

## CAMBRIDGE LIBRARY COLLECTION

*Books of enduring scholarly value*

### Mathematical Sciences

From its pre-historic roots in simple counting to the algorithms powering modern desktop computers, from the genius of Archimedes to the genius of Einstein, advances in mathematical understanding and numerical techniques have been directly responsible for creating the modern world as we know it. This series will provide a library of the most influential publications and writers on mathematics in its broadest sense. As such, it will show not only the deep roots from which modern science and technology have grown, but also the astonishing breadth of application of mathematical techniques in the humanities and social sciences, and in everyday life.

### The Collected Mathematical Papers

Arthur Cayley (1821-1895) was a key figure in the creation of modern algebra. He studied mathematics at Cambridge and published three papers while still an undergraduate. He then qualified as a lawyer and published about 250 mathematical papers during his fourteen years at the Bar. In 1863 he took a significant salary cut to become the first Sadleirian Professor of Pure Mathematics at Cambridge, where he continued to publish at a phenomenal rate on nearly every aspect of the subject, his most important work being in matrices, geometry and abstract groups. In 1882 he spent five months at Johns Hopkins University, and in 1883 he became president of the British Association for the Advancement of Science. Publication of his Collected Papers - 967 papers in 13 volumes plus an index volume - began in 1889 and was completed after his death under the editorship of his successor in the Sadleirian Chair. This volume contains 84 papers, mostly published between 1861 and 1866, but including some earlier items from the British Association Reports and a collection of problems and solutions that had originally appeared in the Educational Times.

Cambridge University Press has long been a pioneer in the reissuing of out-of-print titles from its own backlist, producing digital reprints of books that are still sought after by scholars and students but could not be reprinted economically using traditional technology. The Cambridge Library Collection extends this activity to a wider range of books which are still of importance to researchers and professionals, either for the source material they contain, or as landmarks in the history of their academic discipline.

Drawing from the world-renowned collections in the Cambridge University Library, and guided by the advice of experts in each subject area, Cambridge University Press is using state-of-the-art scanning machines in its own Printing House to capture the content of each book selected for inclusion. The files are processed to give a consistently clear, crisp image, and the books finished to the high quality standard for which the Press is recognised around the world. The latest print-on-demand technology ensures that the books will remain available indefinitely, and that orders for single or multiple copies can quickly be supplied.

The Cambridge Library Collection will bring back to life books of enduring scholarly value across a wide range of disciplines in the humanities and social sciences and in science and technology.

# The Collected Mathematical Papers

VOLUME 5

ARTHUR CAYLEY



CAMBRIDGE  
UNIVERSITY PRESS

Cambridge University Press  
978-1-108-00497-8 - The Collected Mathematical Papers, Volume 5  
Arthur Cayley  
Frontmatter  
[More information](#)

---

CAMBRIDGE UNIVERSITY PRESS

Cambridge New York Melbourne Madrid Cape Town Singapore São Paulo Delhi

Published in the United States of America by Cambridge University Press, New York

[www.cambridge.org](http://www.cambridge.org)

Information on this title: [www.cambridge.org/9781108004978](http://www.cambridge.org/9781108004978)

© in this compilation Cambridge University Press 2009

This edition first published 1892

This digitally printed version 2009

ISBN 978-1-108-00497-8

This book reproduces the text of the original edition. The content and language reflect the beliefs, practices and terminology of their time, and have not been updated.

MATHEMATICAL PAPERS.

Cambridge University Press  
978-1-108-00497-8 - The Collected Mathematical Papers, Volume 5  
Arthur Cayley  
Frontmatter  
[More information](#)

---

London: C. J. CLAY AND SONS,  
CAMBRIDGE UNIVERSITY PRESS WAREHOUSE,  
AVE MARIA LANE.



Cambridge: DEIGHTON, BELL AND CO.  
Leipzig: F. A. BROCKHAUS.  
New York: MACMILLAN AND CO.

THE COLLECTED  
  
MATHEMATICAL PAPERS

OF

ARTHUR CAYLEY, Sc.D., F.R.S.,

SADLERIAN PROFESSOR OF PURE MATHEMATICS IN THE UNIVERSITY OF CAMBRIDGE.

VOL. V.

CAMBRIDGE:  
AT THE UNIVERSITY PRESS.

1892

*[All Rights reserved.]*

CAMBRIDGE :  
PRINTED BY C. J. CLAY, M.A. AND SONS,  
AT THE UNIVERSITY PRESS.



