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### Mathematical and Physical Papers vol.5.

Sir George Stokes (1819-1903) established the science of hydrodynamics with his law of viscosity describing the velocity of a small sphere through a viscous fluid. He published no books, but was a prolific lecturer and writer of papers for the Royal Society, the British Association for the Advancement of Science, the Victoria Institute and other mathematical and scientific institutions. These collected papers (issued between 1880 and 1905) are therefore the only readily available record of the work of an outstanding and influential mathematician, who was Lucasian Professor of Mathematics in Cambridge for over fifty years, Master of Pembroke College, President of the Royal Society (1885-90), Associate Secretary of the Royal Commission on the University of Cambridge and a Member of Parliament for the University.



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# Mathematical and Physical Papers vol.5.

VOLUME 5

GEORGE GABRIEL STOKES





#### CAMBRIDGE UNIVERSITY PRESS

Cambridge New York Melbourne Madrid Cape Town Singapore São Paolo Delhi

Published in the United States of America by Cambridge University Press, New York

 $www. cambridge. org \\ Information on this title: www. cambridge. org/9781108002677$ 

© in this compilation Cambridge University Press 2009

This edition first published 1905 This digitally printed version 2009

ISBN 978-1-108-00267-7

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# MATHEMATICAL

AND

PHYSICAL PAPERS.



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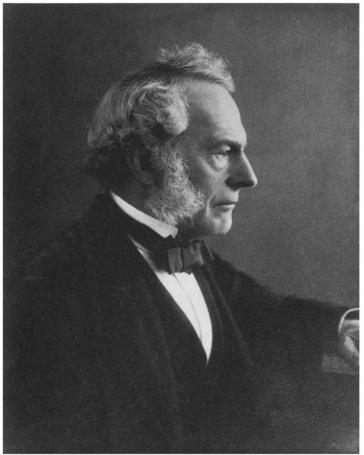
London: AVE MARIA LANE, E.C. Glasgow: 50, WELLINGTON STREET.



Leipzig: F. A. BROCKHAUS. Lew York: THE MACMILLAN COMPANY. Bombay and Calcutta: MACMILLAN AND CO., Ltd.

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# MATHEMATICAL

AND

# PHYSICAL PAPERS

#### BY THE LATE

SIR GEORGE GABRIEL STOKES, BART., Sc.D., LL.D., D.C.L., PAST PRES. R.S.,

KT PRUSSIAN ORDER POUR LE MÉRITE, FOR. ASSOC. INSTITUTE OF FRANCE, ETC.

MASTER OF PEMBROKE COLLEGE AND LUCASIAN PROFESSOR OF MATHEMATICS

IN THE UNIVERSITY OF CAMBRIDGE.

Reprinted from the Original Journals and Transactions, with brief Historical Notes and References.

VOL. V.

CAMBRIDGE:
AT THE UNIVERSITY PRESS.

1905



# Cambridge :

PRINTED BY JOHN CLAY, M.A. AT THE UNIVERSITY PRESS.



#### PREFACE.

THE present volume concludes the reprint of the Scientific Papers of the late Professor Sir George Gabriel Stokes. As in the preceding volume, the brief editorial additions take the form of undated notes enclosed in square brackets.

Various lectures and addresses on scientific subjects remain over, which were considered to be not sufficiently original for inclusion in this collection, in those cases where they are not already represented in other ways. Among them may be mentioned the following:—

Solar Physics, the introductory lecture of a course, delivered in the Theatre of the South Kensington Museum on April 6, 1881, on behalf of the Committee on Solar Research. Reprinted in Nature, Vol. XXIV. pp. 595—598 and 613—618.

The Absorption of Light and the Colours of Natural Bodies, and Fluorescence; two lectures to science teachers at South Kensington. Pp. 43, crown octavo, Macmillan and Co., 1876.

The Presidential Address to the British Association for the Advancement of Science, Exeter Meeting 1869,—mainly recounting recent progress in Science.

Presidential Addresses at the Anniversary Meetings of the Royal Society, 1886 to 1891 inclusive—mainly concerned with the business of the Royal Society.

Annual addresses on scientific topics delivered as President of the Victoria Institute:—The Luminiferous Ether, 1893; The Perception of Light, 1895; The Röntgen Rays, 1896; The Perception of Colour, 1898.

Three volumes of Burnett Lectures delivered at Aberdeen have been published by Macmillan and Co.; On the Nature of Light, Nov. 1883; On Light as a Means of Investigation, Dec. 1884; and On the Beneficial Effects of Light, Nov. 1885. These lectures have also appeared in a German Translation.

The courses of Gifford Lectures on Natural Theology in the University of Edinburgh for the years 1891 and 1893 were delivered by Sir George Stokes, and have been published in two volumes by A. and C. Black; they are in part concerned with the theological aspects of scientific knowledge.



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Some preparations have been made for a volume of biographical character, to be occupied in part by a selection from Sir George Stokes' voluminous scientific correspondence, including some unpublished manuscript material. Stress of other engagements has hitherto delayed this undertaking, but it is hoped that substantial progress will soon be made.

It will be observed that the present volume represents the period in which Sir George Stokes' scientific activities were mainly expended in the work of the Royal Society and of public Scientific Committees, and in giving assistance to the investigations of others. The volume thus consists largely of additions and notes originally appended to memoirs by other authors. Considerable care has been taken in making its contents as complete as possible, and in the elucidation of points of ambiguity or difficulty. It is hoped that no important printed contribution has been overlooked. In one case, pp. 146—159, it has been thought right to insert an unpublished manuscript of considerable length, on the forms of water-waves of finite height, in what appeared to be its natural sequence.

