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Arthur Stanley Ramsey

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A.S. Ramsey (1867-1954) was a distinguished Cambridge mathematician and President of Magdalene College. He wrote several textbooks 'for the use of higher divisions in schools and for first-year students at university'. This book on electricity and magnetism, first published in 1937, and based upon his lectures over many years, was 'adapted more particularly to the needs of candidates for Part I of the Mathematical Tripos'. It covers electrostatics, conductors and condensers, dielectrics, electrical images, currents, magnetism and electromagnetism, and magnetic induction. The book is interspersed with examples for solution, for some of which answers are provided.

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# ELECTRICITY AND MAGNETISM

An Introduction  
to the Mathematical Theory

by

A. S. RAMSEY, M.A.

*President of Magdalene College, Cambridge  
formerly University Lecturer in Mathematics*



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## PREFACE

This book has been written in response to suggestions from friends who have asked for a text-book on the subject adapted more particularly to the needs of candidates for Part I of the Mathematical Tripos.

A complete study of the theory of electricity and magnetism, as a logical mathematical development from experimental data, requires a knowledge of the methods of mathematical analysis far beyond what can reasonably be expected from most readers of an elementary text-book. The knowledge of pure mathematics assumed in the present volume amounts to little more than some elementary calculus and a few properties of vectors. The ground is restricted by this limitation. It covers the schedule for Part I of the Tripos, including the fundamental principles of electrostatics, Gauss's theorem, Laplace's equation, systems of conductors, homogeneous dielectrics and the theory of images; steady currents in wires; elementary theory of the magnetic field and the elementary facts about the magnetic fields of steady currents. There are also short chapters on induced magnetism and induction of currents.

From one standpoint it would be preferable that a book on a branch of Natural Philosophy should consist of a continuous logical development uninterrupted by 'examples'. But experience seems to indicate that mathematical principles are best understood by making attempts to apply them; and, as the purpose of this book is didactic, I have had no hesitation in interspersing examples through the chapters and giving the solutions of some of them. The text is based upon lectures given at intervals over a period of many years, and the examples are part of a collection which I began to make for the use of my pupils about forty years ago, drawn from Tripos and College Examination papers.

As regards notation, I felt much hesitation about abandoning the use of  $V$  for the potential of an electrostatic field; but

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the custom of using a Greek letter to denote the scalar potential of a vector field has become general, and the matter was decided for me when I found ' $E = -\mathit{grad} \phi$ ' in the Cambridge syllabus.

I am greatly indebted to Mr E. Cunningham of St John's College for reading a large part of the text and making many appropriate criticisms and useful suggestions; and also to Dr S. Verblunsky of the University of Manchester for reading and correcting the proofs, and to the printers and readers of the University Press for careful composition and correction.

A. S. R.

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## Table of Units

c.g.s. absolute unit of force = 1 dyne  
 c.g.s. absolute unit of work or energy = 1 erg

## ELECTRICAL UNITS

Practical units		Equivalent absolute c.g.s. units	
		Electrostatic	Electro-magnetic
Charge	1 coulomb	$3 \times 10^9$	$10^{-1}$
Potential or electro- motive force	1 volt	$3^{-1} \times 10^{-2}$	$10^8$
Current	1 ampère	$3 \times 10^9$	$10^{-1}$
Resistance	1 ohm	$3^{-2} \times 10^{-11}$	$10^9$
Capacity	1 farad	$3^2 \times 10^{11}$	$10^{-9}$
Inductance	1 henry	$3^{-2} \times 10^{-11}$	$10^9$
Rate of working	1 watt	$10^7$	$10^7$

One microfarad is one-millionth of a farad.

An electromotive force of 1 volt drives a current of 1 ampère through a resistance of 1 ohm and work is then being done at the rate of 1 watt or  $10^7$  ergs per second.