## ADVERTISEMENT.

THE present volume contains 84 papers numbered 300 to 383 published for the most part in the years 1861 to 1866: No. 378, Report on Catalogue of Philosophical Memoirs, was however published in the *British Association Report for 1856*, and No. 379, Notices of Communications to the British Association, were published in the *British Association Reports*, 1854 to 1864: the concluding Paper 383, Problems and Solutions, contains problems for the most part geometrical ones proposed or solved by me in the *Educational Times* in the years 1863 to 1865.

The Table for the five volumes is

Vol. I.	Numbers	1	to	100.
„ II.	„	101	„	158.
„ III.	„	159	„	222.
„ IV.	„	223	„	299.
„ V.	„	300	„	383.

## CONTENTS.

[An Asterisk denotes that the paper is not printed in full.]

	PAGE
300. <i>Note relative aux droites en involution de M. Sylvester</i> . . . . .	1
Comptes Rendus, Paris, t. LII. (1861), pp. 1039—1042	
301. <i>Sur les cônes du second ordre qui passent par six points donnés</i> . . . . .	4
Comptes Rendus, Paris, t. LII. (1861), pp. 1216—1218	
302. <i>Considérations générales sur les courbes en espace</i> . . . . .	7
Comptes Rendus, Paris, t. LIV. (1862), pp. 55—60, 396—400, 672—678	
303. <i>Sur le problème du polygone inscrit et circonscrit. Lettre à M. Poncelet</i> . . . . .	21
Comptes Rendus, Paris, t. LV. (1862), pp. 700, 701	
304. <i>Sur un mémoire de Jacobi. Extrait d'une lettre à M. J. Bertrand</i>	23
Comptes Rendus, Paris, t. LVI. (1863), p. 43	
305. <i>Considérations générales sur les courbes en espace. Courbes du cinquième ordre</i> . . . . .	24
Comptes Rendus, Paris, t. LVIII. (1864), pp. 994—1000	
306. <i>Sur les coniques qui touchent des courbes d'ordre quelconque. Extrait d'une lettre à M. Chasles</i> . . . . .	31
Comptes Rendus, Paris, t. LIX. (1864), pp. 224, 225	
307. <i>Note sur les fonctions <math>al(x)</math>, &amp;c., de M. Weierstrass</i> . . . . .	33
Liouville, t. VII. (1862), pp. 137—142	
308. <i>On the <math>\Delta</math> faced Polyacrons, in reference to the problem of the enumeration of Polyhedra</i> . . . . .	38
Manchester Memoirs, t. I. (1862), pp. 248—256	

	PAGE
309. <i>Note on the Theory of Determinants</i> . . . . .	45
Phil. Mag. t. XXI. (1861), pp. 180—185	
310. <i>Note on Mr Jerrard's researches on the Equation of the Fifth Order</i> . . . . .	50
Phil. Mag. t. XXI. (1861), pp. 210—214	
311. <i>On a Theorem of Abel's relating to Equations of the Fifth Order</i> . . . . .	55
Phil. Mag. t. XXI. (1861), pp. 257—263	
312. <i>On the Partitions of a Close</i> . . . . .	62
Phil. Mag. t. XXI. (1861), pp. 424—428	
313. <i>On a Surface of the Fourth Order</i> . . . . .	66
Phil. Mag. t. XXI. (1861), pp. 491—495	
314. <i>On the Curves situate on a Surface of the Second Order.</i> . . . .	70
Phil. Mag. t. XXII. (1861), pp. 35—38	
315. <i>On the Cubic Centres of a Line with respect to three lines and a line</i> . . . . .	73
Phil. Mag. t. XXII. (1861), pp. 433—436	
*316. <i>Note on the solution of an Equation of the Fifth Order</i> . . . . .	77
Phil. Mag. t. XXIII. (1862), pp. 195, 196	
317. <i>Note on the transformation of a certain Differential Equation</i> . . . . .	78
Phil. Mag. t. XXIII. (1862), pp. 266, 267	
*318. <i>On a question in the Theory of Probabilities</i> . . . . .	80
Phil. Mag. t. XXIII. (1862), pp. 361—365	
*319. <i>Postscript to the paper, On a question in the Theory of Probabilities</i> . . . . .	85
Phil. Mag. t. XXIII. (1862), pp. 470, 471	
320. <i>On the Transcendent</i> $\text{gd } u = \frac{1}{i} \log \tan \left( \frac{1}{4}\pi + \frac{1}{2}ui \right)$ . . . . .	86
Phil. Mag. t. XXIV. (1862), pp. 19—21	
*321. <i>Final Remarks on Mr Jerrard's theory of Equations of the Fifth Order</i> . . . . .	89
Phil. Mag. t. XXIV. (1862), p. 290	

