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# The Collected Mathematical Papers

VOLUME 2

ARTHUR CAYLEY



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THE COLLECTED  
  
MATHEMATICAL PAPERS

OF

ARTHUR CAYLEY, Sc.D., F.R.S.,

SADLERIAN PROFESSOR OF PURE MATHEMATICS IN THE UNIVERSITY OF CAMBRIDGE.

VOL. II.

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## ADVERTISEMENT.

THE present volume contains fifty-eight papers (numbered 101, 102, ..., 158) originally published, all but two of them, in the years 1851 to 1860: they are here reproduced nearly but not exactly in chronological order. The two excepted papers are 142, Numerical Tables Supplementary to Second Memoir on Quantics, now first published (1889); and, 143, Tables of the Covariants M to W of the Binary Quintic: from the Second, Third, Fourth, Fifth, Eighth, Ninth and Tenth Memoirs on Quantics (arranged in the present form, 1889): the determination of the finite number, 23, of the covariants of the quintic was made by Gordan in the year 1869, and the calculation of them having been completed in my Ninth and Tenth Memoirs, it appeared to me convenient in the present republication to unite together the values of all the covariants: viz. those of A to L are given in the Second Memoir 141, and the remainder M to W in the paper 143. I have added to the Third Memoir 144, in the notation thereof, some formulæ which on account of a difference of notation were omitted from a former paper, 35.

I remark that the present volume comprises the first six of the ten Memoirs on Quantics, viz. these are 139, 141, 144, 155, 156 and 158. I have, in the Notes and References, inserted a discussion of some length in reference to the paper 121, Note on a Question in the Theory of Probabilities: and also some remarks in reference to the theory of Distance developed in the Sixth Memoir on Quantics, 158.

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