CHAPTER I.

MEDIAEVAL MATHEMATICS.[1]

THE subject of this chapter is a sketch of the nature and extent of the mathematics read at Cambridge in the middle ages. The external conditions under which it was carried on are briefly described in the first section of chapter VIII. It is only after considerable hesitation that I have not incorporated that section in this chapter; but I have so far isolated it as to render it possible, for any who may be ignorant of the system of education in a mediæval university, to read it if they feel so inclined, before commencing the history of the development of mathematics at Cambridge.

The period with which I am concerned in this chapter begins towards the end of the twelfth century, and ends with the year 1535. For the history during most of this time of mathematics at Cambridge we are obliged to rely largely on inferences from the condition of other universities. I shall therefore discuss it very briefly referring the reader to the works mentioned below[1] for further details.

[1] Besides the authorities alluded to in the various foot-notes I am indebted for some of the materials for this chapter to *Die Mathematik auf den Universitäten des Mittelalters* by H. Suter, Zurich, 1887; *Die Geschichte des mathematischen Unterrichtes im deutschen Mittelalter bis* 1525, by M. S. Günther, Berlin, 1887; and *Beiträge zur Kenntniss der Mathematik des Mittelalters*, by H. Weissenborn, Berlin, 1888.

Throughout the greater part of this period a student usually proceeded in the faculty of arts; and in that faculty he spent the first four years on the study of the subjects of the trivium, and the next three years on those of the quadrivium. The trivium comprised Latin grammar, logic, and rhetoric; and I have described in chapter VIII. both how they were taught and the manner in which proficiency in them was tested. It must be remembered that students while studying the trivium were treated exactly like school-boys, and regarded in the same light as are the boys of a leading public school at the present time. The title of bachelor was given at the end of this course. A bachelor occupied a position analogous to that of an undergraduate now-a-days. He was required to spend three years in the study of the quadrivium, the subjects of which were mathematics and science. These were divided in the Pythagorean manner into numbers absolute or arithmetic, numbers applied or music, magnitudes at rest or geometry, and magnitudes in motion or astronomy. There was however no test that a student knew anything of the four subjects last named other than his declaration to that effect, and in practice most bachelors left them unread. The degree of master was given at the end of this course.

The quadrivium during **the twelfth and the first half of the thirteenth century**, if studied at all, probably meant about as much science as was to be found in the pages of Boethius, Cassiodorus, and Isidorus. The extent of this is briefly described in the following paragraphs.

The term arithmetic was used in the Greek sense, and meant the study of the properties of numbers; and particularly of ratio, proportion, fractions, and polygonal numbers. It did not include the art of practical calculation, which was generally performed on an abacus; and though symbols were employed to express the result of any numerical computation they were not used in determining it.

The geometry was studied in the text-books either of

Boethius or of Gerbert[1]. The former work, which was the one more commonly used, contains the enunciations of the first book of Euclid, and of a few selected propositions from the third and fourth books. To shew that these are reliable, demonstrations of the first three propositions of the first book are given in an appendix. Some practical applications to the determination of areas were usually added in the form of notes. Even this was too advanced for most students. Thus Roger Bacon, writing towards the close of the thirteenth century, says that at Oxford, there were few, if any, residents who had read more than the definitions and the enunciations of the first five propositions of the first book. In the pages of Cassiodorus and Isidorus a slight sketch of geography is included in geometry.

The two remaining subjects of the quadrivium were music and astronomy. The study of the former had reference to the services of the Church, and included some instruction in metre. The latter was founded on Ptolemy's work, and special attention was supposed to be paid to the rules for finding the moveable festivals of the Church; but it is probable that in practice it generally meant the art of astrology.

By **the middle of the thirteenth century** anyone who was really interested in mathematics had a vastly wider range of reading open to him, but whether students at the English universities availed themselves of it is doubtful.

The mathematical science of modern Europe dates from the thirteenth century, and received its first stimulus[2] from the Moorish schools in Spain and Africa, where the Arab works on arithmetic and algebra, and the Arab translations of Euclid, Archimedes, Apollonius, and Ptolemy were not uncommon. It will be convenient to give here an outline of

[1] Prof. Weissenborn thinks that neither of these books was written by its reputed author, and assigns them both to the eleventh century. This view is not however generally adopted.