Acting on the advice of high authorities, the Smith's Prize Examination Papers set by Sir George Stokes in the University of Cambridge for a long series of years, and also some early papers of Problems set in the Mathematical Tripos, have been reprinted in an Appendix.

The biographical sketch prefixed to this volume was contributed by Lord Rayleigh to the Obituary of the Royal Society. The photographic portrait which illustrated it, taken about the year 1892, has again been made available by kind permission of Mrs F. W. H. Myers.

If this collection serves to bring into due prominence a series of memoirs which have at no time been sufficiently widely known, although some of the greatest of British physicists have traced to them much of their inspiration, any trouble that has been incurred in collecting and editing the final volumes will have been amply rewarded.

J. LARMOR.

St John's College, Cambridge.

March 20, 1905.



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#### OBITUARY NOTICE.

[By LORD RAYLEIGH. From the Proceedings of the Royal Society, 1903.]

SIR GEORGE GABRIEL STOKES, BART. 1819-1903.

In common with so many distinguished men Sir George Stokes was the son of a clergyman. His father, Gabriel Stokes, who was Rector of Skreen, County Sligo, married Elizabeth Haughton, and by her had eight children, of whom George was the youngest. The family can be traced back to Gabriel Stokes, born 1680, a well-known engineer in Dublin and Deputy Surveyor General of Ireland, who wrote a treatise on Hydrostatics and designed the Pigeon House Wall in Dublin Harbour. This Gabriel Stokes married Elizabeth King in 1711 and among his descendants in collateral branches there are several mathematicians, a Regius Professor of Greek, two Regius Professors of Medicine, and a large sprinkling of Scholars of Trinity College, Dublin. In more recent times Margaret Stokes, the Irish antiquary, and the Celtic scholar Whitley Stokes, children of the eminent physician Dr William Stokes, have, among others, shed lustre on the name.

The home at Skreen was a very happy one. In the excellent sea air the children grew up with strong bodies and active minds. Of course great economy had to be practised to meet the educational needs of the family; but in the Arcadian simplicity of a place where chickens cost sixpence and eggs were five or six a penny, it was easy to feed them. They were all deeply attached to their mother, a beautiful and severe woman who made herself feared as well as loved.

Stokes was taught at home; he learnt reading and arithmetic from the Parish Clerk, and Latin from his father who had been a scholar of Trinity College, Dublin. The former used to tell with great delight that Master George had made out for himself new ways of doing sums, better than the book. In 1832, at 13 years of age, he was sent to Dr Wall's school in Dublin; and in 1835 for two years to Bristol College, of which Dr Jerrard was Principal. There is a tradition that he did many of the propositions of Euclid, as problems, without looking at the book. He considered that he owed much to the teaching of Francis Newman, brother of the Cardinal, then mathematical master at Bristol College, and a man of great charm of character as well as of unusual attainments.



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On the first crossing to Bristol the ship nearly foundered; and his brother, who was escorting him, was much impressed by his coolness in face of danger. His habit, often remarked in after life, of answering with a plain "yes" or "no," when something more elaborate was expected, is supposed to date from this time, when his brothers chaffed him and warned him that if he gave "long Irish answers" he would be laughed at by his school-fellows.

It is surprising to learn that as a little boy he was passionate, and liable to violent, if transitory, fits of rage. So completely was this tendency overcome that in after life his temper was remarkably calm and even. He was fond of botany, and when about sixteen or seventeen, collected butterflies and caterpillars. It is narrated that one day while on a walk with a friend he failed to return the salutation of some ladies of his acquaintance, afterwards explaining his conduct by remarking that his hat was full of beetles!

In 1837, the year of Queen Victoria's accession, he commenced residence at Cambridge, where he was to find his home, almost without intermission, for sixty-six years. In those days sports were not the fashion for reading men, but he was a good walker, and astonished his contemporaries by the strength of his swimming. Even at a much later date he enjoyed encounters with wind and waves in his summer holidays on the north coast of Ireland. At Pembroke College his mathematical abilities soon attracted attention, and in 1841 he graduated as Senior Wrangler and first Smith's Prizeman. In the same year he was elected Fellow of his College.

After his degree, Stokes lost little time in applying his mathematical powers to original investigation. During the next three or four years there appeared papers dealing with hydrodynamics, wherein are contained many standard theorems. As an example of these novelties, the use of a stream-function in three dimensions may be cited. It had already been shown by Lagrange and Earnshaw that in the motion of an incompressible fluid in two dimensions the component velocities at any point may be expressed by means of a function known as the stream-function, from the property that it remains constant along any line of motion. It was further shown by Stokes that there is a similar function in three dimensions when the motion is symmetrical with respect to an axis. For many years the papers, now under consideration, were very little known abroad, and some of the results are still attributed by Continental writers to other authors.

A memoir of great importance on the "Friction of Fluids in Motion, etc.," followed a little later (1845). The most general motion of a medium in the neighbourhood of any point is analysed into three constituents—a motion of pure translation, one of pure rotation, and one of pure strain. These results are now very familiar; it may assist us to appreciate their novelty at the time,



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if we recall that when similar conclusions were put forward by Helmholtz twenty-three years later, their validity was disputed by so acute a critic as Bertrand. The splendid edifice, concerning the theory of inviscid fluids, which Helmholtz raised upon these foundations, is the admiration of all students of Hydrodynamics.

