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Elements of Algebra

In 1770, one of the founders of pure mathematics, Swiss mathematician Leonard Euler (1707-1783), published Elements of Algebra, a mathematics textbook for students. This edition of Euler's classic, published in 1822, is an English translation which includes notes added by Euler's tutor, Johann Bernoulli, and additions by Joseph-Louis Lagrange, both giants in eighteenth-century mathematics, as well as a short biography of Euler. Part 1 begins with elementary mathematics of determinate quantities and includes four sections on simple calculations (adding, subtracting, division, multiplication), and then progresses to compound calculations (fractions), ratios and proportions and algebraic equations. Part 2 consists of 15 chapters on analyses of indeterminate quantities. Here, Euler shows the reader several ways to solve polynomial equations up to the fourth degree. This landmark book showed students the beauty of mathematics, and more significantly, how to do it.

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Elements of Algebra

Leonard Euler

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the beliefs, practices and terminology of their time, and have not been updated.

ELEMENTS OF ALGEBRA,

BY

L E O N A R D   E U L E R,

TRANSLATED FROM THE FRENCH;

WITH THE

NOTES OF M BERNOULLI, &c.

AND THE

ADDITIONS OF M. DE LA GRANGE.

THIRD EDITION,
CAREFULLY REVISED AND CORRECTED.

BY THE REV. JOHN HEWLETT, B.D. F.A.S. &c.

TO WHICH IS PREFIXED

A Memoir of the Life and Character of Euler,

BY THE LATE

FRANCIS HORNER, ESQ., M. P.

LONDON:
PRINTED FOR LONGMAN, HURST, REES, ORME, AND CO.
PATERNOSTER-ROW.

1822.

ADVERTISEMENT.

HAVING prefixed my name to the present edition of Euler's Algebra, it may be proper to give some account of the Translation; which I shall do with the greater pleasure, because it furnishes a favorable opportunity of associating my own labors, with those of my distinguished pupil, and most excellent friend, the late FRANCIS HORNER, M. P.

When first placed under my tuition, at the critical and interesting age of seventeen, he soon discovered uncommon powers of intellect, and the most ardent thirst for knowledge, united with a docility of temper, and a sweetness of disposition, which rendered instruction, indeed, a " delightful task." His diligence and attention were such, as to require the frequent interposition of some rational amusement, in order to prevent the intenseness of his application from injuring a constitution, which, though not delicate, had never been robust.

Perceiving that the natural tendency of his mind led to the exercise of reason, rather than to the indulgence of fancy; that he was particularly interested in discussing the merits of some specious theory, in exposing fallacies, and in forming legitimate inductions, from any premises, that were

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supposed to rest on the basis of truth : but finding
also, that, from imitation and habit, he had been
led to think too highly of those metaphysical
speculations, which abound in terms to which we
annex no distinct ideas, and which often require
the admission of principles, that are either unintel-
ligible, or incapable of proof; I recommended to
his notice Euler's Algebra, as affording an ad-
mirable exercise of his reasoning powers, and the
best means of cultivating that talent for analysis,
close investigation, and logical inference, which
he possessed at an early period, and which he after-
wards displayed in so eminent a degree. At the
same time, I was of opinion, that to translate a
part of that excellent work from the French into
English, when he wished to vary his studies, would
improve his knowledge of both languages, and be
the best introduction for him to the mathematics.

He was soon delighted with this occasional em-
ployment, which seemed to supply his mind with
food, that was both solid and nutricious ; and he
generally produced, two or three times a week, as
much as I could find time to revise and correct.
In the course of the first twelvemonth, he had
translated so large a portion of the two volumes,
that it was determined to complete the whole, and
to publish it for the benefit of English students :
but he returned to Scotland before the manuscript
was ready for the press ; and, therefore, the labor
of editing it necessarily devolved on me.

I wished to give this short history of the Trans-

lation at first, without any eulogium on his cha-
racter and talents, while living, of course; but he
modestly, though, at the same time, resolutely
opposed it, saying that whatever merit or emolu-
ment might be attached to the work, it belonged
to me. The same proposal was made to him,
on publishing the second edition*; but he still
persisted in his former determination.