[2] For further details of the introduction of Arab science into Europe, see chapter x. of my *History of mathematics*, London, 1888.

the introduction of the Arab geometry and arithmetic into Europe.

First, for the geometry. As early as 1120 an English monk, named Adelhard (of Bath), had obtained a copy of a Moorish edition of the *Elements* of Euclid; and another specimen was secured by Gerard of Cremona in 1186. The first of these was translated by Adelhard, and a copy of this fell into the hands of Giovanni Campano or Campanus, who in 1260 reproduced it as his own. The first printed edition was taken from it and was issued by Ratdolt at Venice in 1482: of course it is in Latin. This pirated translation was the only one generally known until in 1533 the original Greek text was recovered[1]. Campanus also issued a work founded on Ptolemy's astronomy and entitled the *Theory of the planets*.

The earliest explanation of the Arabic system of arithmetic and algebra, which had any wide circulation in Europe, was that contained in the *Liber abbaci* issued in 1202 by Leonardo of Pisa. In this work Leonardo explained the Arabic system of numeration by means of nine digits and a zero; proved some elementary algebraical formulæ by geometry, as in the second book of Euclid; and solved a few algebraical equations. The reasoning was expressed at full length in words and without the use of any symbols. This was followed in 1220 by a work in which he shewed how algebraical formulæ could be applied to practical geometrical problems, such as the determination of the area of a triangle in terms of the lengths of the sides.

Some ten or twelve years later, circ. 1230, the emperor Frederick II. engaged a staff of Jews to translate into Latin all the Arab works on science which were obtainable; and manuscript transcripts of these were widely distributed. Most of the mediæval editions of the writings of Ptolemy, Archimedes, and Apollonius were derived from these copies.

One branch of this science of the Moors was almost at once adopted throughout Europe. This was their arithmetic, which

[1] See p. 23, hereafter; and also the article *Eucleides*, by A. De Morgan, in Smith's *Dictionary of Greek and Roman biography*, London, 1849.

was commonly known as algorism, or the art of Alkarismi, to distinguish it from the arithmetic founded on the work of Boethius. From the middle of the thirteenth century this was used in nearly all mathematical tables, whether astronomical, astrological, or otherwise. It was generally employed for trade purposes by the Italian merchants at or about the same time, and from them spread through the rest of Europe. It would however seem that this rapid adoption of the Arabic numerals and arithmetic was at least as largely due to the calculators of calendars as to merchants and men of science. Perhaps the oriental origin of the symbols gave them an attractive flavour of magic, but there seem to have been very few almanacks after the year 1300 in which an explanation of the system was not included.

The earliest lectures on the subjects of algebra and algorism at any university, of which I can find mention, are some given by Holywood, who is perhaps better known by the latinized name of Sacrobosco. *John de Holywood* was born in Yorkshire and educated at Oxford, but after taking his master's degree he moved to Paris and taught there till his death in 1244 or 1246. His work on arithmetic[1] was for many years a standard authority. He further wrote a treatise on the sphere, which was made public in 1256: this had a wide circulation, and indicates how rapidly a knowledge of mathematics was spreading. Besides these, two pamphlets by him, entitled respectively *De computo ecclesiastico* and *De astrolabio*, are still extant.

Towards **the end of the thirteenth century** a strong effort was made to introduce this science, as studied in Italy, into the curriculum of the English universities. This was due to *Roger Bacon*[2]. Bacon, who was educated at Oxford and Paris

[1] This was printed at Paris in 1496 under the title *De algorithmo;* and has been reissued in Halliwell's *Rara mathematica*, London, second edition, 1841. See also pp. 13—15 of *Arithmetical books*, by A. De Morgan, London, 1847.

[2] See *Roger Bacon, sa vie, ses ouvrages...* by E. Charles, Paris, 1861; and *Roger Bacon, eine Monographie*, by Schneider, Augsburg, 1873. The first of these is very eulogistic, the latter somewhat severely critical. An

and taught at both universities, declared that divine mathematics was not only the alphabet of all philosophy, but should form the foundation of all liberal education, since it alone could fit the student for the acquirement of other knowledge, and enable him to detect the false from the true. He urged that it should be followed by linguistic or scientific studies. These seem also to have been the views of Grosseteste, the statesmanlike bishop of Lincoln. But the power of the schoolmen in the universities was too strong to allow of such a change, and not only did they prevent any alteration of the curriculum but even the works of Bacon on physical science (which might have been included in the quadrivium) were condemned as tending to lead men's thoughts away from the problems of philosophy. It is clear, however, that henceforth a student, who was desirous of reading beyond the narrow limits of the schools, had it in his power to do so: and if I say nothing more about the science of this time it is because I think it probable that no such students were to be found in Cambridge.