In applying the above purely kinematical analysis to viscous fluids, Stokes lays down the following principle:—"That the difference between the pressure on a plane passing through any point P of a fluid in motion and the pressure which would exist in all directions about P if the fluid in its neighbourhood were in a state of relative equilibrium depends only on the relative motion of the fluid immediately about P; and that the relative motion due to any motion of rotation may be eliminated without affecting the differences of the pressures above mentioned." This leads him to general dynamical equations, such as had already been obtained by Navier and Poisson, starting from more special hypotheses as to the constitution of matter.

Among the varied examples of the application of the general equations two may be noted. In one of these, relating to the motion of fluid between two coaxial revolving cylinders, an error of Newton's is corrected. In the other, the propagation of sound, as influenced by viscosity, is examined. It is shown that the action of viscosity  $(\mu)$  is to make the intensity of the sound diminish as the time increases, and to render the velocity of propagation less than it would otherwise be. Both effects are greater for high than for low notes; but the former depends on the first power of  $\mu$ , while the latter depends only on  $\mu^2$ , and may usually be neglected.

In the same paragraph there occur two lines in which a question, which has recently been discussed on both sides, and treated as a novelty, is disposed of. The words are—"we may represent an arbitrary disturbance of the medium as the aggregate

of series of plane waves propagated in all directions."

In the third section of the memoir under consideration, Stokes applies the same principles to find the equations for an elastic solid. In his view the two elastic constants are independent and not reducible to one, as in Poisson's theory of the constitution of matter. He refers to indiarubber as hopelessly violating Poisson's condition. Stokes' position, powerfully supported by Lord Kelvin, seems now to be generally accepted. Otherwise, many familiar materials must be excluded from the category of elastic solids.

In 1846 he communicated to the British Association a Report on Recent Researches in Hydrodynamics. This is a model of what such a survey should be, and the suggestions contained in it have inspired many subsequent investigations. He greatly admired the work of Green, and his comparison of opposite styles may often recur to the reader of mathematical lucubrations. Speaking of the Reflection and Refraction of Sound, he remarks that "this problem

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had been previously considered by Poisson in an elaborate memoir. Poisson treats the subject with extreme generality, and his analysis is consequently very complicated. Mr Green, on the contrary, restricts himself to the case of plane waves, a case evidently comprising nearly all the phenomena connected with this subject which are of interest in a physical point of view, and thus is enabled to obtain his results by a very simple analysis. Indeed Mr Green's memoirs are very remarkable, both for the elegance and rigour of the analysis, and for the ease with which he arrives at most important results. This arises in a great measure from his divesting the problems he considers of all unnecessary generality; where generality is really of importance he does not shrink from it. In the present instance there is one important respect in which Mr Green's investigation is more general than Poisson's, which is, that Mr Green has taken the case of any two fluids, whereas Poisson considered the case of two elastic fluids, in which equal condensations produce equal increments of pressure. It is curious that Poisson, forgetting this restriction, applied his formulae to the case of air and water. course his numerical result is quite erroneous. Mr Green easily arrives at the ordinary laws of reflection and refraction. obtains also a very simple expression for the intensity of its reflected sound...." As regards Poisson's work in general there was no lack of appreciation. Indeed, both Green and Stokes may be regarded as followers of the French school of mathematicians.

The most cursory notice of Stokes' hydrodynamical researches cannot close without allusion to two important memoirs of somewhat later date. In 1847 he investigated anew the theory of oscillatory waves, as on the surface of the sea, pursuing the approximation so as to cover the case where the height is not very small in comparison with the wave-length. To the reprint in *Math. and Phys. Papers* are added valuable appendices pushing the approximation further by a new method, and showing that the slopes which meet at the crest of the highest possible wave (capable of propagation without change of type) enclose an

angle of  $120^{\circ}$ .

The other is the great treatise on the Effect of Internal Friction of Fluids on the Motion of Pendulums. Here are given the solutions of difficult mathematical problems relating to the motion of fluid about vibrating solid masses of spherical or cylindrical form; also, as a limiting case, the motion of a viscous fluid in the neighbourhood of a uniformly advancing solid sphere, and a calculation of the resistance experienced by the latter. In the application of the results to actual pendulum observations, Stokes very naturally assumed that the viscosity of air was proportional to density. After Maxwell's great discovery that viscosity is independent of density within wide limits, the question assumed a different aspect; and in the reprint of the memoir Stokes



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explains how it happened that the comparison with theory was not more prejudiced by the use of an erroneous law.

In 1849 appeared another great memoir, on the Dynamical Theory of Diffraction, in which the luminiferous aether is treated as an elastic solid so constituted as to behave as if it were nearly or quite incompressible. Many fundamental propositions respecting the vibration of an elastic solid medium are given here for the first time. For example, there is an investigation of the disturbance due to the operation at one point of the medium of a The waves emitted are of course symmetrical periodic force. with respect to the direction of the force as axis. At a distance, the displacement is transverse to the ray and in the plane which includes the axis, while along the axis itself there is no disturb-Incidentally a general theorem is formulated connecting the disturbances due to initial displacements and velocities. any material system in which the forces acting depend only on the positions of the particles be slightly disturbed from a position of equilibrium, and then left to itself, the part of the subsequent motion which depends on the initial displacements may be obtained from the part which depends upon the initial velocities by replacing the arbitrary functions, or arbitrary constants, which express the initial velocities by those which express the corresponding initial displacements, and differentiating with respect to