From the pleasure and instruction which he re-
ceived from Euler's Algebra, it was natural for
him to wish to know something more of the life
and character of that profound mathematician.
Having therefore in some measure satisfied his
curiosity, and collected the necessary materials,
by consulting the ordinary sources of information,
I advised him, by way of literary exercise, to draw
up a biographical Memoir on the subject. He
readily complied with my wishes; and this may
be considered as one of his earliest productions.
Its merits would do credit, in my opinion, to any
writer; and therefore in appreciating them, the
reader will not deem any apology necessary on
account of the author's youth.

I have been led into this short detail of circum-
stances, first, because I disdain the contemptible
vanity of shining in what may be thought bor-
rowed plumes, and because I feel a melancholy
pleasure in speaking of my highly valued, and

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* The care of correcting the press for this edition was en-
trusted to Mr. P. Barlow, being engaged myself, at that time,
in the laborious employment of editing the Bible.

much lamented friend. The English nation will long remember, and ever estimate, as they ought, his manly eloquence in the senate; his lofty spirit of independence, which had no mixture of pride, or affectation; his enlarged views and inflexible integrity; his vigilance and activity in the discharge of public duties; his fairness and liberality, his temperance and firmness in debate; his accurate, various, and extensive knowledge; the soundness of his argumentation, and the sagacity with which he unveiled deception, without coveting any triumph, or wishing to inflict disgrace; and his calm, but dignified opposition, which often confuted the errors, and exposed the misapprehensions of his opponents; but without ever provoking resentment, or making an enemy.

All these qualities, however rare when united, it is well known, he possessed; and, on this subject, many members on both sides of the House of Commons have borne the most ample testimony: but those only who enjoyed the happiness of being numbered among his intimate friends, could form any adequate idea of the uncommon *affectionateness* of his character; his lasting, disinterested, and sincere attachments; his gentle, unassuming manners; and his readiness, at all times, to do good, and to relieve the distressed, without the slightest tincture of vanity, or ostentation. In the discharge of his duties as a son and a brother, it is almost needless to add, that his conduct was most exemplary.

His loss as a public character will be long felt,

and deplored; and, in private life, it has produced a chasm, that can never be filled up. To have had some share in directing the studies, cultivating the talents, and forming the taste of such a man, will always be to me a source of the greatest satisfaction. That he should have fallen a victim to lingering disease, in the prime of manhood, and before he had reached the meridian of his brilliant and useful career, is truly deplorable; yet we should be thankful for what we once possessed. He is indeed gone; but "though dead he still liveth." All regret for his premature death is vain; and it should be remembered, that humble resignation to the Divine Will is one of the first duties of every human being.

> " His saltem accumulem donis, et fungar inani
> " Munere."

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In this third edition, the two volumes have been compressed into one; the whole has been very carefully revised and corrected; the Notes will be found at the foot of the pages, to which they respectively belong; the Questions for Practice, which were omitted in the last edition, have been restored; and though it is scarcely possible to print a mathematical work, of any extent, without some *errata;* yet it is hoped, that few can be named, which will be found more correct than the present.

JOHN HEWLETT.

Hunter-street, March, 1822.

MEMOIR

OF THE

LIFE AND CHARACTER OF EULER,

BY THE LATE

FRANCIS HORNER, ESQ., M. P.

————

LEONARD EULER was the son of a clergyman in the neighbourhood of Basil, and was born on the 15th of April, 1707. His natural turn for mathematics soon appeared, from the eagerness and facility with which he became master of the elements under the instructions of his father, by whom he was sent to the university of Basil at an early age. There, his abilities and his application were so distinguished, that he attracted the particular notice of John Bernoulli. That excellent mathematician seemed to look forward to the youth's future achievements in science, while his own kind care strengthened the powers by which they were to be accomplished. In order to superintend his studies, which far outstripped the usual routine of the public lecture, he gave him a private lesson regularly once a week; when they conversed together on the acquisitions, which the pupil had been making since their last interview, considered whatever difficulties might have occurred in his

progress, and arranged the reading and exercises for the ensuing week.