The only notable English mathematician in **the early half of the fourteenth century** of whom I find any mention is *Thomas Bradwardine*[1], archbishop of Canterbury. Bradwardine was born at Chichester about 1290. He was educated at Merton College, Oxford, and subsequently lectured in that university. From 1335 to the time of his death he was chiefly occupied with the politics of the church and state: he took a prominent part in the invasion of France, the capture of Calais, and the victory of Cressy. He died at Lambeth in 1349. His mathematical works, which were probably written when he was at Oxford, are (i) the *Tractatus de proportionibus*, printed at Paris in 1495; (ii) the *Arithmetica speculativa*, printed

account of his life by J. S. Brewer is prefixed to the Rolls Series edition of the *Opera inedita*, London, 1859.

[1] See vol. IV. of the *Lives of the Archbishops of Canterbury*, by W. F. Hook, London, 1860—68; see also pp. 480, 487, 521—24 of the *Aperçu historique sur...géométrie* by M. Chasles (first edition).

at Paris in 1502; (iii) the *Geometria speculativa*, printed at Paris in 1511; and (iv) the *De quadratura circuli*, printed at Paris in 1495. They probably give a fair idea of the nature of the mathematics then read at an English university.

By the middle of this century Euclidean geometry (as expounded by Campanus) and algorism were fairly familiar to all professed mathematicians, and the Ptolemaic astronomy was also generally known. About this time the almanacks began to add to the explanation of the Arabic symbols the rules of addition, subtraction, multiplication, and division, "de algorismo." The more important calendars and other treatises also inserted a statement of the rules of proportion, illustrated by various practical questions; such books usually concluded with algebraic formulæ (expressed in words) for most of the common problems of commerce. Of course the fundamental rules of this algorism were not strictly proved—that is the work of advanced thought—but it is important to note that there was some discussion of the principles involved.

I should add that next to the Italians the English took the most prominent part in the early development and improvement of algorism[1], a fact which the backward condition of the country makes rather surprising. Most merchants continued however to keep their accounts in Roman numerals till about 1550, and monasteries and colleges till about 1650: though in both cases it is probable that the processes of arithmetic were performed in the algoristic manner. No instance in a parish register of a date or number being written in Arabic numerals is known to exist before the seventeenth century.

In **the latter half of the fourteenth century** an attempt was made to include in the quadrivium these new works on the elements of mathematics. The stimulus came from Paris, where a statute to that effect was passed in 1366, and a year or two later similar regulations were made at Oxford and Cam-

[1] An English treatise by John Norfolk, written about 1340, is still extant. It was printed in 1445 and reissued by Halliwell in his *Rara mathematica*, London, second edition, 1841.

bridge; unfortunately no text-books[1] are mentioned. We can however form a reasonable estimate of the range of mathematical reading required, by looking at the statutes of the universities of Prague founded in 1350, of Vienna founded in 1364, and of Leipzig founded in 1389[2].

By the statutes of Prague[3], dated 1384, candidates for the bachelor's degree were required to have read Holywood's treatise on the sphere, and candidates for the master's degree to be acquainted with the first six books of Euclid, optics, hydrostatics, the theory of the lever, and astronomy. Lectures were actually delivered on arithmetic, the art of reckoning with the fingers, and the algorism of integers; on almanacks, which probably meant elementary astrology; and on the *Almagest*, that is on Ptolemaic astronomy. There is however some reason for thinking that mathematics received there far more attention than was then usual at other universities.

At Vienna in 1389 the candidate for a master's degree was required[4] to have read five books of Euclid, common perspective, proportional parts, the measurement of superficies, and the *Theory of the planets*. The book last named is the treatise by Campanus, which was founded on that by Ptolemy. This was a fairly respectable mathematical standard, but I would remind the reader that there was no such thing as "plucking" in a mediæval university. The student had to keep an act or give a lecture on certain subjects, but whether he did it well or badly he got his degree, and it is probable that it was only the few students whose interests were mathematical who really mastered the subjects mentioned above.