CONTENTS.		ix
		PAGE
322.	<i>On a Skew Surface of the Third Order</i> . . . . .	90
	Phil. Mag. t. xxiv. (1862), pp. 514—519	
323.	<i>On a tactical Theorem relating to the Triads of Fifteen Things</i> . . . . .	95
	Phil. Mag. t. xxv. (1863), pp. 59—61	
324.	<i>Note on a Theorem relating to Surfaces</i> . . . . .	98
	Phil. Mag. t. xxv. (1863), pp. 61, 62	
325.	<i>Note on a Theorem relating to a Triangle, Line, and Conic</i> . . . . .	100
	Phil. Mag. t. xxv. (1863), pp. 181—183	
326.	<i>Theorems relating to the Canonic Roots of a Binary Quantic of an odd order</i> . . . . .	103
	Phil. Mag. t. xxv. (1863), pp. 206—208	
327.	<i>On the Stereographic Projection of the Spherical Conic</i> . . . . .	106
	Phil. Mag. t. xxv. (1863), pp. 350—353	
328.	<i>On the delineation of a Cubic Scroll</i> . . . . .	110
	Phil. Mag. t. xxv. (1863), pp. 528—530	
329.	<i>Note on the Problem of Pedal Curves</i> . . . . .	113
	Phil. Mag. t. xxvi. (1863), pp. 20, 21	
330.	<i>On Differential Equations and Umbilici</i> . . . . .	115
	Phil. Mag. t. xxvi. (1863), pp. 373—379 and 441—452	
331.	<i>Analytical Theorem relating to the four Conics inscribed in the same Conic and passing through the same three Points</i>	131
	Phil. Mag. t. xxvii. (1864), pp. 42, 43	
332.	<i>Analytical Theorem relating to the sections of a Quadric Surface</i>	133
	Phil. Mag. t. xxvii. (1864), pp. 43, 44	
333.	<i>Note on the Nodal Curve of the Developable derived from the Quartic Equation <math>(a, b, c, d, e\sqrt{t}, 1)^4 = 0</math></i> . . . . .	135
	Phil. Mag. t. xxvii. (1864), pp. 437—440	
334.	<i>Note on the Theory of Cubic Surfaces</i> . . . . .	138
	Phil. Mag. t. xxvii. (1864), pp. 493—496	
c. v.		b

	PAGE
335. <i>Tables des formes quadratiques binaires pour les déterminants négatifs depuis <math>D = -1</math> jusqu'à <math>D = -100</math>, pour les déterminants positifs non carrés depuis <math>D = 2</math> jusqu'à <math>D = 99</math>, et pour les treize déterminants négatifs irréguliers qui se trouvent dans le premier millier . . . . .</i>	141
Crelle, t. LX. (1862), pp. 357—372	
336. <i>Note sur l'élimination . . . . .</i>	157
Crelle, t. LX. (1862), pp. 373, 374	
337. <i>Note sur la réalité des racines d'une équation quadratique . . . . .</i>	160
Crelle, t. LXI. (1863), pp. 367, 368	
338. <i>Nouvelles recherches sur l'élimination et la théorie des courbes . . . . .</i>	162
Crelle, t. LXIII. (1864), pp. 34—39	
339. <i>On Skew Surfaces, otherwise Scrolls . . . . .</i>	168
Phil. Trans. t. CLIII. (for 1863), pp. 453—483	
340. <i>A Second Memoir on Skew Surfaces, otherwise Scrolls . . . . .</i>	201
Phil. Trans. t. CLIV. (for 1864), pp. 559—576	
341. <i>On the Sextactic Points of a Plane Curve . . . . .</i>	221
Phil. Trans. t. CLV. (for 1865), pp. 545—578	
342. <i>On the Conics which pass through three given points and touch a given line . . . . .</i>	258
Quart. Math. Jour. t. VI. (1864), pp. 24—30	
343. <i>On the Cusp of the second kind or Nodecusp . . . . .</i>	265
Quart. Math. Jour. t. VI. (1864), pp. 74, 75	
344. <i>On Certain Developable Surfaces . . . . .</i>	267
Quart. Math. Jour. t. VI. (1864), pp. 108—126	
345. <i>On the Inflexions of the Cubical Divergent Parabolas . . . . .</i>	284
Quart. Math. Jour. t. VI. (1864), pp. 199—203	
346. <i>Note on an expression for the Resultant of two Binary Cubics . . . . .</i>	289
Quart. Math. Jour. t. VI. (1864), pp. 380—382	
347. <i>On the Notion and Boundaries of Algebra . . . . .</i>	292
Quart. Math. Jour. t. VI. (1864), pp. 382—384	