Under such eminent advantages, the capacity of Euler did not fail to make rapid improvements; and in his seventeenth year, the degree of Master of Arts was conferred on him. On this occasion, he received high applause for his probationary discourse, the subject of which was a comparison between the Cartesian and Newtonian systems.

His father, having all along intended him for his successor, enjoined him now to relinquish his mathematical studies, and to prepare himself by those of theology, and general erudition, for the ministerial functions. After some time, however, had been consumed, this plan was given up. The father, himself a man of learning and liberality, abandoned his own views for those, to which the inclination and talents of his son were of themselves so powerfully directed; persuaded, that in thwarting the propensities of genius, there is a sort of impiety against nature, and that there would be real injustice to mankind in smothering those abilities, which were evidently destined to extend the boundaries of science. Leonard was permitted, therefore, to resume his favorite pursuits; and, at the age of nineteen, transmitting two dissertations to the Academy of Sciences at Paris, one on the masting of ships, and the other on the philosophy of sound, he commenced that splendid career, which continued, for so long a period, the admiration and the glory of Europe.

About the same time, he stood candidate for a

vacant professorship in the university of Basil;
but having lost the election, he resolved, in con-
sequence of this disappointment, to leave his na-
tive country; and in 1727 he set out for Peters-
burg, where his friends, the young Bernoullis, had
settled about two years before, and where he
flattered himself with prospects of literary success
under the patronage of Catherine I. Those pro-
spects, however, were not immediately realised;
nor was it till after he had been frequently and
long disappointed, that he obtained any prefer-
ment. His first appointment appears to have been
to the chair of natural philosophy; and when
Daniel Bernoulli removed from Petersburg, Euler
succeeded him as professor of mathematics.

In this situation he remained for several years,
engaged in the most laborious researches, enrich-
ing the academical collections of the continent
with papers of the highest value, and producing
almost daily improvements in the various branches
of physical, and, more particularly, analytical
science. In 1741, he complied with a very press-
ing invitation from Frederic the Great, and re-
sided at Berlin till 1766. Throughout this pe-
riod, he continued the same literary labors, di-
rected by the same wonderful sagacity and com-
prehension of intellect. As he advanced with his
own discoveries and inventions, the field of know-
ledge seemed to widen before his view, and new
subjects still multiplied on him for further specula-
tion. The toils of intense study, with him, seemed
only to invigorate his future exertions. Nor did

the energies of Euler's mind give way, even when
the organs of the body were overpowered : for in
the year 1735, having completed, in three days,
certain astronomical calculations, which the aca-
demy called for in haste ; but which several ma-
thematicians of eminence had declared could not
be performed within a shorter period than some
months, the intense application threw him into a
fever, in which he lost the sight of one eye.

Shortly after his return to Petersburg, in 1766,
he became totally blind. His passion for science,
however, suffered no decline ; the powers of his
mind were not impaired, and he continued as in-
defatigable as ever. Though the distresses of age
likewise were now crowding fast upon him, for he
had passed his sixtieth year ; yet it was in this
latter period of his life, under infirmity, bodily
pain, and loss of sight, that he produced some of
his most valuable works ; such as command our
astonishment, independently of the situation of
the author, from the labor and originality which
they display. In fact, his habits of study and
composition, his inventions and discoveries, closed
only with his life. The very day on which he
died, he had been engaged in calculating the orbit
of Herschel's planet, and the motions of aërostatic
machines. His death happened suddenly in Sep-
tember 1783, from a fit of apoplexy, when he was
in the seventy-sixth year of his age.