[1] See p. 81 of *De l'organisation de l'enseignement...au moyen âge* by C. Thurot, Paris, 1850.

[2] The following account is taken from *Die Geschichte der Mathematik*, by H. Hankel, Leipzig, 1874.

[3] See vol. I. pp. 49, 56, 77, 83, 92, 108, 126, of the *Historia universitatis Pragensis*, Prag, 1830.

[4] See vol. I. p. 237 of the *Statuta universitatis Wiennensis* by V. Kollar, Vienna, 1839: quoted in vol. I. pp. 283 and 351 of the *University of Cambridge*, by J. Bass Mullinger, Cambridge, 1873.

At any rate no test of proficiency was imposed; and a few facts gleaned from the history of the next century tend to shew that the regulations about the study of the quadrivium were not seriously enforced. The lecture lists for the years 1437 and 1438 of the university of Leipzig (the statutes of which are almost identical with those of Prague as quoted above) are extant, and shew that the only lectures given there on mathematics in those years were confined to astrology. The records[1] of Bologna, Padua, and Pisa seem to imply that there also astrology was the only scientific subject taught in the fifteenth century, and even as late as 1598 the professor of mathematics at Pisa was required to lecture on the *Quadripartitum*, a spurious astrological work attributed to Ptolemy. According to the registers[2] of the university of Oxford the only mathematical subjects read there between the years 1449 and 1463 were Ptolemy's astronomy (or some commentary on it) and the first two books of Euclid. Whether most students got as far as this is doubtful. It would seem, from an edition of Euclid published at Paris in 1536, that after 1452 candidates for the master's degree at that university had to take an oath that they had attended lectures on the first six books of Euclid.

The only Cambridge mathematicians of **the fifteenth century** of whom I can find any mention were Holbroke, Marshall, and Hodgkins. No details of their lives and works are known. **John Holbroke**, master of Peterhouse and chancellor of the university for the years 1428 and 1429, who died in 1437, is reputed to have been a distinguished astronomer and astrologer. **Roger Marshall**, who was a fellow of Pembroke, taught mathematics and medicine; he subsequently moved to London and became physician to Edward IV. **John Hodgkins**, a fellow of King's, who died in 1485 is mentioned as a celebrated mathematician.

[1] See pp. 15, 20 of *Die Geschichte der mathematischen Facultät in Bologna* by S. Gherardi, edited by M. Kurtze, Berlin, 1871.

[2] Quoted in the *Life of bishop Smyth* (the founder of Brazenose College) by Ralph Churton, Oxford, 1800.

At **the beginning of the sixteenth century** the names of Master, Paynell, and Tonstall occur. Of these **Richard Master**, a fellow of King's, is said to have been famous for his knowledge of natural philosophy. He entered at King's in 1502, and was proctor in 1511. He took up the cause of the holy maid of Kent and was executed for treason in April, 1534. **Nicholas Paynell**, a fellow of Pembroke Hall, graduated B.A. in 1515. In 1530 he was appointed mathematical lecturer to the university. The date of his death is unknown.

Cuthbert Tonstall[1] was born at Hackforth, Yorkshire, in 1474 and died in 1559. He had entered at Balliol College, Oxford, but finding the philosophers dominant in the university (see p. 243), he migrated to King's Hall, Cambridge. We must not attach too much importance to this step for such migrations were then very common, and his action only meant that he could continue his studies better at Cambridge than at Oxford. He subsequently went to Padua, where he studied the writings of Regiomontanus and Pacioli. His arithmetic termed *De arte supputandi* was published in 1522 as a "farewell to the sciences" on his appointment to the bishopric of London. A presentation copy on vellum with the author's autograph is in the university library at Cambridge. The work is described by De Morgan in his *Arithmetical Books* as one of the best which has been written both in matter, style, and for the historical knowledge displayed. Few critics would agree with this estimate, but it was undoubtedly the best arithmetic then issued, and forms a not unworthy conclusion to the mediæval history of Cambridge. It is particularly valuable as containing illustrations of the mediæval processes of computation. Several extracts from it are given in the *Philosophy of arithmetic* by J. Leslie, second edition, Edinburgh, 1820.

That brings me to the close of the middle ages, and the above account—meagre though it is—contains all that I have

[1] See vol. I. p. 198 of the *Athenae Cantabrigienses* by C. H. and T. Cooper, Cambridge, 1858—61.