ELEMENTS OF THE MATHEMATICAL THEORY OF ELECTRICITY AND MAGNETISM

CHAPTER I

GENERAL PRINCIPLES OF ELECTROSTATICS

1. **Example of Electric Phenomena. Electrification. Electric Field.** A stick of sealing-wax after being rubbed with a well dried piece of flannel attracts light bodies such as small pieces of paper or pith balls covered with gold leaf. If such a ball be suspended by a silk thread, it will be attracted towards the sealing-wax, and, if the silk thread is long enough, the ball will move towards the wax until it strikes against it. When it has done this, however, it immediately flies away from the wax; and the pith ball is now repelled from the wax instead of being attracted towards it as it was before the two had been in contact. The piece of flannel used to rub the sealing-wax also exhibits similar attractions for the pith balls, and these attractions are also changed into repulsions after the balls have been in contact with the flannel.

The effects we have described are called 'electric' phenomena, a title which as we shall see includes an enormous number of effects of the most varied kinds. The example we have selected, where electrical effects are produced by rubbing two dissimilar bodies against each other, is the oldest electrical experiment known to science.

The sealing-wax and the flannel are said to be *electrified*, or to be in a state of *electrification*, or to be charged with *electricity*; and the region in which the attractions and repulsions are observed is called the *electric field*.

2. Positive and Negative Electrification. If we take two pith balls A and B, coated with gold leaf and suspended by silk threads, and let them strike against the stick of sealing-wax which has been rubbed with a piece of flannel, they will be found to be repelled, not merely from the sealing-wax but also from each other. To observe this most conveniently remove the pith balls to such a distance from the sealing-wax and the flannel that the effects due to these are inappreciable. Now take another pair of similar balls, C and D, and let them strike against the flannel; C and D will be found to be repelled from each other when they are placed close together. Now take the ball A and place it near C; A and C will be found to be *attracted* towards each other. Thus, a ball which has touched the sealing-wax is repelled from another ball which has been similarly treated, but is attracted towards a ball which has been in contact with the flannel. The electricity on the balls A and B is thus of a kind different from that on the balls C and D, for while the ball A is repelled from B it is attracted towards D, while the ball C is attracted towards B and repelled from D; thus when the ball A is attracted the ball C is repelled and *vice versâ*.

The state of the ball which has touched the flannel is said to be one of *positive electrification*, or the ball is said to be *positively electrified*; the state of the ball which has touched the sealing-wax is said to be one of *negative electrification*, or the ball is said to be *negatively electrified*.

We may for the present regard 'positive' and 'negative' as conventional terms, which when applied to electric phenomena denote nothing more than the two states of electrification described above. As we proceed in the subject, however, we shall see that the choice of these terms is justified, since the properties of positive and negative electrification are, over a wide range of phenomena, contrasted like the properties of the signs *plus* and *minus* in Algebra.

The two balls A and B must be in similar states of electrification since they have been similarly treated; the two balls C and D will also for the same reason be in similar states of electrification. Now A and B are repelled from each other, as are also C and D; hence we see that *two bodies in similar states of electrification are repelled from each other:* while, since one of the pair A, B is attracted towards either of the pair C, D, we see that *two bodies, one in a positive state of electrification, the other in a negative state, are attracted towards each other.*

In whatever way a state of electrification is produced on a body, it is found to be one or other of the preceding kinds; i.e. the ball A is either repelled from the electrified body or attracted towards it. In the former case the electrification is negative, in the latter positive.

A method, which is sometimes convenient, of detecting whether the electrification of a body is positive or negative is to dust it with a mixture of powdered red lead and yellow sulphur which has been well shaken; the friction of the one powder against the other electrifies both powders, the sulphur becoming negatively, the red lead positively electrified. If now we dust a negatively electrified surface with this mixture, the positively electrified red lead will stick to the surface, while the negatively electrified sulphur will be easily detached, so that if we blow on the powdered surface the sulphur will come off while the red lead will remain, and thus the surface will be coloured red: if a positively electrified surface is treated in this way it will become yellow in consequence of the sulphur sticking to it.