Such is the short history of this illustrious man.
The incidents of his life, like that of most other

laborious students, afford very scanty materials for
biography; little more than a journal of studies
and a catalogue of publications : but curiosity may
find ample compensation in surveying the charac-
ter of his mind.   An object of such magnitude,
so far elevated above the ordinary range of human
intellect, cannot be approached without reverence,
nor nearly inspected, perhaps, without some de-
gree of presumption.   Should an apology be ne-
cessary, therefore, for attempting the following
estimate of Euler's character, let it be considered,
that we can neither feel that admiration, nor offer
that homage, which is worthy of genius, unless,
aiming at something more than the dazzled sensa-
tions of mere wonder, we subject it to actual ex-
amination, and compare it with the standards of
human nature in general.

Whoever is acquainted with the memoirs of
those great men, to whom the human race is in-
debted for the progress of knowledge, must have
perceived, that, while mathematical genius is di-
stinct from the other departments of intellectual
excellence, it likewise admits in itself of much di-
versity. The subjects of its speculation are become
so extensive and so various, especially in modern
times, and present so many interesting aspects, that
it is natural for a person, whose talents are of this
cast, to devote his principal curiosity and attention
to particular views of the science.   When this hap-
pens, the faculties of the mind acquire a superior
facility of operation, with respect to the objects

towards which they are most frequently directed,
and the invention becomes habitually most active
and most acute in that channel of inquiry.

The truth of these observations is strikingly
illustrated by the character of Euler. His studies
and discoveries lay not among the lines and figures
of geometry, those characters, to use an expres-
sion of Galileo, in which the great book of the
universe is written; nor does he appear to have
had a turn for philosophising by experiment, and
advancing to discovery through the rules of in-
ductive investigation. The region, in which he
delighted to speculate, was that of pure intellect.
He surveyed the properties and affections of
quantity under their most abstracted forms. With
the same rapidity of perception, as a geometrician
ascertains the relative position of portions of exten-
sion, Euler ranges among those of abstract quan-
tity, unfolding their most involved combinations,
and tracing their most intricate proportions. That
admirable system of mathematical logic and lan-
guage, which at once teaches the rules of just
inference, and furnishes an instrument for prose-
cuting deductions, free from the defects which
obscure and often falsify our reasonings on other
subjects; the different species of quantity, whether
formed in the understanding by its own abstrac-
tions, or derived from modifications of the repre-
sentative system of signs; the investigation of the
various properties of these, their laws of genesis,
the limits of comparison among the different

species, and the method of applying all this to the solution of physical problems; these were the researches on which the mind of Euler delighted to dwell, and in which he never engaged without finding new objects of curiosity, detecting sources of inquiry, which had passed unobserved, and exploring fields of speculation and discovery, which before were unknown.

The subjects, which we have here slightly enumerated, form, when taken together, what is called the Modern Analysis: a science eminent for the profound discoveries which it has revealed; for the refined artifices that have been devised, in order to bring the most abstruse parts of mathematics within the compass of our reasoning powers, and for applying them to the solution of actual phænomena, as well as for the remarkable degree of systematic simplicity, with which the various methods of investigation are employed and combined, so as to confirm and throw light on one another. The materials, indeed, had been collecting for years, from about the middle of the seventeenth century; the foundations had been laid by Newton, Leibnitz, the elder Bernoullis, and a few others; but Euler raised the superstructure: it was reserved for him to work upon the materials, and to arrange this noble monument of human industry and genius in its present symmetry. Through the whole course of his scientific labors, the ultimate and the constant aim on which he set his mind, was the perfection of Calculus

and Analysis. Whatever physical inquiry he be-
gan with, this always came in view, and very fre-
quently received more of his attention than that
which was professedly the main subject. His
ideas ran so naturally in this train, that even in
the perusal of Virgil's poetry, he met with images
that would recall the associations of his more fa-
miliar studies, and lead him back, from the fairy
scenes of fiction, to mathematical abstraction, as
to the element, more congenial to his nature.

That the sources of analysis might be ascertained
in their full extent, as well as the various modifica-
tions of form and restrictions of rule that become
necessary in applying it to different views of
nature; he appears to have nearly gone through a
complete course of philosophy. The theory of
rational mechanics, the whole range of physical
astronomy, the vibrations of elastic fluids, as well
as the movements of those which are incom-
pressible, naval architecture and tactics, the doc-
trine of chances, probabilities, and political arith-
metic, were successively subjected to the analytical
method; and all these sciences received from him
fresh confirmation and further improvement *.

