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OF

ARTHUR CAYLEY, Sc.D., F.R.S.,

LATE SADLERIAN PROFESSOR OF PURE MATHEMATICS IN THE UNIVERSITY OF CAMBRIDGE.

VOL. XII.

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The Table for the twelve volumes is

<table>
<thead>
<tr>
<th>Vol.</th>
<th>Numbers</th>
<th>1 to 100,</th>
</tr>
</thead>
<tbody>
<tr>
<td>I.</td>
<td></td>
<td>101</td>
</tr>
<tr>
<td>II.</td>
<td></td>
<td>158</td>
</tr>
<tr>
<td>III.</td>
<td></td>
<td>222</td>
</tr>
<tr>
<td>IV.</td>
<td></td>
<td>299</td>
</tr>
<tr>
<td>V.</td>
<td></td>
<td>383</td>
</tr>
<tr>
<td>VI.</td>
<td></td>
<td>416</td>
</tr>
<tr>
<td>VII.</td>
<td></td>
<td>485</td>
</tr>
<tr>
<td>VIII.</td>
<td></td>
<td>555</td>
</tr>
<tr>
<td>IX.</td>
<td></td>
<td>629</td>
</tr>
<tr>
<td>X.</td>
<td></td>
<td>705</td>
</tr>
<tr>
<td>XI.</td>
<td></td>
<td>798</td>
</tr>
<tr>
<td>XII.</td>
<td></td>
<td>887</td>
</tr>
</tbody>
</table>

A. R. FORSYTH.

19 May, 1897.
CONTENTS.

[An Asterisk means that the paper is not printed in full.]

799. On curvilinear coordinates. . . . . . . . . . . . 1
    Quart. Math. Journ., t. xix. (1883), pp. 1—22

800. Note on the standard solutions of a system of linear equations 19

801. On seminvariants . . . . . . . . . . . . . . . . . . 22

802. Note on Captain MacMahon's paper "On the differential equation \(X^{-\frac{1}{2}}dx + Y^{-\frac{1}{2}}dy + Z^{-\frac{1}{2}}dz = 0\)". . . . . . . 30

803. On Mr Anglin's formula for the successive powers of the root of an algebraical equation . . . . . . . . . . . . . 33
    Quart. Math. Journ., t. xix. (1883), pp. 223, 224

804. On the elliptic-function solution of the equation \(x^2 + y^2 - 1 = 0\) . 35

805. Note on Abel's theorem . . . . . . . . . . . . . . . . 38

806. Determination of the order of a surface . . . . . . . . . 42
    Messenger of Mathematics, t. xii. (1883), pp. 29—32

