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OF

ARTHUR CAYLEY, Sc.D., F.R.S.,

LATE SADLERIAN PROFESSOR OF PURE MATHEMATICS IN THE UNIVERSITY OF CAMBRIDGE.

VOL. XIII.

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THE present volume contains 80 papers, numbered 888 to 967, published for the most part in the years 1889 to 1895, the year in which Professor Cayley died. The volume completes the collection of his mathematical papers.

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A. R. FORSYTH.

30 *November*, 1897.

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