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THE COLLECTED  
  
MATHEMATICAL PAPERS

OF

ARTHUR CAYLEY, Sc.D., F.R.S.,

SADLERIAN PROFESSOR OF PURE MATHEMATICS IN THE UNIVERSITY OF CAMBRIDGE.

VOL. I.

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## PREFACE.

RATHER more than a year ago I was requested by the Syndics of the University Press to allow my mathematical papers to be reprinted in a collected form: I had great pleasure in acceding to a request so complimentary to myself, and I willingly undertook the work of superintending the impression of them, and of adding such notes and references as might appear to me desirable.

The present volume contains one hundred papers (numbered 1, 2, 3, ..., 100) originally published in the years 1841 to 1853. They are here reproduced nearly, but not exactly, in chronological order: and as nearly as may be in their original forms; but in a few cases where the paper is controversial, or where it is a translation (into French or English) of an English or French paper, only the title is given: there are in some few cases omissions which are indicated where they occur. The number is printed in the upper inside corner of the page; it is intended that the numbers shall run consecutively through all the volumes; and thus a paper can be referred to simply by its number.

I have of course corrected obvious typographical errors, and in particular have freely altered punctuation, but I have not attempted to verify formulæ. Additions are made in square brackets [ ]; to avoid confusion with these, square brackets occurring in the original papers have in general been changed into twisted ones { }, but where they occur in a formula it was not always possible to make the alteration. The addition in a square bracket is very frequently that of a date: it appears to me that the proper reference to a serial work is by the number of the volume, accompanied by the date on the title page: the date is always useful, and, in the case of two or more series of a Journal or set of Transactions, we avoid the necessity of a reference to the series; Liouville t. I. (1850) is better than Liouville, Série 2, t. I. I regret that this rule has not been strictly followed as regards the titles of some of the earlier papers, see the remark at the end of the Contents.

A. CAYLEY.

*January 23, 1889.*

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