3. Electrification by Induction. If the negatively electrified stick of sealing-wax used in the preceding experiments is held near to, but not touching, one end of an elongated piece of metal supported entirely on glass or ebonite stems, and if the metal is dusted over with the mixture of red lead and sulphur, it will be found, after blowing off the loose powder, that the end of the metal nearest to the sealing-wax is covered with the yellow sulphur, while the end furthest away is covered with red lead, showing that the end of the metal nearest the negatively electrified stick of sealing-wax is positively, the end remote from it negatively, electrified. In this experiment the metal, which has neither been rubbed nor been in contact with an electrified body, is said to be electrified by *induction*; the electrification on the metal is said to be *induced* by the electrification on the stick of sealing-wax. The electrification on the part of the metal nearest the wax is of the kind opposite to that on the wax, while the electrification on the more remote parts of the metal is of the same kind as that on the wax. The electrification on the metal disappears as soon as the stick of sealing-wax is removed.

4. **Electroscope.** An instrument by which the presence of electrification can be detected is called an *electroscope*. All electroscopes give some indication of the amount of the electrification, but if accurate measurements are required a special form of electroscope or a more elaborate instrument, called an electrometer (Art. 60), is generally used.

A simple form of electroscope, called the gold leaf electroscope, is represented in Fig. 1. It consists of a

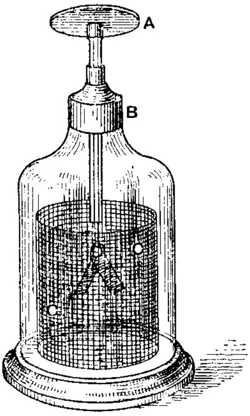

Fig. 1.

glass vessel fitting into a stand; a metal rod, with a disc of metal at the top and terminating below in two strips of gold leaf, passes through the neck of the vessel the rod passing through a glass tube covered inside and out with sealing-wax or shellac varnish and fitting tightly into a plug in the mouth of the vessel.

When the gold leaves are electrified they are repelled from each other and diverge, the amount of the divergence giving some indication of the degree of electrification. It is desirable to protect the gold leaves from the influence of electrified bodies which may happen to be near the electroscope, and from any electrification there may be on the surface of the glass. To do this we take advantage of the property of electrical action (proved in Art. 33), that a closed metallic vessel completely protects bodies inside it from the electrical action of bodies outside. Thus if the gold leaves could be completely surrounded by a metal vessel, they would be perfectly shielded from extraneous electrical influence: this however is not practicable, as the metal case would hide the gold leaves from observation. In practice, sufficient protection is afforded by a cylinder of metal gauze connected to earth, such as is shown in Fig. 1, care being taken that the top of the gauze cylinder reaches above the gold leaves.

If the disc of the electroscope is touched by an electrified body, part of the electrification will go to the gold leaves; these will be electrified in the same way, and therefore will be repelled from each other. In this case the electrification on the gold leaves is of the same sign as that on the electrified body. When the electrified body does not touch the disc but is held near to it, the metal parts of the electroscope will be electrified by induction; the disc, being the part nearest the electrified body, will have electrification opposite to that of the body, while the gold leaves, being the parts furthest from the electrified body, will have the same kind of electrification as the body, and will repel each other. This repulsion will cease as soon as the electrified body is removed.

If, when the electrified body is near the electroscope, the disc is connected to the ground by a metal wire, then the metal of the electroscope, the wire and the ground, will correspond to the elongated piece of metal in the experiment described in Art. 3. Thus, supposing the body to be negatively electrified, the positive electrification will be on the disc, while the negative will go to the most remote part of the system consisting of the metal of the electroscope, the wire and the ground, i.e. the negative electrification will go to the ground and the gold leaves will be free from electrification. They cease then to repel each other and remain closed. If the wire is removed from the disc while the electrified body remains in the neighbourhood, the gold leaves will remain closed as long as the electrified body remains stationary, but if this is removed far away from the electroscope the gold leaves diverge. The positive electrification, which, when the electrified body was close to the electroscope, concentrated itself on the disc so as to be as near the electrified body as possible, when this body is removed spreads to the gold leaves and causes them to diverge.