## CONTENTS.

xi

	PAGE
348. <i>On the Theory of Involution</i> . . . . .	295
Camb. Phil. Trans. t. xi. Part I. (1866), pp. 21—38	
349. <i>On a case of the Involution of Cubic Curves</i> . . . . .	313
Camb. Phil. Trans. t. xi. Part I. (1866), pp. 39—80	
350. <i>On the Classification of Cubic Curves</i> . . . . .	354
Camb. Phil. Trans. t. xi. Part I. (1866), pp. 81—128	
351. <i>On Cubic Cones and Curves</i> . . . . .	401
Camb. Phil. Trans. t. xi. Part I. (1866), pp. 129—144	
352. <i>Suite des recherches sur l'élimination et la théorie des courbes</i>	416
Crelle, t. LXIV. (1865), pp. 167—171	
353. <i>Note sur la surface du quatrième ordre de Steiner</i> . . . . .	421
Crelle, t. LXIV. (1865), pp. 172—174	
354. <i>Note sur les singularités supérieures des courbes planes</i> . . . . .	424
Crelle, t. LXIV. (1865), pp. 369—371	
355. <i>Sur un théorème relatif à huit points situés sur une conique</i> . . . . .	427
Crelle, t. LXV. (1866), pp. 180—184	
356. <i>Sur un cas particulier de la surface du quatrième ordre avec seize points singuliers</i> . . . . .	431
Crelle, t. LXV. (1866), pp. 284—291	
357. <i>A Supplementary Memoir on the Theory of Matrices</i> . . . . .	438
Phil. Trans. t. CLVI. (for 1866), pp. 25—35	
358. <i>Addition to the Memoir on Tschirnhausen's Transformation</i> . . . . .	449
Phil. Trans. t. CLVI. (for 1866), pp. 97—100	
359. <i>A Supplementary Memoir on Caustics</i> . . . . .	454
Phil. Trans. t. CLVII. (for 1867), pp. 7—16	
360. <i>Note on a Quartic Surface</i> . . . . .	465
Phil. Mag. t. XXIX. (1865), pp. 19—22	
361. <i>On Quartic Curves</i> . . . . .	468
Phil. Mag. t. XXIX. (1865), pp. 105—108	

b 2

	PAGE
362. <i>Note on Lobatschewsky's Imaginary Geometry</i> . . . . .	471
Phil. Mag. t. xxix. (1865), pp. 231—233	
363. <i>On the theory of the Evolute</i> . . . . .	473
Phil. Mag. t. xxix. (1865), pp. 344—350	
364. <i>On a Theorem relating to Five Points in a plane</i> . . . . .	480
Phil. Mag. t. xxix. (1865), pp. 460—464	
365. <i>On the Intersections of a Pencil of four lines by a Pencil of two lines</i> . . . . .	484
Phil. Mag. t. xxix. (1865), pp. 501—503	
366. <i>Note on the Projection of the Ellipsoid</i> . . . . .	487
Phil. Mag. t. xxx. (1865), pp. 50—52	
367. <i>On a Triangle in-and-circumscribed to a Quartic Curve</i> . . . . .	489
Phil. Mag. t. xxx. (1865), pp. 340—342	
368. <i>On a problem of Geometrical Permutation</i> . . . . .	493
Phil. Mag. t. xxx. (1865), pp. 370—372	
369. <i>On a property of Commutants</i> . . . . .	495
Phil. Mag. t. xxx. (1865), pp. 411—413	
370. <i>On the signification of an elementary formula of Solid Geometry</i>	498
Phil. Mag. t. xxx. (1865), pp. 413, 414	
371. <i>On a Formula for the intersections of a Line and Conic, and on an Integral Formula connected therewith</i> . . . . .	500
Quart. Math. Jour. t. vii. (1866), pp. 1—6	
372. <i>On the Reciprocation of a Quartic Developable</i> . . . . .	505
Quart. Math. Jour. t. vii. (1866), pp. 87—92	
373. <i>On a Special Sextic Developable</i> . . . . .	511
Quart. Math. Jour. t. vii. (1866), pp. 105—113	
374. <i>On the Higher Singularities of a Plane Curve</i> . . . . .	520
Quart. Math. Jour. t. vii. (1866), pp. 212—223	