807. A proof of Wilson's theorem . . . . . . . . . . . . 45
    Messenger of Mathematics, t. xii. (1883), p. 41
<table>
<thead>
<tr>
<th>Number</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>808</td>
<td>Note on a form of the modular equation in the transformation of the third order</td>
<td>46</td>
</tr>
<tr>
<td></td>
<td>Messenger of Mathematics, t. xii. (1883), pp. 173, 174</td>
<td></td>
</tr>
<tr>
<td>809</td>
<td>Schröter’s construction of the regular pentagon</td>
<td>47</td>
</tr>
<tr>
<td></td>
<td>Messenger of Mathematics, t. xii. (1883), p. 177</td>
<td></td>
</tr>
<tr>
<td>810</td>
<td>Note on a system of equations</td>
<td>48</td>
</tr>
<tr>
<td></td>
<td>Messenger of Mathematics, t. xii. (1883), pp. 191, 192</td>
<td></td>
</tr>
<tr>
<td>811</td>
<td>On the linear transformation of the theta-functions</td>
<td>50</td>
</tr>
<tr>
<td></td>
<td>Messenger of Mathematics, t. xiii. (1884), pp. 54–60</td>
<td></td>
</tr>
<tr>
<td>812</td>
<td>On Archimedes’ theorem for the surface of a cylinder</td>
<td>56</td>
</tr>
<tr>
<td></td>
<td>Messenger of Mathematics, t. xiii. (1884), pp. 107, 108</td>
<td></td>
</tr>
<tr>
<td>813</td>
<td>[Note on Mr Griffiths’ paper “On a deduction from the elliptic-integral formula ( y = \sin (A + B + C + \ldots) ) ”]</td>
<td>58</td>
</tr>
<tr>
<td>814</td>
<td>On double algebra</td>
<td>60</td>
</tr>
<tr>
<td>815</td>
<td>The binomial equation ( x^n - 1 = 0 ); quinquisection. Second part</td>
<td>72</td>
</tr>
<tr>
<td>816</td>
<td>On the bitangents of a plane quartic</td>
<td>74</td>
</tr>
<tr>
<td></td>
<td>Crelle’s Journal der Mathem., t. xciv. (1883), pp. 93–115; Camb.</td>
<td></td>
</tr>
<tr>
<td>817</td>
<td>On the sixteen-nodal quartic surface</td>
<td>95</td>
</tr>
<tr>
<td></td>
<td>Crelle’s Journal der Mathem., t. xciv. (1883), pp. 270–272</td>
<td></td>
</tr>
<tr>
<td>818</td>
<td>Note on hyperelliptic integrals of the first order</td>
<td>98</td>
</tr>
<tr>
<td></td>
<td>Crelle’s Journal der Mathem., t. xcviii. (1885), pp. 95, 96</td>
<td></td>
</tr>
<tr>
<td>819</td>
<td>On two cases of the quadric transformation between two planes</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>Johns Hopkins University Circulars, No. 13 (1882), pp. 178, 179</td>
<td></td>
</tr>
<tr>
<td>820</td>
<td>On a problem of analytical geometry</td>
<td>102</td>
</tr>
<tr>
<td></td>
<td>Johns Hopkins University Circulars, No. 15 (1882), p. 209</td>
<td></td>
</tr>
</tbody>
</table>
CONTENTS.

821. *On the geometrical representation of an equation between two variables* .......................... 104

822. *On associative imaginaries* ................................................................. 105
    Johns Hopkins University Circulars, No. 15 (1882), pp. 211, 212

823. *On the geometrical interpretation of certain formula in elliptic functions* ................. 107
    Johns Hopkins University Circulars, No. 17 (1882), p. 238

824. *Note on the formula of trigonometry* ..................................................... 108
    Johns Hopkins University Circulars, No. 17 (1882), p. 241

825. *A memoir on the Abelian and Theta Functions* ........................................... 109

826. *Note on a partition series* ........................................................................... 217
    American Journal of Mathematics, t. vi. (1884), pp. 63, 64

827. *On the non-Euclidian plane geometry* ....................................................... 220

828. *A memoir on seminvariants* ........................................................................... 239
    American Journal of Mathematics, t. vii. (1885), pp. 1–25

829. *Tables of the symmetric functions of the roots, to the degree 10, for the form* 1 + bx + cx² + ... = (1 - ax)(1 - bx)(1 - γx)........................................................................... 263

830. *Non-unitary partition tables* ........................................................................... 273
    American Journal of Mathematics, t. vii. (1885), pp. 57, 58

831. *Seminvariant tables* ..................................................................................... 275

832. *Note on an apparent difficulty in the theory of curves, when the coordinates of a point are given as functions of a variable parameter* ..................................................... 290
    Messenger of Mathematics, t. xiv. (1885), pp. 12–14

