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Leipzig: F. A. BROCkHAUS.
New York: THE MACMILLAN COMPANY.
Bombay: E. SEYMOUR HALE.
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MATHEMATICAL PAPERS

OF

ARTHUR CAYLEY, Sc.D., F.R.S.,

LATE SADLERIAN PROFESSOR OF PURE MATHEMATICS IN THE UNIVERSITY OF CAMBRIDGE.

VOL. XIII.

CAMBRIDGE:
AT THE UNIVERSITY PRESS.
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A. R. FORSYTH.

30 November, 1897.
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