most modern books on the Theory of Functions devote some pages to the establishment of certain results belonging to our subject, and required for the special purposes in hand. There is moreover in existence the first half of Schoenflies's useful *Bericht über die Mengenlehre.* The philosophical point of view is discussed to some extent in Russell's *Principles of Mathematics.* But we may fairly claim that the present work is the first attempt at a systematic exposition of the subject as a whole.

The difficulties in arrangement which this fact suggests have been enhanced by the nature of the subject itself and by the tentative character of some of its results. The writing of the book has necessarily involved attempts to extend the frontier of existing knowledge, and to fill in gaps which broke the connexion between isolated parts of the subject. The references in the text which do not give the name of the author always refer to my own papers; in this connexion, however, I should like to point out that the citations in the text are not to be regarded as by any means complete, and are supplemented by the list of literature at the end of the volume.

On the other hand, imperfect though the book is felt to be, it is hoped that it may prove of use to a somewhat large class of readers. As far as the professional mathematician is concerned, it may be confidently asserted that a grasp of the Theory of Sets of Points is indispensable. Wherever he has to deal—and where does he not?—with an infinite number of operations, he is treading on ground full of pitfalls, one or more of which may well prove fatal to him, if he is unprovided by the clue to furnish which is the object of the present volume.

In subjects as wide apart as Projective Geometry, Theory of Functions of a Complex Variable, the Expansions of Astronomy, Calculus of Variations, Differential Equations, mistakes have in fact been made by mathematicians of standing, which even a slender grasp of the Theory of Sets of Points would have enabled them to avoid. It can scarcely be doubted that the near future will see a marked influence exerted by our theory on the language and conceptions of Applied Mathematics and Physics. To the philosophical reader on the other hand and to the general public with mathematical interests the subject presents the advantage, as compared with other of the more recent developments of mathematical science, that it is less technical and requires a smaller mathematical equipment than most of them.

I should like to take this opportunity of acknowledging my indebtedness to Professor Vivanti of Messina, who has most carefully read all the proof-sheets, and considered various points submitted to him; his help and criticism have throughout been invaluable. Dr Felix Bernstein of Halle has also been so good as to read the proof-sheets of the first eight chapters with especial reference to the arithmetical portions of the subject. Mr Philip Jourdain, who read Chapters VI and VII, and also looked through the earlier proof-sheets, and Professor Oswald Veblen of Princeton, who undertook Chapters IX and X, have also been of the greatest help with criticisms and suggestions. Any reference to the constant assistance which I have received during my work from my wife is superfluous, since, with the consent of the Syndics of the Press, her name has been associated with mine on the title-page.

In spite, however, of the greatest care to avoid error, clerical or otherwise, mistakes are sure to have escaped notice. The reader is recommended not to overlook the Appendix, in which some mistakes, discovered too late for correction, and some points in the text which seemed to require elucidation are discussed.

W. H. YOUNG.

HESWALL.
May, 1906.

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