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Arthur Stanley Ramsey

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A.S. Ramsey (1867-1954) was a distinguished Cambridge mathematician and President of Magdalene College. He wrote several textbooks 'for the use of higher divisions in schools and for first-year students at university'. This book on dynamics, published in 1929, was based upon his lectures to students of the mathematical tripos, and reflects the way in which this branch of mathematics had expanded in the first three decades of the twentieth century. It assumes some knowledge of elementary dynamics, and contains an extensive collection of examples for solution, taken from scholarship and examination papers of the period. The subjects covered include vectors, rectilinear motion, harmonic motion, motion under constraint, impulsive motion, moments of inertia and motion of a rigid body. Ramsey published a companion volume, Statics, in 1934.

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Dynamics

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the Universities*

ARTHUR STANLEY RAMSEY



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CAMBRIDGE UNIVERSITY PRESS

Cambridge New York Melbourne Madrid Cape Town Singapore São Paulo Delhi

Published in the United States of America by Cambridge University Press, New York

www.cambridge.org

Information on this title: www.cambridge.org/9781108003148

© in this compilation Cambridge University Press 2009

This edition first published 1929

This digitally printed version 2009

ISBN 978-1-108-00314-8

This book reproduces the text of the original edition. The content and language reflect
the beliefs, practices and terminology of their time, and have not been updated.

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Cambridge University Press

Fetter Lane, London

New York

Bombay, Calcutta, Madras

Toronto

Macmillan

Tokyo

Maruzen-Kabushiki-Kaisha

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DYNAMICS
A Text-Book for the use of the
Higher Divisions in Schools
and for
First Year Students at the Universities

by

A. S. RAMSEY, M.A.

*President of Magdalene College,
Cambridge; and University Lecturer
in Mathematics*



CAMBRIDGE
AT THE UNIVERSITY PRESS
1929

Cambridge University Press

978-1-108-00314-8 - Dynamics: A Text-Book for the Use of the Higher Divisions in
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PRINTED IN GREAT BRITAIN

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P R E F A C E

This book is intended primarily for the use of students in the higher divisions in schools, particularly for those who intend to take an Honours Course of Mathematics at a University, and also for University students preparing for a first Honours Examination. It is based upon courses of lectures given during many years to first-year students preparing for the Mathematical Tripos, and it is assumed that the majority of readers will already have acquired some knowledge of elementary dynamics. Although the book contains chapters on Orbits and the dynamics of Rigid Bodies, none the less it may claim to be a text-book on *Elementary Dynamics*, for there is probably no branch of elementary Mathematics the content of which has expanded so greatly in the last twenty years.

One of the changes that accompanied the reform of the Mathematical Tripos was the removal of the restriction that Elementary Mechanics meant Mechanics without the Calculus. This restriction set well-defined and narrow bounds to the subject and the new regulations which gave teachers and students freedom to use any analytical methods in their work have been far reaching in their effect. Though the schedule in Dynamics for Part I of the new Tripos has remained unaltered, successive Examiners have added considerably to the interpretation of its contents. To give one instance only—the phrase ‘motion under gravity’ is now understood to mean ‘in a resisting medium’—and it would be easy to give other examples of the elasticity of interpretation to which the schedule lends itself. The result of this change is that a first-year course in Dynamics at the University now includes all the easier problems of two-dimensional dynamics stopping short of the use of moving axes and Lagrange’s Equations. This growth in the content of Elementary Dynamics has been a gradual process and undoubtedly beneficial to the study of the subject and stimulating to the average student. It is inevitable that its effect will extend to the schools, if it has not already done so; and it is not unreasonable to suppose that before many

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PREFACE

years have passed, candidates for Scholarships in Mathematics will be expected to possess a wider knowledge of dynamics embracing such parts of the subject as 'motion under simple central forces' and the elements of uniplanar rigid dynamics. The object of this book is to assist in this development. It is hoped that the presentation of the subject will prove sufficiently simple. An attempt has been made to preserve the conciseness of lecture notes and at the same time to give detailed explanations where experience has shewn that students find difficulties. Besides examples for solution the book contains a large number of worked examples; some of these are of purpose very simple illustrations of the theory, while others are of a more difficult kind for the assistance of readers who wish to learn how to work harder examples. The examples are nearly all taken from Scholarship papers or Tripos papers and the source is indicated by the letters S. and M. T. No attempt is made to exhaust the subject and the later chapters are only intended to be suggestive of the kinds of problems that can be solved, without elaborate analysis, as examples of the fundamental theorems; some few of these may prove to be too difficult for weaker students and they are intended rather to introduce abler students to more advanced work.

In conclusion I desire to express my thanks to the printers and readers of the University Press for their excellent work in the setting up of the book and the elimination of mistakes, and also to say that if the book contains errors I shall be grateful to anyone who will point them out.

A. S. RAMSEY

30 Nov. 1928
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