It cannot be denied that, in general, his at-
tention is more occupied with the analysis itself,

* A complete edition of his works, comprising the numerous
papers which he sent to the academies of St. Petersburg,
Berlin, Paris, and other public societies, his separate Treatises
on Curves, the Analysis of Infinites, the differential and integral
Calculus, &c. would occupy, at least, forty quarto volumes.

than with the subject to which he is applying it;
and that he seems more taken up with his instru-
ments, than with the work, which they are to assist
him in executing.   But this can hardly be made a
ground of censure, or regret, since it is the very
circumstance to which we owe the present per-
fection of those instruments; a perfection to which
he could never have brought them, but by the un-
remitted attention and enthusiastic preference
which he gave to his favorite object.   If he now
and then exercised his ingenuity on a physical, or
perhaps metaphysical, hypothesis, he must have
been aware, as well as any one, that his conclusions
would of course perish with that from which they
were derived.   What he regarded, was the proper
means of arriving at those conclusions; the new
views of analysis, which the investigation might
open; and the new expedients of calculus, to which
it might eventually give birth.   This was his uni-
form pursuit; all other inquiries were prosecuted
with reference to it; and in this consisted the
peculiar character of his mathematical genius.

The faculties that are subservient to invention
he possessed in a very remarkable degree.   His
memory was at once so retentive and so ready,
that he had perfectly at command all those nu-
merous and complex formulæ, which enunciate
the rules and more important theorems of analysis.
As is reported of Leibnitz, he could also repeat
the Æneid from beginning to end; and could
trust his recollection for the first and last lines in

every page of the edition, which he had been accustomed to use. These are instances of a kind of memory, more frequently to be found where the capacity is inferior to the ordinary standard, than accompanying original, scientific genius. But in Euler, they seem to have been not so much the result of natural constitution, as of his most wonderful attention; a faculty, which, if we consider the testimony of Newton * sufficient evidence, is the great constituent of inventive power. It is that complete retirement of the mind within itself, during which the senses are locked up; that intense meditation, on which no extraneous idea can intrude; that firm, straight-forward progress of thought, deviating into no irregular sally, which can alone place mathematical objects in a light sufficiently strong to illuminate them fully, and preserve the perceptions of " the mind's eye" in the same order that it moves along.

Two of Euler's pupils (we are told by M. Fuss, a pupil himself) had calculated a converging series as far as the seventeenth term; but found, on comparing the written results, that they differed one unit at the fiftieth figure: they communicated this difference to their master, who went over the whole calculation by head, and his decision was found to be the true one.—For the purpose of exercising his little grandson in the extraction of roots, he has been known to form to

* This opinion of Sir Isaac Newton is recorded by Dr. Pemberton.

himself the table of the six first powers of all numbers, from 1 to 100, and to have preserved it actually in his memory.

The dexterity which he had acquired in analysis and calculation, is remarkably exemplified by the manner in which he manages formulæ of the greatest length and intricacy. He perceives, almost at a glance, the factors from which they may have been composed; the particular system of factors belonging to the question under present consideration; the various artifices by which that system may be simplified and reduced; and the relation of the several factors to the conditions of the hypothesis. His expertness in this particular probably resulted, in a great measure, from the ease with which he performed mathematical investigations by head. He had always accustomed himself to that exercise; and having practised it with assiduity, even before the loss of sight, which afterwards rendered it a matter of necessity, he is an instance to what an astonishing degree of perfection that talent may be cultivated, and how much it improves the intellectual powers. No other discipline is so effectual in strengthening the faculty of attention; it gives a facility of apprehension, an accuracy and steadiness to the conceptions; and, what is a still more valuable acquisition, it habituates the mind to arrangement in its reasonings and reflections.