## CONTENTS.

xiii

	PAGE
375. <i>Notes on Polyhedra</i> . . . . .	529
Quart. Math. Jour. t. VII. (1866), pp. 304—316	
376. <i>Théorème relatif à l'équilibre de quatre forces</i> . . . . .	540
Comptes Rendus, t. LXI. (1865), pp. 829, 830	
377. <i>Note sur la correspondance de deux points sur une courbe</i> . . . . .	542
Comptes Rendus, t. LXII. (1866), pp. 586—590	
378. <i>Report of a Committee appointed by the British Association to consider the formation of a Catalogue of Philosophical Memoirs (A. Cayley, R. Grant, G. G. Stokes)</i> . . . . .	546
Report of the British Association (1856), pp. 463, 464	
379. <i>Notices of Communications to the British Association for the Advancement of Science</i> . . . . .	549
Brit. Assoc. Reports, Notices and Abstracts of Communications to the Sections (1854 to 1864)	
380. <i>Note on the Rectangular Hyperbola</i> . . . . .	554
Oxford, Camb. and Dubl. Messenger of Mathematics, t. I. (1862), p. 77	
381. <i>Note on Bezout's Method of Elimination</i> . . . . .	555
Oxford, Camb. and Dubl. Messenger of Mathematics, t. II. (1864), pp. 88, 89	
382. <i>Note on the Tetrahedron</i> . . . . .	557
Oxford, Camb. and Dubl. Messenger of Mathematics, t. III. (1866), pp. 8—10	
383. <i>Problems and Solutions</i> . . . . .	560
Mathematical Questions with their Solutions from the Educational Times, vols. I. to IV. (1863 to 1865); for contents, see p. 612	
-----	
<i>Notes and References</i> . . . . .	613
<i>Plates</i> . . . . .	<i>to face pp.</i> 44, 400



## CLASSIFICATION.

### GEOMETRY

The Node-cusp, 343 ; Higher Singularities of Plane Curves, 354, 374  
 Curves in space (as defined by Cone and Monoid Surface), 302, 305  
 Correspondence of Points on Plane Curve, 377  
 Sextactic Points of Plane Curve, 341  
 Lines in Involution, 300 ; Involution, 348, 349  
 Elimination and Theory of Curves, 338, 352  
 Conics touching given Curves, 306  
 Conics, 325, 331, 342, 355, 371, 380 ; Spherical, 327  
 Cubic Curves and Cones, 345, 350, 351  
 Cubic Surfaces 322, 328, 333  
 Quadric Cones and Surfaces, 301, 314, 332, 366  
 Quartic Curves, 361, 367  
 Quartic Surfaces, 313, 360 ; Steiner's, 353 ; 16-nodal, 356  
 Caustics, 359  
 Equilibrium of Four Forces, 376  
 Evolutes, 363  
 In-and-circumscribed Polygon, 303  
 Surfaces, 324 ; Developable, 344, 372, 373 ; Skew, 339, 340  
 Miscellaneous, 315, 325, 364, 365, 368, 370, 382 ; and 383, (see contents, p. 612)  
 Polyhedra, Notes on, 375 : the  $\Delta$ -faced polyacra, 308  
 Lobatschewsky's Imaginary Geometry, 362  
 Partitions of Close, 312  
 Pedal Curves, 329  
 Umbilici, 330

### ANALYSIS

Algebra, the Notion and Boundaries of, 347  
 Differential Equations, 317 ; theorem of Jacobi's, 304 ; for Umbilici, 330  
 Determinants, 309 ; Commutants, 369  
 Binary Quadratic Forms (numerical), tables of, 335  
 Binary Forms, 326, 346  
 Roots of quadratic equation, 337  
 Elimination, 336, 338, 352, 381

- Functions  $al(x)$  of Weierstrass, 307  
Gudermannian, 320  
Integral Formula connected with Line and Conic, 371  
Matrices, 357  
Probabilities, 318\*, 319\*  
Quintic Equations, 310, 311, 316\*, 321\*  
Tschirnhausen's Transformation, 358  
Triads of 15 things, 323
- Report On Catalogue of Scientific Memoirs, 378  
Notices of Communications to the British Association, 378  
Problems and Solutions, 383, see contents, p. 612