C. XII.
## CONTENTS.

833. On a formula in elliptic functions . . . . . . 292

Message of Mathematics, t. xiv. (1885), pp. 21, 22

834. On the addition of the elliptic functions . . . . 294

Message of Mathematics, t. xiv. (1885), pp. 56—61

835. On Cardan's solution of a cubic equation . . . . 299

Message of Mathematics, t. xiv. (1885), pp. 96, 97

836. On the quaternion equation $qQ - Qq' = 0$ . . . . 300

Message of Mathematics, t. xiv. (1885), pp. 108—112

837. On the so-called D'Alembert Carnot geometrical paradox . . . 305

Message of Mathematics, t. xiv. (1885), pp. 113, 114

838. On the twisted cubics upon a quadric surface . . . . 307

Message of Mathematics, t. xiv. (1885), pp. 129—132

839. On the matrical equation $qQ - Qq' = 0$ . . . . 311

Message of Mathematics, t. xiv. (1885), pp. 176—178

840. On Mascheroni's geometry of the compass . . . . 314

Message of Mathematics, t. xiv. (1885), pp. 179—181

841. On a differential operator . . . . . . . . . . 318

Message of Mathematics, t. xiv. (1885), pp. 190, 191

842. On the value of $\tan(\sin \theta) - \sin(\tan \theta)$ . . . . 319

Message of Mathematics, t. xiv. (1885), pp. 191, 192

843. On the quadri-quadric curve in connexion with the theory of elliptic functions . . . . . . . 321

Mathematische Annalen, t. xxv. (1885), pp. 152—156

844. On a theorem relating to semiinvariants . . . . . 326


845. On the orthomorphosis of the circle into the parabola . . . . 328


846. A verification in regard to the linear transformation of the theta-functions . . . . . . . . . 337

Quart. Math. Journ., t. xxi. (1886), pp. 77—84
CONTENTS.

847. On the theory of seminvariants . . . . . . . . . 344


848. On the transformation of the double theta-functions . . 358


849. On the invariants of a linear differential equation . . 390


850. On linear differential equations . . . . . . . . 394


851. On linear differential equations: the theory of decomposition . 403

Quart. Math. Journ., t. xxi. (1886), pp. 331—335

852. Note sur le mémoire de M. Picard “Sur les intégrales de différentielles totales algébriques de première espèce” . 408

Bull. des Sciences Math., 2eme Sér., t. x. (1886), pp. 75—78

853. Note on a formula for \( \Delta^n 0/n^t \) when \( n, i \) are very large numbers . . . . . . . . . . . . . . . . . . . . . . . . 412


854. An algebraical transformation . . . . . . . . 416

Messenger of Mathematics, t. xv. (1886), pp. 58, 59

855. Solution of \((a, b, c, d) = (a', b', c', d')\) . . . . 418

Messenger of Mathematics, t. xv. (1886), pp. 59—61

856. Note on a cubic equation . . . . . . . . . . . 421

Messenger of Mathematics, t. xv. (1886), pp. 62—64

857. Analytical geometrical note on the conic . . . . . . . . 424

Messenger of Mathematics, t. xv. (1886), p. 192

858. Comparison of the Weierstrassian and Jacobian elliptic functions . . . . . . . . . . . . . . . . . . . . . . . . 425

Messenger of Mathematics, t. xvi. (1887), pp. 129—132

859. On the complex of lines which meet a unicursal quartic curve 428

CONTENTS.

860. On Briot and Bouquets theory of the differential equation
\[ F(u, \frac{du}{ds}) = 0 \]

861. Note on a formula relating to the zero-value of a theta-function
Crelle's Journal der Mathehm., t. c. (1887), pp. 87, 88

862. Note on the theory of linear differential equations
Crelle's Journal der Mathem., t. c. (1887), pp. 286—295

863. Note on the theory of linear differential equations
Crelle's Journal der Mathem., t. ci. (1887), pp. 209—213

864. On Rudio's inverse centro-surface

865. On multiple algebra

866. Note on Kiepert's L-equations, in the transformation of elliptic functions
Mathematische Annalen, t. xxx. (1887), pp. 75—77

867. Note on the Jacobian sextic equation
Mathematische Annalen, t. xxx. (1887), pp. 78—84

868. On the intersection of curves
Mathematische Annalen, t. xxx. (1887), pp. 85—90

869. On the transformation of elliptic functions

870. On the transformation of elliptic functions (sequel)
American Journal of Mathematics, t. x. (1888), pp. 71—93

871. A case of complex multiplication with imaginary modulus arising out of the cubic transformation in elliptic functions

872. On the finite number of the covariants of a binary quantic
Mathematische Annalen, t. xxxx. (1889), pp. 319, 320
CONTENTS.