If the reader wants a further commentary on its advantages, let him proceed to the work of

Euler, of which we here offer a Translation; and if he has any taste for the beauties of method, and of what is properly called *composition*, we venture to promise him the highest satisfaction and pleasure. The subject is so aptly divided, the order is so luminous, the connected parts seem so truly to grow one out of the other, and are disposed altogether in a manner so suitable to their relative importance, and so conducive to their mutual illustration, that, when added to the precision, as well as clearness with which every thing is explained, and the judicious selection of examples, we do not hesitate to consider it, next to Euclid's Geometry, the most perfect model of elementary writing, of which the scientific world is in possession.

When our reader shall have studied so much of these volumes as to relish their admirable style, he will be the better qualified to reflect on the circumstances under which they were composed. They were drawn up soon after our author was deprived of sight, and were dictated to his servant, who had originally been a tailor's apprentice; and, without being distinguished for more than ordinary parts, was completely ignorant of mathematics. But Euler, blind as he was, had a mind to teach his amanuensis, as he went on with the subject. Perhaps, he undertook this task by way of exercise, with the view of conforming the operation of his faculties to the change, which the loss of sight had produced. Whatever was the

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motive, his Treatise had the advantage of being
composed under an immediate experience of the
method best adapted to the natural progress of a
learner's ideas: from the want of which, men of
the most profound knowledge are often awkward,
and unsatisfactory, when they attempt elementary
instruction. It is not improbable, that we may
be farther indebted to the circumstance of our
Author's blindness; for the loss of this sense is
generally succeeded by the improvement of other
faculties. As the surviving organs, in particular,
acquire a degree of sensibility, which they did not
previously possess; so the most charming visions
of poetical fancy have been the offspring of minds,
on which external scenes had long been closed.
And perhaps a philosopher, familiarly acquainted
with Euler's writings, might trace some improve-
ment in perspicuity of method, and in the flowing
progress of his deductions, after this calamity had
befallen him; which, leaving " an universal blank
of nature's works," favors that entire seclusion of
the mind, which concentrates attention, and gives
liveliness and vigor to the conceptions.

In men devoted to study, we are not to look for
those strong, complicated passions, which are con-
tracted amidst the vicissitudes and tumult of public
life. To delineate the character of Euler, requires
no contrasts of coloring. Sweetness of disposition,
moderation in the passions, and simplicity of man-
ners, were his leading features. Susceptible of the
domestic affections, he was open to all their amiable

impressions, and was remarkably fond of children. His manners were simple, without being singular, and seemed to flow naturally from a heart that could dispense with those habits, by which many must be trained to artificial mildness, and with the forms that are often necessary for concealment. Nor did the equability and calmness of his temper indicate any defect of energy, but the serenity of a soul that overlooked the frivolous provocations, the petulant caprices, and jarring humours of ordinary mortals.

Possessing a mind of such wonderful comprehension, and dispositions so admirably formed to virtue and to happiness, Euler found no difficulty in being a Christian : accordingly, "his faith was unfeigned," and his love "was that of a pure and undefiled heart." The advocates for the truth of revealed religion, therefore, may rejoice to add to the bright catalogue, which already claims a Bacon, a Newton, a Locke, and a Hale, the illustrious name of Euler. But, on this subject, we shall permit one of his learned and grateful pupils* to sum up the character of his venerable master. " His piety was rational and sincere ; his devotion " was fervent. He was fully persuaded of the " truth of Christianity ; he felt its importance to " the dignity and happiness of human nature ; " and looked upon its detractors, and opposers, as " the most pernicious enemies of man."

The length to which this account has been ex-

* M. Fuss, Eulogy of M. L. Euler.

tended may require some apology; but the character of Euler is an object so interesting, that, when reflections are once indulged, it is difficult to prescribe limits to them. One is attracted by a sentiment of admiration, that rises almost to the emotion of sublimity; and curiosity becomes eager to examine what talents and qualities and habits belonged to a mind of such superior power. We hope, therefore, the student will not deem this an improper introduction to the work which he is about to peruse; as we trust he is prepared to enter on it with that temper and disposition, which will open his mind both to the perception of excellence, and to the ambition of emulating what he cannot but admire.

