I

Millions and Billions and Trillions

A MILLION is a very big number. A man with a million pounds is a man to be envied; he is master, or slave, of a very big income. Let us look for one or two comparisons that will help us to realize just how big a million is.

A man's pace is about a yard. A million paces would take him a million yards, or 1,000,000 ÷ 1760 miles:

```
           568 miles
1760)1000000 yards
      8800
      -----
      12000
      10560
      -----
       14400
       14080
       -----
         320
```

So that a million paces would take a man about 568 miles, which is about the distance in a straight line from Land's End to John o' Groats. At 40 miles per day, which is a considerable speed for a long journey, the million paces would take more than 14 days.

Suppose you had a million counters to count out into packets of a hundred. The counters are piled up, more than a foot high, on a table 5 feet by 10 feet. The task is evidently a formidable one. You will probably be able to count one packet in a minute. To find the number of minutes required to count the million we divide by 100, or knock off two noughts:

10,000.

Divide by 60 to change the minutes to hours:

$$60\overline{)10000}$$
$$166 \text{ hr. } 40 \text{ min.}$$

Allowing for an 8 hour day this would be nearly 21 days; and with a 5 day working week the time would be more than 4 weeks.

Pause for a moment to imagine yourself working hour after hour at this perpetual counting, day after day, and week after week, until the last hundred counters had been put into its packet. You may begin to have a considerable respect for big numbers, and to look at the string of six o's in 1,000,000 with some sense of what they indicate.

A million days! When we wish a man long life we sometimes cry out "May he live a thousand years!" An extravagant wish, but it would be still more extravagant to wish him a million days.

$$\begin{array}{r} 2740 \text{ years} \\ 365\overline{)1000000} \text{ days} \\ 730 \\ \hline 2700 \\ 2555 \\ \hline 1450 \\ 1460 \\ \hline \end{array}$$

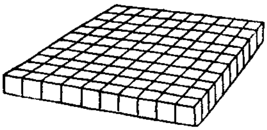
A million days is just about 2700 years. (For this purpose we need not trouble about leap years, or getting the end figures exactly.) It is not quite three-quarters of a million days since Julius Caesar landed in Britain and found it peopled with blue-painted Celts.

Here is a tiny cube with its edges one-tenth of an inch long. Suppose a million of them were placed side by side. They would stretch 100,000 inches or

$$100{,}000 \div 12 = 8333\tfrac{1}{3} \text{ feet,}$$

which is well over a mile and a half. Imagine a mile and a half of these tiny cubes!

We can arrange the cubes to take up less space. Look at this square. It contains ten rows of small cubes each holding ten, that is $10 \times 10 = 100$ altogether. To make up a million we should want 10,000 of these one-inch squares. $10{,}000 = 100 \times 100$. So we should want a space 100 inches long and 100 inches wide, completely covered with the tiny dice. The total number of dice is



$$1000 \times 1000 = 1{,}000{,}000.$$

(There are 1000 rows each of 1000 dice.)

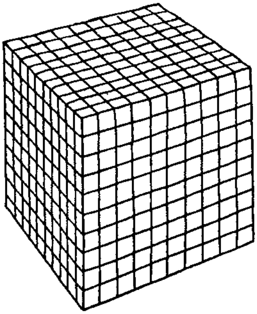
Draw on a wall a square with sides 8 ft. 4 in. long. Divide the sides into inches and join across; there are 198 of these lines to draw across the square, besides the 4 outside lines—560 yards of lines. We now have the big square divided into 1 inch squares. Begin at the top left-hand corner, divide the first square into a hundred small squares each with sides a tenth of an inch long. Then proceed to the next. And so on.

What a task! It would be futile to ask anyone to perform it. It might be sufficient to divide one of the 1 inch squares into hundredths, and then to realize that there are still 9999 to be done.

A film firm with which I was connected was asked to represent a 40,000 horse-power turbine by means of a drawing showing 40,000 horses simultaneously on the screen. When we politely pointed out the absurdity of this, the turbine firm insisted on 40,000 dots. The artist was accordingly told to draw 40,000 dots. He drew 400 dots, and then refused to draw any more when I pointed out to him that he had only accomplished one-hundredth of his task. Finally the drawing of 400 dots was reproduced as a block, a hundred careful prints were made from it, and these were pasted in a square. Even this was a long and tedious job. The word "dotty" acquired a considerable vogue in those days.

If we had been asked for a million dots we should have had to paste down twenty-five times as many squares, 2500 of them.