One of the principal objects of the memoir was to determine the law of vibration of the secondary waves into which, in accordance with Huygens' principle, a primary wave may be resolved, and thence, by a comparison with phenomena observed with gratings, to answer a question then much agitated but now (unless restated) almost destitute of meaning, viz., whether the vibrations of light are parallel or perpendicular to the plane of polarisation. the law of the secondary wave Stokes' conclusion is expressed in the following theorem: "Let  $\xi = 0$ ,  $\eta = 0$ ,  $\zeta = f(bt - x)$  be the displacements corresponding to the incident light; let  $O_1$  be any point in the plane P, dS an element of that plane adjacent to  $O_1$ ; and consider the disturbance due to that portion only of the incident disturbance which passes continually across dS. Let O be any point in the medium situated at a distance from the point O, which is large in comparison with the length of a wave; let  $OO_1 = r$ , and let this line make angles  $\theta$  with the direction of propagation of the incident light, or the axis of x, and  $\phi$  with the direction of vibration, or the axis of z. Then the displacement at O will take place in a direction perpendicular to  $OO_1$ , and lying in the plane  $ZO_1O$ ; and if  $\zeta'$  be the displacement at O, reckoned positive in the direction nearest to that in which the incident vibrations are reckoned positive,

$$\zeta' = \frac{dS}{4\pi r} (1 + \cos \theta) \sin \phi \cdot f' (bt - r).$$

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In particular, if

$$f(bt - x) = c \sin \frac{2\pi}{\lambda} (bt - x),$$

we shall have

$$\zeta' = \frac{cdS}{2\lambda r} (1 + \cos\theta) \sin\phi \cdot \cos\frac{2\pi}{\lambda} (bt - r).$$

Stokes' own experiments on the polarisation of light diffracted by a grating led him to the conclusion that the vibrations of light

are perpendicular to the plane of polarisation.

The law of the secondary wave here deduced is doubtless a possible one, but it seems questionable whether the problem is really so definite as Stokes regarded it. A merely mathematical resolution may be effected in an infinite number of ways; and if the problem is regarded as a physical one, it then becomes a question of the character of the obstruction offered by an actual screen.

As regards the application of the phenomena of diffraction to the question of the direction of vibration, Stokes' criterion finds a better subject in the case of diffraction by very small *particles* disturbing an otherwise uniform medium, as when a fine precipitate of sulphur falls from an aqueous solution.

The work already referred to, as well as his general reputation, naturally marked out Stokes for the Lucasian Professorship, which fell vacant at this time (1849). It is characterised throughout by accuracy of thought and lucidity of statement. Analytical results are fully interpreted, and are applied to questions of physical interest. Arithmetic is never shirked.

Among the papers which at this time flowed plentifully from his pen, one "On Attractions, and on Clairaut's Theorem," deserves special mention. In the writings of earlier authors the law of gravity at the various points of the earth's surface had been deduced from more or less doubtful hypotheses as to the distribution of matter in the interior. It was reserved for Stokes to point out that, in virtue of a simple theorem relating to the potential, the law of gravity follows immediately from the form of the surface, assumed to be one of equilibrium, and that no conclusion can be drawn concerning the internal distribution of attracting matter.

From an early date he had interested himself in Optics, and especially in the Wave Theory. Although, not long before, Herschel had written ambiguously, and Brewster, the greatest living authority, was distinctly hostile, the magnificent achievements of Fresnel had converted the younger generation; and, in his own University, Airy had made important applications of the theory, e.g., to the explanation of the rainbow, and to the diffraction of object-glasses. There is no sign of any reserve in the attitude of Stokes. He threw himself without misgiving into the discussion of outstanding difficulties, such as those connected with



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the aberration of light, and by further investigations succeeded in

bringing new groups of phenomena within the scope of the theory.

An early example of the latter is the paper "On the Theory of certain Bands seen in the Spectrum." These bands, now known after the name of Talbot, are seen when a spectrum is viewed through an aperture half covered by a thin plate of mica or glass. In Talbot's view the bands are produced by the interference of the two beams which traverse the two halves of the aperture, darkness resulting whenever the relative retardation amounts to an odd number of half wave-lengths. This explanation cannot be accepted as it stands, being open to the same objection as Arago's theory of stellar scintillation. A body emitting homogeneous light would not become invisible on merely covering half the aperture of vision with a half wave-plate. That Talbot's view is insufficient is proved by the remarkable observation of Brewster—that the bands are seen only when the retarding plate is held

akin to the phenomena of diffraction.

A little later we have an application of the general principle of reversion to explain the perfect blackness of the central spot in Newton's rings, which requires that when light passes from a second medium to a first the coefficient of reflection shall be numerically the same as when the propagation is in the opposite sense, but be affected with the reverse sign—the celebrated "loss of half an undulation." The result is obtained by expressing the conditions that the refracted and reflected rays, due to a given incident ray, shall on reversal reproduce that ray and no other.

towards the blue side of the spectrum. By Stokes' theory this polarity is fully explained, and the formation of the bands is shown to be connected with the limitation of the aperture, viz., to be

It may be remarked that on any mechanical theory the reflection from an infinitely thin plate must tend to vanish, and therefore that a contrary conclusion can only mean that the theory has been applied incorrectly.

A not uncommon defect of the eye, known as astigmatism, was first noticed by Airy. It is due to the eye refracting the light with different power in different planes, so that the eye, regarded as an optical instrument, is not symmetrical about its axis. consequence, lines drawn upon a plane perpendicular to the line of vision are differently focussed according to their direction in It may happen, for example, that vertical lines are that plane. well seen under conditions where horizontal lines are wholly confused, and vice versa. Airy had shown that the defect could be cured by cylindrical lenses, such as are now common; but no convenient method of testing had been proposed. For this purpose Stokes introduced a pair of plano-cylindrical lenses of equal cylindrical curvatures, one convex and the other concave, and so mounted as to admit of relative rotation. However the components may be situated, the combination is upon the whole



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neither convex nor concave. If the cylindrical axes are parallel, the one lens is entirely compensated by the other, but as the axes diverge the combination forms an astigmatic lens of gradually increasing power, reaching a maximum when the axes are perpendicular. With the aid of this instrument, an eye, already focussed as well as possible by means (if necessary) of a suitable spherical lens, convex or concave, may be corrected for any degree or direction of astigmatism; and from the positions of the axes of the cylindrical lenses may be calculated, by a simple rule, the curvatures of a single lens which will produce the same result. It is now known that there are comparatively few eyes whose vision

may not be more or less improved by an astigmatic lens.