If, when the electroscope is charged, we wish to determine whether the charge is positive or negative, all we have to do is to bring near to the disc of the electroscope a stick of sealing-wax, which has been negatively electrified by friction with flannel; the proximity of the negatively electrified wax, in consequence of the induction (Art. 3), increases the negative electrification on the gold leaves. Hence, if the presence of the sealing-wax increases the divergence of the leaves, the original electrification was negative, but if it diminishes the divergence the original electrification was positive.

5. Charge on an electrified body. Definition of equal charges. Place on the disc of the electroscope a metal vessel as nearly closed as possible, the opening being only just wide enough to allow electrified

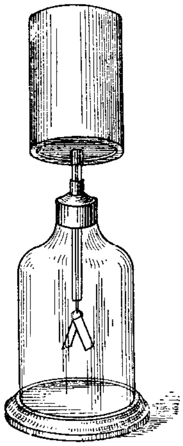

Fig. 2.

bodies to be placed inside. Then introduce into this vessel a charged body suspended by a silk thread, and let it sink well below the opening. The gold leaves of the electroscope will diverge, since they will be electrified by induction (see Art. 3), but the divergence will remain the same however the body is moved about in the vessel. If two or more electrified bodies are placed in the vessel the divergence of the gold leaves is the same however the electrified bodies are moved about relatively to each other or to the vessel. The divergence of the gold leaves thus measures some property of the electrified body which remains constant however the body is moved about within the vessel. This property is called the charge on the body, and two bodies, A and B, have equal charges when the divergence of the gold leaves is the same when A is inside the vessel placed on the disc of the electroscope and B far away, as when B is inside and A far away. A and B are each supposed to be suspended by dry silk threads, for such threads do not allow the electricity to escape along them; see Art. 6. Again, the charge on a body C is twice that on A if, when C is introduced into the vessel, it produces the same effect on the electroscope as that produced by A and B when introduced together. B is a body whose charge has been proved equal to that on A in the way just described. Proceeding in this way we can test what multiple the charge on any given electrified body is of the charge on another body, so that if we take the latter charge as the unit charge we can express any charge in terms of this unit.

Two bodies have equal and opposite charges if when introduced simultaneously into the metal vessel they produce no effect on the divergence of the gold leaves.

6. **Insulators and Conductors.** Introduce into the vessel described in the preceding experiment an electrified pith ball coated with gold leaf and suspended by a dry silk thread: this will cause the gold leaves to diverge. If now the electrified pith ball is touched with a stick of sealing-wax, an ebonite rod or a dry piece of glass tube, no effect is produced on the electroscope, the divergence of the gold leaves is the same after the pith ball has been touched as it was before. If, however, the pith ball is touched with a metal wire held in the hand or by the hand itself, the gold leaves of the electroscope immediately fall together and remain closed after the wire has been

withdrawn from the ball. Thus the pith ball loses its charge when touched with a metal wire, though not when touched with a piece of sealing-wax. We may thus divide bodies into two classes, (1) those which, when placed in contact with a charged body, can discharge the electrification, these are called *conductors*; (2) those which can not discharge the electrification of a charged body with which they are in contact, these are called *insulators.* The metals, the human body, solutions of salts or acids are examples of conductors, while the air, dry silk threads, dry glass, ebonite, sulphur, paraffin wax, sealing-wax, shellac are examples of insulators.

When a body is entirely surrounded by insulators it is said to be *insulated.*

7. *When electrification is excited by friction or by any other process, equal charges of positive and negative electricity are always produced.* To show this, when the electrification is excited by friction, take a piece of sealing-wax and electrify it by friction with a piece of flannel; then, though both the wax and the flannel are charged with electricity, they will, if introduced together into the metal vessel on the disc of the electroscope (Art. 5), produce no effect on the electroscope, thus showing that the charge of negative electricity on the wax is equal to the charge of positive electricity on the flannel. This can be shown in a more striking way by working a frictional electrical machine, insulated and placed inside a large insulated metal vessel in metallic connexion with the disc of an electroscope; then, although the most vigorous electrical effects can be observed near the machine inside the vessel, the leaves of the electroscope remain unaffected.