The million small dice, that stretched a distance of more than a mile and a half when placed side by side, were tucked into a square that was only 8 ft. 4 in. wide. Let us see how we can tuck them away still more snugly. Look at this 1 inch cube. Look at the front layer of small dice; there are 10 rows of 10; and altogether there are 10 layers. So we have $10 \times 10 \times 10 = 1000$ small dice in the cube. We have packed 1000 dice into the small space of an inch cube.



To make up a million dice we should want 1000 of these inch cubes, that is a thousand thousand dice, or $1000 \times 1000 = 1,000,000$. We can pack the inch cubes in the same way, into a bigger cube 10 inches by 10 inches by 10 inches. In each edge of this bigger cube there are 100 dice, so that there are $100 \times 100 = 10{,}000$ dice in each of 100 layers. That is $100 \times 100 \times 100 = 1{,}000{,}000$ altogether.

See again what we have done. Placed side by side a million tenth-of-an-inch dice stretch more than a mile and a half. Arranged in a square they fill a space 8 ft. 4 in. square. Arranged in a cube they occupy a cube with 10 inch edges. It really is extraordinary how the biggest numbers climb down when we begin to arrange them. It is all because a million is $1{,}000{,}000 \times 1$, or 1000×1000, or $100 \times 100 \times 100$.

The distance round the world is 25,000 miles—quite a considerable distance in its way. An express train running at 50 miles per hour would take $25{,}000 \div 50 = 500$ hours, or nearly 21 days to do this distance. But the distance is a mere fortieth part of a million miles. Steaming ceaselessly night and day at 50 miles per hour an express train would take 20,000 hours, or 833 days, or nearly $2\frac{1}{3}$ years to go a million miles.

The moon is nearly ten times as far away as the distance round the earth—238,000 miles (not far short of a quarter of a million miles); so that our express train would take more than six months to perform a journey equal to the distance of the moon.

The distance of the sun is about 93 million miles. Let us see how many times greater that is than the distance of the moon:

$$93{,}000{,}000 \div 238{,}000 = 93{,}000 \div 238$$

$$\begin{array}{r} 390 \\ 238\overline{)93000} \\ \underline{714} \\ 2160 \\ \underline{2142} \end{array}$$

We need not trouble about the end figure. The sun is about 390 times as far off as the moon. To travel the distance to the sun our express train would take:

$$93,000,000 \div 50 = 1,860,000 \text{ hours},$$
$$= 77,500 \text{ days},$$
$$= \text{about } 212 \text{ years}.$$

So we have the comparison:

Round the earth:	25,000 miles—	21 days.
To the moon:	238,000 miles—	6 months.
To the sun:	93,000,000 miles—	212 years.

Here is another comparison for these distances. Sound travels with extreme rapidity when compared with a railway train. The speed is about a mile in 5 seconds, as every child knows who has counted the distance of a lightning flash. A mile in 5 seconds is 12 miles per minute or 720 miles per hour. To go the distance round the world sound would take 25,000 ÷ 720 = about 35 hours, so that sound would take nearly a day and a half to travel the distance round the world.

No sound can ever reach us from the sun or the moon, because there is no air to carry the sound. But suppose there were, how long would sound take to reach us from the moon? The answer is easy enough:

$$238,000 \div 720 \text{ hours}$$

```
        330 hours
  72 ) 23800
       216
       ---
        220
        216
        ---
          40
        ---
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(The end figure does not matter.) 330 hours is $13\frac{3}{4}$ days. So that if we *could* hear moon sounds we should hear them $13\frac{3}{4}$ days after they were made.

And what about the sun?

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        129000 hours
   72)9300000
      72
      ---
      210
      144
      ---
       660
       648
      ---
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(There is no need to carry the division further; indeed we might have called the number 130,000.)

129,000 hours = about 5000 days = about 14 years.

If we *could* hear explosions in the sun we should hear them 14 years after they happened.

Millions do very well for the larger affairs of everyday life. The population of Greater London is more than 8 millions, Greater New York is about the same, and we begin to consider a town really large when it passes the million mark. Just as we consider a man really wealthy when he becomes a millionaire. National revenues are reckoned in hundreds of millions. Distances in the solar system are equally well expressed in millions of miles, from the quarter of a million miles distance of the moon from the earth, to the 2794 millions of miles that Neptune lies out from the sun. A million, with its string of six noughts, is indeed an admirable and useful number.

And now for a jump! We jump suddenly from a million to a billion, from 1,000,000 to 1,000,000,000,000. The English (and German) billion is a million times a million, a million squared as we say; the number of noughts is doubled, a billion has twelve noughts. In the United States and on most of the Continent a billion is a mere thousand millions —1,000,000,000, the thousandth part of our billion. Allowing five dollars to the pound a dollar billionaire has the equivalent of 200 million pounds. A sterling billionaire would have to possess the impossible sum of a billion pounds; if he could exist he could buy out five thousand dollar billionaires. Which shows that there is something in a name.

Millions do well enough for distances in the solar system. When we come to star distances millions are no longer big enough; we need billions. The distance of the nearest star is 26 billion miles. No human mind can realize what a billion miles means, but we can get some faint idea of the immensity of the distance by finding how long light takes to come from the star.

Light is the speediest thing we know of. Its speed is so great that in ordinary conditions it seems to take no time at all to move from one place to another; it is only across immense distances that the time it takes becomes apparent. The speed of light has been measured many times and in many ways. The results agree in giving the speed as about 186,000 miles per second.

The immensity of that speed may be judged by comparing it with the speed of an express train. At 50 miles per hour an express train would do 186,000 miles in

$$\begin{aligned} &186{,}000 \div 50 \text{ hours,} \\ &= 3720 \text{ hours,} \\ &= 155 \text{ days,} \\ &= 5 \text{ months of } 31 \text{ days.} \end{aligned}$$

So that a train, travelling night and day continuously at 50 miles per hour, would take five months to travel the distance that light leaps across in a second.

$$186{,}000 \div 25{,}000 = \text{about } 7\tfrac{1}{2}.$$

So that light travels in a second a distance equal to $7\frac{1}{2}$ times the distance round the world. To come from the moon, light takes $238{,}000 \div 186{,}000 = $ about $1\frac{1}{4}$ seconds. To come from the sun it takes $93{,}000{,}000 \div 186{,}000 = 500$ seconds, or rather more than 8 minutes.

How far does light travel in a year? It sounds as though the number of miles would be extravagantly large, but the calculation is easy enough.

$$\begin{aligned} &186{,}000 \text{ miles in a second,}\\ &= 186{,}000 \times 3600 \text{ miles in an hour,}\\ &= 186{,}000 \times 3600 \times 24 \text{ miles in a day,}\\ &= 186{,}000 \times 3600 \times 24 \times 365\tfrac{1}{4} \text{ miles in a year.}\end{aligned}$$

When those four numbers are multiplied out we get a number that is about 5·9 billion miles. This distance is called a light-year. A light-year is a length, not a time; it is the distance that light travels in a year; it is about 5·9 billion miles.

When we give the distance of a star we often express it in light-years. The distance of the nearest star (α Centauri—the brightest star in the Centaur in the southern heavens) is 4·4 light years. The North Star is 47 light-years away. And there are other stars hundreds and thousands of light-years from us. If for any reason we want to turn light-years into billions of miles we have only to multiply the number of light-years by 5·9. If we know the distance in billions of miles we can change it into light-years by dividing by 5·9.

A billion miles is an admirable unit for star distances. We use the light-year because of the imaginative suggestion of light, leaping at almost incredible speed across immense distances, taking four and a half years to reach us at 186,000 miles a second.

We saw how a million tenth-inch dice could be arranged to form a line more than a mile and a half long, or compressed into a square with 8 ft. 4 in. sides, or tucked away snugly into a cube with 10 inch edges. We can do the same sort of things with a billion tenth-inch dice. Spread out in a line we should have a line a million times as long—the line would stretch about one and a half million miles, or sixty times round the world. The sides of the square would be a thousand times as long as $8\frac{1}{3}$ feet—more than a mile and a half long; think of an immense square field—about 1600 acres—and then think of the whole of this immense space divided into small squares with sides a tenth of an inch long. The cube would have edges a hundred times as long as the 10 inch cube used for a million dice:

$$\begin{aligned} 10 \times 100 \text{ in.} &= 1000 \text{ in.} \\ &= 83\tfrac{1}{3} \text{ feet.} \end{aligned}$$

We have seen how numbers climb down when we arrange the objects in squares, and still more when we arrange them in cubes. Now let us see how they jump up when we multiply.

A mile is 5280 feet—a quite comprehensible number.

A square mile is 5280^2 or 5280×5280 square feet $= 27{,}878{,}400$ square feet, or nearly 28 million square feet.

A cubic mile is 5280^3 or $5280 \times 5280 \times 5280$ cubic feet $= 147{,}197{,}952{,}000$ cubic feet or more than a seventh of a billion cubic feet. A big enough number! But then you must remember what a cubic mile is. We have to think of