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MATHEMATICAL PAPERS

OF

ARTHUR CAYLEY, Sc.D., F.R.S.,

SADLERIAN PROFESSOR OF PURE MATHEMATICS IN THE UNIVERSITY OF CAMBRIDGE.

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THE present volume contains 69 papers numbered 417 to 485 published for the most part in the years 1866 to 1872; they include a series of astronomical papers published in the *Memoirs* and *Monthly Notices* of the Royal Astronomical Society.

The Portrait in Volume VI. is from the Painting in Oil by Mr Lowes Dickenson in the year 1874, presented by the Subscribers to Trinity College, Cambridge, and now in the Hall of the College: the portrait in the present volume is a photograph of a pencil sketch by Mr Lowes Dickenson in the year 1893.

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