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OF

ARTHUR CAYLEY, Sc.D., F.R.S.,

SADLERIAN PROFESSOR OF PURE MATHEMATICS IN THE UNIVERSITY OF CAMBRIDGE.

VOL. IV.

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## ADVERTISEMENT.

THE present volume contains 77 papers numbered 223 to 299 originally published all but one of them in the years 1856 to 1862, the excepted paper, No. 265, Addition to the Memoir on an Extension of Arbogast's Method of Derivations, being now first published.

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