Passing over other investigations of considerable importance in themselves, especially that on the composition and resolution of streams of polarised light from different sources, we come to the great memoir on what is now called Fluorescence, the most farreaching of Stokes' experimental discoveries. He "was led into the researches detailed in this paper by considering a very singular phenomenon which Sir J. Herschel had discovered in the case of a weak solution of sulphate of quinine and various other salts of the same alkaloid. This fluid appears colourless and transparent, like water, when viewed by transmitted light, but exhibits in certain aspects a peculiar blue colour. Sir J. Herschel found that when the fluid was illuminated by a beam of ordinary daylight, the blue light was produced only throughout a very thin stratum of fluid adjacent to the surface by which the light entered. It was unpolarised. It passed freely through many inches of the fluid. The incident beam after having passed through the stratum from which the blue light came, was not sensibly enfeebled or coloured, but yet it had lost the power of producing the usual blue colour when admitted into a solution of sulphate of quinine. A beam of light modified in this mysterious manner was called by Sir J. Herschel epipolised.

Several years before, Sir D. Brewster had discovered in the case of an alcoholic solution of the green colouring matter of leaves a very remarkable phenomenon, which he has designated as internal dispersion. On admitting into this fluid a beam of sunlight condensed by a lens, he was surprised by finding the path of the rays within the fluid marked by a bright light of a blood-red colour, strangely contrasting with the beautiful green of the fluid itself when seen in moderate thickness. Sir David afterwards observed the same phenomenon in various vegetable solutions and essential oils, and in some solids. He conceived it to be due to coloured particles held in suspension. But there was one circumstance attending the phenomenon which seemed very difficult of explanation on such a supposition, namely, that the whole or a great part of the dispersed beam was unpolarised, whereas a beam reflected from suspended particles might be expected to be polarised



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by reflection. And such was, in fact, the case with those beams which were plainly due to nothing but particles held in suspension. From the general identity of the circumstances attending the two phenomena, Sir D. Brewster was led to conclude that epipolic was merely a particular case of internal dispersion, peculiar only in this respect, that the rays capable of dispersion were dispersed with unusual rapidity. But what rays they were which were capable of affecting a solution of sulphate of quinine, why the active rays were so quickly used up, while the dispersed rays which they produced passed freely through the fluid, why the transmitted light when subjected to prismatic analysis showed no deficiencies in those regions to which, with respect to refrangibility, the dispersed rays chiefly belonged, were questions to which the answers appeared

to be involved in as much mystery as ever."

Such a situation was well calculated to arouse the curiosity and enthusiasm of a young investigator. A little consideration showed that it was hardly possible to explain the facts without admitting that in undergoing dispersion the light changed its refrangibility, but that if this rather startling supposition was allowed, there was no further difficulty; and experiment soon placed the fact of a change of refrangibility beyond doubt. pure spectrum from sunlight having been formed in air in the usual manner, a glass vessel containing a weak solution of sulphate of quinine was placed in it. The rays belonging to the greater part of the visible spectrum passed freely through the fluid, just as if it had been water, being merely reflected here and there from motes. But from a point about halfway between the fixed lines G and H to far beyond the extreme violet, the incident rays gave rise to a light of a sky-blue colour, which emanated in all directions from the portion of the fluid which was under the influence of the incident rays. The anterior surface of the blue space coincided, of course, with the inner surface of the vessel in which the fluid was contained. The posterior surface marked the distance to which the incident rays were able to penetrate before they were absorbed. This distance was at first considerable, greater than the diameter of the vessel, but it decreased with great rapidity as the refrangibility of the incident rays increased, so that from a little beyond the extreme violet to the end, the blue space was reduced to an excessively thin stratum adjacent to the surface by which the incident rays entered. It appears, therefore, that this fluid, which is so transparent with respect to nearly the whole of the visible rays, is of an inky blackness with respect to the invisible rays, more refrangible than the extreme violet. The fixed lines belonging to the violet and the invisible region beyond were beautifully represented by dark planes interrupting the blue space. When the eye was properly placed these planes were, of course, projected into lines.'

At a time when photography was of much less convenient



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application than at present—even wet collodion was then a novelty—the method of investigating the ultra-violet region of the spectrum by means of fluorescence was of great value. The obstacle presented by the imperfect transparency of glass soon made itself apparent, and this material was replaced by quartz in the lenses and prisms, and in the mirror of the heliostat. When the electric arc was substituted for sunlight a great extension of the spectrum in the direction of shorter waves became manifest.

Among the substances found "active" were the salts of uranium—an observation destined after nearly half a century to become in the hands of Becquerel the starting point of a most interesting scientific advance, of which we can hardly yet foresee the development.

In a great variety of cases the refrangibility of the dispersed light was found to be *less* than that of the incident. That light is always degraded by fluorescence is sometimes referred to as Stokes' law. Its universality has been called in question, and the doubt is perhaps still unresolved. The point is of considerable interest in connection with theories of radiation and the second law of Thermodynamics.

