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# The Collected Mathematical Papers

VOLUME 10

ARTHUR CAYLEY



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OF

ARTHUR CAYLEY, Sc.D., F.R.S.,

LATE SADLERIAN PROFESSOR OF PURE MATHEMATICS IN THE UNIVERSITY OF CAMBRIDGE.

VOL. X.

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A. R. FORSYTH.

11 *June*, 1896.

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