ADVERTISEMENT BY THE EDITORS OF THE ORIGINAL, IN GERMAN.

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WE present to the lovers of Algebra a work, of which a Russian translation appeared two years ago. The object of the celebrated author was to compose an Elementary Treatise, by which a beginner, without any other assistance, might make himself complete master of Algebra. The loss of sight had suggested the idea to him, and his activity of mind did not suffer him to defer the execution of it. For this purpose M. Euler pitched on a young man, whom he had engaged as a servant on his departure from Berlin, sufficiently master of arithmetic, but in other respects without the least knowledge of mathematics. He had learned the trade of a tailor; and, with regard to his capacity, was not above mediocrity. This young man, however, has not only retained what his illustrious master taught and dictated to him, but in a short time was able to perform the most difficult algebraic calculations, and to resolve with readiness whatever analytical questions were proposed to him.

This fact must be a strong recommendation of the manner in which this work is composed, as the young man who wrote it down, who performed the calculations, and whose proficiency was so striking, received no instructions whatever but from this master, a superior one indeed, but deprived of sight.

Independently of so great an advantage, men of science will perceive, with pleasure and admiration, the manner in which the doctrine of logarithms is explained, and its connexion with other branches of calculus pointed out, as well

as the methods which are given for resolving equations of the third and fourth degrees.

Lastly, those who are fond of *Diophantine* problems will be pleased to find, in the last Section of the Second Part, all these problems reduced to a system, and all the processes of calculation, which are necessary for the solution of them, fully explained.

ADVERTISEMENT BY M. BERNOULLI, THE FRENCH TRANSLATOR.

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THE Treatise of Algebra, which I have undertaken to translate, was published in German, 1770, by the Royal Academy of Sciences at Petersburg. To praise its merits, would almost be injurious to the celebrated name of its author; it is sufficient to read a few pages, to perceive, from the perspicuity with which every thing is explained, what advantage beginners may derive from it. Other subjects are the purpose of this advertisement.

I have departed from the division which is followed in the original, by introducing, in the first volume of the French translation, the first Section of the Second Volume of the original, because it completes the analysis of determinate quantities. The reason for this change is obvious: it not only favours the natural division of Algebra into determinate and indeterminate analysis; but it was necessary to preserve some equality in the size of the two volumes, on account of the additions which are subjoined to the Second Part.

The reader will easily perceive that those additions come from the pen of M. De la Grange; indeed, they formed one of the principal reasons that engaged me in this translation. I am happy in being the first to shew more generally to mathematicians, to what a pitch of perfection two of our most illustrious mathematicians have lately carried a branch of analysis but little known, the researches of which are attended with many difficulties, and, on the confession even of these great men, present the most difficult problems that they have ever resolved.

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I have endeavoured to translate this algebra in the style best suited to works of the kind. My chief anxiety was to enter into the sense of the original, and to render it with the greatest perspicuity. Perhaps I may presume to give my translation some superiority over the original, because that work having been dictated, and admitting of no revision from the author himself, it is easy to conceive that in many passages it would stand in need of correction. If I have not submitted to translate literally, I have not failed to follow my author step by step; I have preserved the same divisions in the articles, and it is only in so few places that I have taken the liberty of suppressing some details of calculation, and inserting one or two lines of illustration in the text, that I believe it unnecessary to enter into an explanation of the reasons by which I was justified in doing so.

Nor shall I take any more notice of the notes which I have added to the first part. They are not so numerous as to make me fear the reproach of having unnecessarily increased the volume; and they may throw light on several points of mathematical history, as well as make known a great number of Tables that are of subsidiary utility.

With respect to the correctness of the press, I believe it will not yield to that of the original. I have carefully compared all the calculations, and having repeated a great number of them myself, have by those means been enabled to correct several faults beside those which are indicated in the *Errata.*

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PART I. *Containing the* Analysis *of* Determinate Quantities.

SECTION I.

Of the Different Methods *of calculating* Simple Quantities.

SECTION II.

Of the different Methods *of calculating* Compound Quantities.