Associated with fluorescence there is frequently seen a "false dispersion," due to suspended particles, sometimes of extreme minuteness. When a horizontal beam of falsely dispersed light was viewed from above in a vertical direction, and analysed, it was found to consist chiefly of light polarised in the plane of reflection. On this fact Stokes founded an important argument as to the direction of vibration of polarised light. For "if the diameters of the (suspended) particles be small compared with the length of a wave of light, it seems plain that the vibrations in a reflected ray cannot be perpendicular to the vibrations in the incident ray." From this it follows that the direction of vibration must be perpendicular to the plane of polarisation, as Fresnel supposed, and the test seems to be simpler and more direct than the analogous test with light diffracted from a grating. It should not be overlooked that the argument involves the supposition that the effect of a particle is to load the aether.

It was about this time that Lord Kelvin learned from Stokes "Solar and Stellar Chemistry." "I used always to show [in lectures at Glasgow] a spirit lamp flame with salt on it, behind a slit prolonging the dark line D by bright continuation. I always gave your dynamical explanation, always asserted that certainly there was sodium vapour in the sun's atmosphere and in the atmospheres of stars which show presence of the D's, and always pointed out that the way to find other substances besides sodium in the sun and stars was to compare bright lines produced by them in artificial flames with dark lines of the spectra of the lights of the distant bodies\*."

<sup>\*</sup> Letter to Stokes, published in Edinburgh address, 1871.



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Stokes always deprecated the ascription to him of much credit in this matter; but what is certain is, that had the scientific world been acquainted with the correspondence of 1854, it could not have greeted the early memoir of Kirchhoff (1859) as a new revelation. This correspondence will appear in Vol. Iv. of Stokes' Collected Papers, now being prepared under the editorship of Prof. Larmor. The following is from a letter of Kelvin, dated March 9, 1854: "It was Miller's experiment (which you told me about a long time ago) which first convinced me that there must be a physical connection between agency going on in and near the sun, and in the flame of a spirit lamp with salt on it. doubted, after I learned Miller's experiment, that there must be such a connection, nor can I conceive of any one knowing Miller's experiment and doubting....If it could only be made out that the bright line D never occurs without soda, I should consider it perfectly certain that there is soda or sodium in some state in or about the sun. If bright lines in any other flames can be traced, as perfectly as Miller did in his case, to agreement with dark lines in the solar spectrum, the connection would be equally certain, to my mind. I quite expect a qualitative analysis of the sun's atmosphere by experiments like Miller's on other flames."

By temperament, Stokes was over-cautious. "We must not go too fast," he wrote. He felt doubts whether the effects might not be due to some constituent of sodium, supposed to be broken up in the electric arc or flame, rather than to sodium itself. But his facts and theories, if insufficient to satisfy himself, were abundantly enough for Kelvin, and would doubtless have convinced others. If Stokes hung back, his correspondent was ready enough

It is difficult to restrain a feeling of regret that these important advances were no further published than in Lord Kelvin's Glasgow lectures. Possibly want of time prevented Stokes from

to push the application and to formulate the conclusions.

giving his attention to the question. Prof. Larmor significantly remarks that he became Secretary of the Royal Society in 1854. And the reader of the Collected Papers can hardly fail to notice a marked falling off in the speed of production after this time. The reflection suggests itself that scientific men should be kept to scientific work, and should not be tempted to assume heavy administrative duties, at any rate until such time as they have

delivered their more important messages to the world.

But if there was less original work, science benefited by the assistance which, in his position as Secretary of the Royal Society, he was ever willing to give to his fellow-workers. The pages of the "Proceedings" and "Transactions" abound with grateful recognitions of help thus rendered, and in many cases his suggestions or comments form not the least valuable part of memoirs which appear under the names of others. It is not in human nature for an author to be equally grateful when his mistakes are indicated, but from the point of view of the Society and of science



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in general, the service may be very great. It is known that in not a few cases the criticism of Stokes was instrumental in sup-

pressing the publication of serious errors.

No one could be more free than he was from anything like an unworthy jealousy of his comrades. Perhaps he would have been the better for a little more wholesome desire for reputation. As happened in the case of Cavendish, too great an indifference in this respect, especially if combined with a morbid dread of mistakes, may easily lead to the withholding of valuable ideas and even to the suppression of elaborate experimental work, which it is

often a labour to prepare for publication.

In 1857 he married Miss Robinson, daughter of Dr Romney Robinson, F.R.S., astronomer of Armagh. Their first residence was in rooms over a nursery gardener's in the Trumpington Road, where they received visits from Whewell and Sedgwick. Afterwards they took Lensfield Cottage, where they resided until her death in 1899. Though of an unusually quiet and silent disposition, he did not like being alone. He was often to be seen at parties and public functions, and, indeed, rarely declined invitations. In later life, after he had become President of the Royal Society, the hardihood and impunity with which he attended public dinners were matters of general admiration. The nonsense of fools, or rash statements by men of higher calibre, rarely provoked him to speech; but if directly appealed to, he would often explain his view at length with characteristic moderation and lucidity.

