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# The Collected Mathematical Papers

VOLUME 6

ARTHUR CAYLEY



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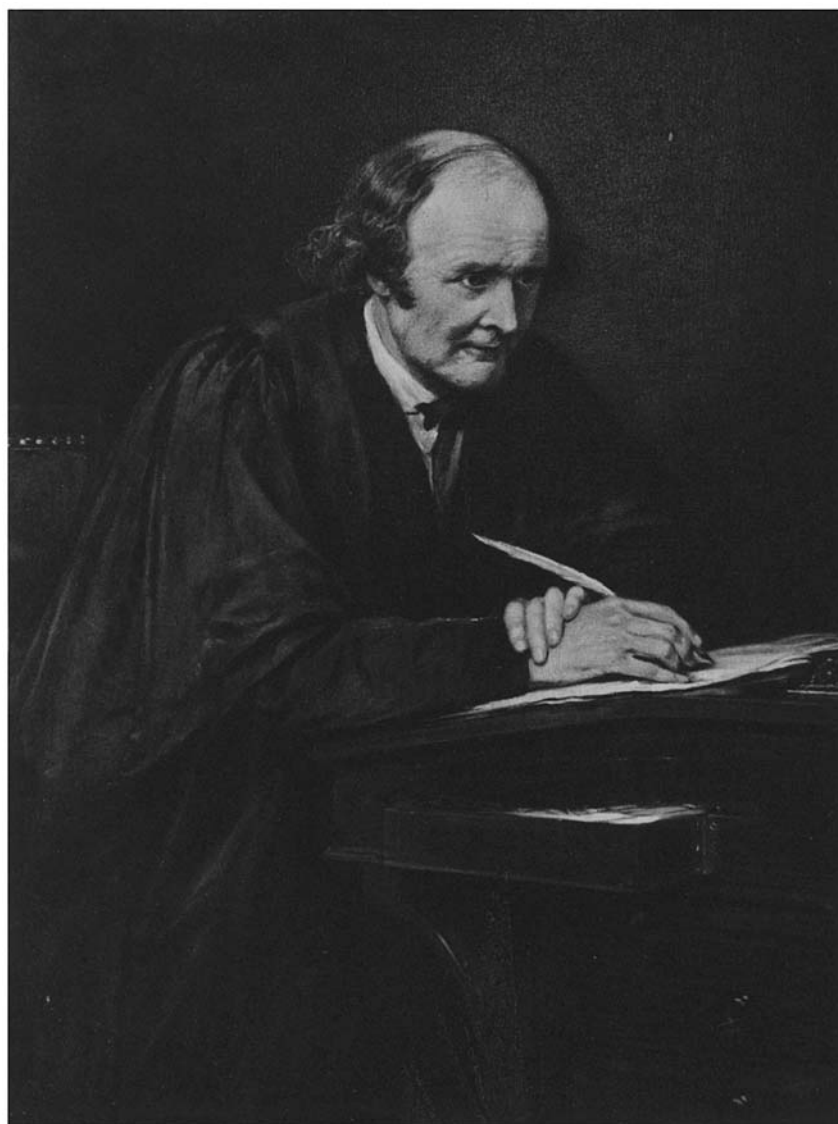
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OF

ARTHUR CAYLEY, Sc.D., F.R.S.,

SADLERIAN PROFESSOR OF PURE MATHEMATICS IN THE UNIVERSITY OF CAMBRIDGE.

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