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BY

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PREFACE.

IT IS now some years since I was requested by the Syndics of the University Press to allow my papers on mathematical and physical subjects, which are scattered over various Transactions and scientific Journals, to be reprinted in a collected form. Many of these were written a long time ago, and science has in the mean time progressed, and it seemed to me doubtful whether it was worth while now to reprint a series of papers the interest of which may in good measure be regarded as having passed away. However, several of my scientific friends, and among them those to whose opinions I naturally pay the greatest deference, strongly urged me to have the papers reprinted, and I have accordingly acceded to the request of the Syndics. I regret that in consequence of the pressure of other engagements the preparation of the first volume has been so long in hand.

The arrangement of the papers and the mode of treating them in other respects were left entirely to myself, but both the Syndics and my friends advised me to make the reprint full, leaning rather to the inclusion than exclusion of a paper in doubtful cases. I have acted on this advice, and in the first volume, now presented to the public, I have omitted nothing but a few papers which were merely controversial.

As to the arrangement of the papers, it seemed to me that the chronological order was the simplest and in many respects the

best. Had an arrangement by subjects been attempted, not only would it have been difficult in some cases to say under what head a particular paper should come, but also a later paper on some one subject would in many cases have depended on a paper on some different subject which would come perhaps in some later volume, whereas in the chronological arrangement each paper reaches up to the level of the author's knowledge at the time, so that forward reference is not required.

Although notes are added here and there, I have not attempted to bring the various papers up to the level of the present time. I have not accordingly as a rule alluded to later researches on the same subject, unless for some special reason. The notes introduced in the reprint are enclosed in square brackets in order to distinguish them from notes belonging to the original papers. To the extent of these notes therefore, which were specially written for the reprint, the chronological arrangement is departed from. The same is the case as regards the last paper in the first volume, which suggested itself during the preparation for press of the paper to which it relates. In reprinting the papers, any errors of inadvertence which may have been discovered are of course corrected. Mere corrections of this kind are not specified, but any substantial change or omission is noticed in a foot-note or otherwise.

After full consideration, I determined to introduce an innovation in notation which was proposed a great many years ago, for at least partial use, by the late Professor De Morgan, in his article on the Calculus of Functions in the *Encyclopædia Metropolitana*, though the proposal seems never to have been taken up. Mathematicians have been too little in the habit of considering the mechanical difficulty of setting up in type the expressions which they so freely write with the pen; and where the setting up can be facilitated with only a trifling departure from existing usage as regards the appearance of the expression, it seems advisable to make the change.

Now it seems to me preposterous that a compositor should be called on to go through the troublesome process of what printers call *justification*, merely because an author has occasion to name

some simple fraction or differential coefficient in the text, in which term I do not include the formal equations which are usually printed in the middle of the page. The difficulty may be avoided by using, in lieu of the bar between the numerator and denominator, some symbol which may be printed on a line with the type. The symbol ":" is frequently used in expressing ratios; but for employment in the text it has the fatal objection that it is appropriated to mean a colon. The symbol "$\div$" is certainly distinctive, but it is inconveniently long, and $dy \div dx$ for a differential coefficient would hardly be tolerated. Now simple fractions are frequently written with a slant line instead of the horizontal bar separating the numerator from the denominator, merely for the sake of rapidity of writing. If we simply consent to allow the same to appear *in print*, the difficulty will be got over, and a differential coefficient which we have occasion to name in the text may be printed as dy/dx. The type for the slant line already exists, being called a *solidus*.

On mentioning to some of my friends my intention to use the "solidus" notation, it met with a good deal of approval, and some of them expressed their readiness to join me in the use of it, amongst whom I may name Sir William Thomson and the late Professor Clerk Maxwell.

In the formal equations I have mostly preserved the ordinary notation. There is however one exception. It frequently happens that we have to deal with fractions of which the numerator and denominator involve exponentials the indices of which are fractions themselves. Such expressions are extremely troublesome to set up in type in the ordinary notation. But by merely using the solidus for the fractions which form the indices, the setting up of the expression is made comparatively easy, while yet there is not much departure from the appearance of the expressions according to the ordinary notation. Such exponential expressions are commonly associated with circular functions; and though it would not otherwise have been necessary, it seemed desirable to employ the solidus notation for the fraction under the symbol "sin" or "cos," in order to preserve the similarity of appearance between the exponential and circular functions.

In the use of the solidus it seems convenient to enact that it shall as far as possible take the place of the horizontal bar for which it stands, and accordingly that what stands immediately on the two sides of it shall be regarded as welded into one. Thus $\sin n\pi x/a$ means $\sin (n\pi x \div a)$, and not $(\sin n\pi x) \div a$. This welding action may be arrested when necessary by a stop: thus $\sin n\theta\,./r^n$ means $(\sin n\theta) \div r^n$ and not $\sin (n\theta \div r^n)$.

The only objection that I have heard suggested against the solidus notation on the ground of its being already appropriated to something else, relates to a condensed notation sometimes employed for factorials, according to which $x\,(x+a)\,\ldots$ to n factors is expressed by $x^{n|a}$ or by $x^{n/a}$. I do not think the objection is a serious one. There is no risk of the solidus notation, as I have employed it, being mistaken for the expression of factorials; of the two factorial notations just given, that with the separating line vertical seems to be the more common, and might be adhered to when factorials are intended; and if a greater distinction were desired, a factorial might be printed in the condensed notation as $x^{n(a}$, where the "(" would serve to recall the parentheses in the expression written at length.

G. G. STOKES.

CAMBRIDGE,
August 16, 1880.

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ERRATA.

P. 103, l. 14, *for* their *read* there.
P. 193, l. 3, for $p^{y=b}$ read $p_{y=b}$.