His experimental work was executed with the most modest appliances. Many of his discoveries were made in a narrow passage behind the pantry of his house, into the window of which he had a shutter fixed with a slit in it and a bracket on which to place crystals and prisms. It was much the same in lecture. many years he gave an annual course on Physical Optics, which was pretty generally attended by candidates for mathematical honours. To some of these, at any rate, it was a delight to be taught by a master of his subject, who was able to introduce The present into his lectures matter fresh from the anvil. writer well remembers the experiments on the spectra of blood, communicated in the same year (1864) to the Royal Society. There was no elaborate apparatus of tanks and "spectroscopes." A test-tube contained the liquid and was held at arm's length The prism was a small one of 60°, and was held to behind a slit. the eye without the intervention of lenses. The blood in a fresh condition showed the characteristic double band in the green. On reduction by ferrous salt, the double band gave place to a single one, to re-assert itself after agitation with air. By such simple means was a fundamental reaction established. The impression left upon the hearer was that Stokes felt himself as much at home in chemical and botanical questions as in Mathematics and Physics.

At this time the scientific world expected from him a systematic



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treatise on Light, and indeed a book was actually advertised as in preparation. Pressure of work, and perhaps a growing habit of procrastination, interfered. Many years later (1884—1887) the Burnett Lectures were published. Simple and accurate, these lectures are a model of what such lectures should be, but they hardly take the place of the treatise hoped for in the sixties. There was, however, a valuable report on Double Refraction, communicated to the British Association in 1862, in which are correlated the work of Cauchy, MacCullagh and Green. To the theory of MacCullagh, Stokes, imbued with the ideas of the elastic solid theory, did less than justice. Following Green, he took too much for granted that the elasticity of aether must have its origin in deformation, and was led to pronounce the incompatibility of MacCullagh's theory with the laws of Mechanics. It has recently been shown at length by Prof. Larmor that MacCullagh's equations may be interpreted on the supposition that what is resisted is not deformation, but rotation. It is interesting to note that Stokes here expressed his belief that the true dynamical theory of double refraction was yet to be found.

In 1885 he communicated to the Society his observations upon one of the most curious phenomena in the whole range of Optics—a peculiar internal coloured reflection from certain crystals of chlorate of potash. The seat of the colour was found to be a narrow layer, perhaps one-thousandth of an inch in thickness, apparently constituting a twin stratum. Some of the leading

features were described as follows:-

(1) If one of the crystalline plates be turned round in its own plane, without alteration of the angle of incidence, the peculiar reflection vanishes twice in a revolution, viz., when the plane of incidence coincides with the plane of symmetry of the crystal.

(2) As the angle of incidence is increased, the reflected light

becomes brighter, and rises in refrangibility.

(3) The colours are not due to absorption, the transmitted light being strictly complementary to the reflected.

(4) The coloured light is not polarised.

(5) The spectrum of the reflected light is frequently found to consist almost entirely of a comparatively narrow band. In many

cases the reflection appears to be almost total.

Some of these peculiarities, such, for example, as the evanescence of the reflection at perpendicular incidence, could easily be connected with the properties of a twin plane, but the copiousness of the reflection at moderate angles, as well as the high degree of selection, were highly mysterious. There is reason to think that they depend upon a regular, or nearly regular, alternation of twinning many times repeated.

It is impossible here to give anything more than a rough sketch of Stokes' optical work, and many minor papers must be passed over without even mention. But there are two or three contributions to other subjects as to which a word must be said.



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Dating as far back as 1857 there is a short but important discussion on the effect of wind upon the intensity of sound. That sound is usually ill heard up wind is a common observation, but the explanation is less simple than is often supposed. The velocity of moderate winds in comparison with that of sound is too small to be of direct importance. The effect is attributed by Stokes to the fact that winds usually increase overhead, so that the front of a wave proceeding up wind is more retarded above than below. The front is thus tilted; and since a wave is propagated normally to its front, sound proceeding up wind tends to rise, and so to pass over the heads of observers situated at the level of the source, who find themselves, in fact, in a sound shadow.

In a more elaborate memoir (1868) he discusses the important subject of the communication of vibration from a vibrating body to a surrounding gas. In most cases a solid body vibrates without much change of volume, so that the effect is represented by a distribution of sources over the surface, of which the components are as much negative as positive. The resultant is thus largely a question of interference, and it would vanish altogether were it not for the different situations and distances of the positive and negative elements. In any case it depends greatly upon the wave-length (in the gas) of the vibration in progress. Stokes calculates in detail the theory for vibrating spheres and cylinders, showing that when the wave-length is large relatively to the dimensions of the vibrating segments, the resultant effect is enormously diminished by interference. Thus the vibrations of a piano-string are communicated to the air scarcely at all directly, but only through the intervention of the sounding board\*.

On the foundation of these principles he easily explains a curious observation by Leslie, which had much mystified earlier writers. When a bell is sounded in hydrogen, the intensity is greatly reduced. Not only so, but reduction accompanies the actual addition of hydrogen to rarefied air. The fact is that the hydrogen increases the wave-length, and so renders more complete the interference between the sounds originating in the positively

and negatively vibrating segments.

The determination of the laws of viscosity in gases was much advanced by him. Largely through his assistance and advice, the first decisive determinations at ordinary temperatures and pressures were effected by Tomlinson in 1886. In 1881 he brilliantly took advantage of Crookes' observations on the decrement of oscillation of a vibrator in a partially exhausted space, to prove that Maxwell's law holds up to very high exhaustion and to trace the mode of subsequent departure from it. Throughout the course of Crookes' investigations on the electric discharge in vacuum tubes, in which he was keenly interested and closely concerned, he upheld the British

<sup>\*</sup> It may be worth notice that similar conclusions are more simply reached by considering the particular case of a plane vibrating surface.



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view that the cathode stream consists of projected particles which excite phosphorescence in obstacles by impact: and accordingly, after the discovery of the Röntgen rays, he came forward with the view that they consisted of very concentrated spherical pulses travelling through the aether, but distributed quite fortuitously because excited by the random collisions of the cathode particles.