873. System of equations for three circles which cut each other at given angles ... 559
    Messenger of Mathematics, t. xvii. (1888), pp. 18—21

874. Note on the Legendrian coefficients of the second kind ... 562
    Messenger of Mathematics, t. xvii. (1888), pp. 21—23

875. On the system of three circles which cut each other at given angles and have their centres in a line ... 564
    Messenger of Mathematics, t. xvii. (1888), pp. 60—69

876. On systems of rays ... 571
    Messenger of Mathematics, t. xvii. (1888), pp. 73—78

877. Note on the two relations connecting the distances of four points on a circle ... 576
    Messenger of Mathematics, t. xvii. (1888), pp. 94, 95

878. Note on the anharmonic ratio equation ... 578
    Messenger of Mathematics, t. xvii. (1888), pp. 95, 96

879. Note on the differential equation \( \frac{dx}{\sqrt{(1-x^2)}} + \frac{dy}{\sqrt{(1-y^2)}} = 0 \) ... 580
    Messenger of Mathematics, t. xiii. (1889), p. 90

880. Note on the relation between the distance of five points in space ... 581
    Messenger of Mathematics, t. xiii. (1889), pp. 100—102

881. On Hermite's H-product theorem ... 584
    Messenger of Mathematics, t. xiii. (1889), pp. 104—107

882. A correspondence of confocal Cartesians with the right lines of a hyperboloid ... 587
    Messenger of Mathematics, t. xiii. (1889), pp. 128—130

883. Analytical formula in regard to an octad of points ... 590
    Messenger of Mathematics, t. xiii. (1889), pp. 149—152

884. Note sur les surfaces minima et le théorème de Joachimsthal ... 594
    Comptes Rendus, t. cvi. (1888), pp. 995, 996
CONTENTS.

885. On the Diophantine relation, \( z^2 + y^2 = \text{Square} \) . . . . . 596

886. On the surfaces with plane or spherical curves of curvature . . . . . . . 601
          293—306

887. On the theory of groups . . . . . . . . . . . . . . . . . . . . . . . . . 639
     American Journal of Mathematics, t. xi. (1889), pp. 139—157
CLASSIFICATION.

ANALYSIS.

Equations, 801, 803, 835, 855, 856, 867, 878.
Theory of numbers, 807, 815.
Groups, 887.
Quaternions, 836.
Matrices, 839.
Trigonometry, 824, 842.
Symmetric functions, 829, 841.
Partitions, 826, 830.
Invariants and covariants, 800, 801, 828, 831, 844, 847, *872.
Multiple Algebra, 814, 822, 855.
Integral calculus, 852.
Legendre's coefficients, 874.
Finite differences, 853.
Differential equations (general), 802, 860, 879, 885.
Differential equations (linear), 849, 850, 851, 862, 863.
Elliptic functions, 804, 813, 823, 833, 834, 843, 858, 881.
Transformation of elliptic functions, 808, 854, 866, 869, 870, 871.
Abel's theorem, 805.
Abelian functions, 818, 825.
Theta functions, 811, 825, 846, 848, 861.
CLASSIFICATION.

GEOMETRY.

Properties of circles, 873, 875, 877.
Non-Euclidian geometry, 827.
General theory of curves, 820, 832, 868.
Correspondence, 819, 821.
Orthomorphosis, 821, 845.
Plane Quartics, 816.
Groups of points, 880, 883.
General theory of surfaces, 799, 806.
Quadrics, 882.
Quartic surfaces, 817.
Centro-surfaces, 864.
Minimal surfaces, 884.
Rays, 876.
Curves of curvature, 886.
Twisted cubic curves, 838.
Twisted quartic curves, 843, 859.

MISCELLANEOUS, 809, 810, 812, 837, 840, 857.