A complete estimate of Stokes' position in scientific history would need a consideration of his more purely mathematical writings, especially of those on Fourier series and the discontinuity of arbitrary constants in semi-convergent expansions over a plane, but this would demand much space and another pen. The present inadequate survey may close with an allusion to another of those "notes," suggested by the work of others, where Stokes in a few pages illuminated a subject hitherto obscure. By an adaptation of Maxwell's colour diagram he showed (1891) how to represent the results of experiments upon ternary mixtures, with reference to the work of Alder Wright. If three points in the plane represent the pure substances, all associations of them are quantitatively represented by points lying within the triangle so defined. example, if two points represent water and aether, all points on the intermediate line represent associations of these substances, but only small parts of the line near the two ends correspond to mixture. If the proportions be more nearly equal, the association separates into two parts. If a third point (off the line) represents alcohol, which is a solvent for both, the triangle may be divided into two regions, one of which corresponds to simple mixtures of the three components, and the other to proportions for which a

simple mixture is not possible.

A consideration of Stokes' work, even though limited to what has here been touched upon, can lead to no other conclusion than that in many subjects, and especially in Hydrodynamics and Optics, the advances which we owe to him are fundamental. Instinct, amounting to genius, and accuracy of workmanship are everywhere apparent; and in scarcely a single instance can it be said that he has failed to lead in the right direction. But, much as he did, one can hardly repress a feeling that he might have done still more. If the activity in original research of the first fifteen years had been maintained for twenty years longer, much additional harvest might have been gathered in. No doubt distractions of all kinds multiplied, and he was very punctilious in the performance of duties more or less formal. During the sitting of the last Cambridge Commission he interrupted his holiday in Ireland to attend a single meeting, at which however, as was remarked, he scarcely opened his mouth. His many friends and admirers usually took a different view from his of the relative urgency of competing claims. Anything for which a date was not fixed by the nature of the case, stood a poor chance. For example, owing to projected improvements and additions, the third volume



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of his Collected Works was delayed until eighteen years after the second, and fifty years after the first appearance of any paper it included. Even this measure of promptitude was only achieved under much pressure, private and official.

But his interest in matters scientific never failed. The intelligence of new advances made by others gave him the greatest joy. Notably was this the case in late years with regard to the Röntgen rays. He was delighted at seeing a picture of the arm which he had broken sixty years before, and finding that it showed clearly the united fracture.

Although this is not the place to dilate upon it, no sketch of Stokes can omit to allude to the earnestness of his religious life. In early years he seems to have been oppressed by certain theological difficulties, and was not exactly what was then considered orthodox. Afterwards he saw his way more clearly. In later life he took part in the work of the Victoria Institute: the spirit which actuated him may be judged from the concluding words of an Address on Science and Revelation. "But whether we agree or cannot agree with the conclusions at which a scientific investigator may have arrived, let us, above all things, beware of imputing evil motives to him, of charging him with adopting his conclusions for the purpose of opposing what is revealed. Scientific investigation is eminently truthful. The investigator may be wrong, but it does not follow he is other than truth-loving. If on some subjects which we deem of the highest importance he does not agree with us—and yet he may agree with us more nearly than we suppose-let us, remembering our own imperfections, both of understanding and of practice, bear in mind that caution of the Apostle: 'Who art thou that judgest another man's servant? To his own master he standeth or falleth."

Scientific honours were showered upon him. He was Foreign Associate of the French Institute, and Knight of the Prussian Order Pour le Mérite. He was awarded the Gauss Medal in 1877, the Arago on the occasion of the Jubilee Celebration in 1899, and the Helmholtz in 1901. In 1889 he was made a Baronet on the recommendation of Lord Salisbury. From 1887 to 1891 he represented the University of Cambridge in Parliament, in this, as in the Presidency of the Society, following the example of his illustrious predecessor in the Lucasian Chair. He was Secretary of the Society from 1854 to 1885, President from 1885 to 1890, received the Rumford Medal in 1852, and the Copley in 1893.

But the most remarkable testimony by far to the estimation in which he was held by his scientific contemporaries was the gathering at Cambridge in 1899, in celebration of the Jubilee of his Professorship. Men of renown flocked from all parts of the world to do him homage, and were as much struck by the modesty and simplicity of his demeanour as they had previously been by the brilliancy of his scientific achievements. The beautiful lines



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by his colleague, Sir R. Jebb, cited below, were written upon this occasion.

There is little more to tell. In 1902 he was chosen Master of Pembroke. But he did not long survive. At the annual dinner of the Cambridge Philosophical Society, held in the College about a month before his death, he managed to attend though very ill, and made an admirable speech, recalling with charming simplicity and courtesy his lifelong intimate connection with the College, to the Mastership of which he had recently been called, and with the Society through which he had published much of his scientific work. Near the end, while conscious that he had not long to live, he retained his faculties unimpaired; only during the last few hours he wandered slightly, and imagined that he was addressing the undergraduates of his College, exhorting them to purity of life. He died on the first of February, 1903.

Clear mind, strong heart, true servant of the light, True to that light within the soul, whose ray Pure and serene, hath brightened on thy way, Honour and praise now crown thee on the height Of tranquil years. Forgetfulness and night Shall spare thy fame, when in some larger day Of knowledge yet undream'd, Time makes a prey Of many a deed and name that once were bright.

Thou, without haste or pause, from youth to age, Hast moved with sure steps to thy goal. And thine That sure renown which sage confirms to sage, Borne from afar. Yet wisdom shows a sign Greater, through all thy life, than glory's wage; Thy strength hath rested on the Love Divine.

R.