

PRELIMINARY DISCOURSE.

PRIMARY causes are unknown to us; but are subject to simple and constant laws, which may be discovered by observation, the study of them being the object of natural philosophy.

Heat, like gravity, penetrates every substance of the universe, its rays occupy all parts of space. The object of our work is to set forth the mathematical laws which this element obeys. The theory of heat will hereafter form one of the most important branches of general physics.

The knowledge of rational mechanics, which the most ancient nations had been able to acquire, has not come down to us, and the history of this science, if we except the first theorems in harmony, is not traced up beyond the discoveries of Archimedes. This great geometer explained the mathematical principles of the equilibrium of solids and fluids. About eighteen centuries elapsed before Galileo, the originator of dynamical theories, discovered the laws of motion of heavy bodies. Within this new science Newton comprised the whole system of the universe. The successors of these philosophers have extended these theories, and given them an admirable perfection: they have taught us that the most diverse phenomena are subject to a small number of fundamental laws which are reproduced in all the acts of nature. It is recognised that the same principles regulate all the movements of the stars, their form, the inequalities of their courses, the equilibrium and the oscillations of the seas, the harmonic vibrations of air and sonorous bodies, the transmission of light, capillary actions, the undulations of fluids, in fine the most complex effects of all the natural forces, and thus has the thought

of Newton been confirmed: *quod tam paucis tam multa præstet geometria gloriatur*¹.

But whatever may be the range of mechanical theories, they do not apply to the effects of heat. These make up a special order of phenomena, which cannot be explained by the principles of motion and equilibrium. We have for a long time been in possession of ingenious instruments adapted to measure many of these effects; valuable observations have been collected; but in this manner partial results only have become known, and not the mathematical demonstration of the laws which include them all.

I have deduced these laws from prolonged study and attentive comparison of the facts known up to this time: all these facts I have observed afresh in the course of several years with the most exact instruments that have hitherto been used.

To found the theory, it was in the first place necessary to distinguish and define with precision the elementary properties which determine the action of heat. I then perceived that all the phenomena which depend on this action resolve themselves into a very small number of general and simple facts; whereby every physical problem of this kind is brought back to an investigation of mathematical analysis. From these general facts I have concluded that to determine numerically the most varied movements of heat, it is sufficient to submit each substance to three fundamental observations. Different bodies in fact do not possess in the same degree the power to *contain* heat, to *receive or transmit it across their surfaces*, nor to *conduct* it through the interior of their masses. These are the three specific qualities which our theory clearly distinguishes and shews how to measure.

It is easy to judge how much these researches concern the physical sciences and civil economy, and what may be their influence on the progress of the arts which require the employment and distribution of heat. They have also a necessary connection with the system of the world, and their relations become known when we consider the grand phenomena which take place near the surface of the terrestrial globe.

¹ *Philosophiæ naturalis principia mathematica. Auctoris præfatio ad lectorem.* Ac gloriatur geometria quod tam paucis principiis aliunde petitis tam multa præstet. [A. F.]

In fact, the radiation of the sun in which this planet is incessantly plunged, penetrates the air, the earth, and the waters; its elements are divided, change in direction every way, and, penetrating the mass of the globe, would raise its mean temperature more and more, if the heat acquired were not exactly balanced by that which escapes in rays from all points of the surface and expands through the sky.

Different climates, unequally exposed to the action of solar heat, have, after an immense time, acquired the temperatures proper to their situation. This effect is modified by several accessory causes, such as elevation, the form of the ground, the neighbourhood and extent of continents and seas, the state of the surface, the direction of the winds.

The succession of day and night, the alternations of the seasons occasion in the solid earth periodic variations, which are repeated every day or every year: but these changes become less and less sensible as the point at which they are measured recedes from the surface. No diurnal variation can be detected at the depth of about three metres [ten feet]; and the annual variations cease to be appreciable at a depth much less than sixty metres. The temperature at great depths is then sensibly fixed at a given place: but it is not the same at all points of the same meridian; in general it rises as the equator is approached.

The heat which the sun has communicated to the terrestrial globe, and which has produced the diversity of climates, is now subject to a movement which has become uniform. It advances within the interior of the mass which it penetrates throughout, and at the same time recedes from the plane of the equator, and proceeds to lose itself across the polar regions.

In the higher regions of the atmosphere the air is very rare and transparent, and retains but a minute part of the heat of the solar rays: this is the cause of the excessive cold of elevated places. The lower layers, denser and more heated by the land and water, expand and rise up: they are cooled by the very fact of expansion. The great movements of the air, such as the trade winds which blow between the tropics, are not determined by the attractive forces of the moon and sun. The action of these celestial bodies produces scarcely perceptible oscillations in a fluid so rare and at so great a distance. It

is the changes of temperature which periodically displace every part of the atmosphere.

The waters of the ocean are differently exposed at their surface to the rays of the sun, and the bottom of the basin which contains them is heated very unequally from the poles to the equator. These two causes, ever present, and combined with gravity and the centrifugal force, keep up vast movements in the interior of the seas. They displace and mingle all the parts, and produce those general and regular currents which navigators have noticed.

Radiant heat which escapes from the surface of all bodies, and traverses elastic media, or spaces void of air, has special laws, and occurs with widely varied phenomena. The physical explanation of many of these facts is already known; the mathematical theory which I have formed gives an exact measure of them. It consists, in a manner, in a new catoptrics which has its own theorems, and serves to determine by analysis all the effects of heat direct or reflected.

The enumeration of the chief objects of the theory sufficiently shews the nature of the questions which I have proposed to myself. What are the elementary properties which it is requisite to observe in each substance, and what are the experiments most suitable to determine them exactly? If the distribution of heat in solid matter is regulated by constant laws, what is the mathematical expression of those laws, and by what analysis may we derive from this expression the complete solution of the principal problems? Why do terrestrial temperatures cease to be variable at a depth so small with respect to the radius of the earth? Every inequality in the movement of this planet necessarily occasioning an oscillation of the solar heat beneath the surface, what relation is there between the duration of its period, and the depth at which the temperatures become constant?

What time must have elapsed before the climates could acquire the different temperatures which they now maintain; and what are the different causes which can now vary their mean heat? Why do not the annual changes alone in the distance of the sun from the earth, produce at the surface of the earth very considerable changes in the temperatures?

From what characteristic can we ascertain that the earth has not entirely lost its original heat; and what are the exact laws of the loss?

If, as several observations indicate, this fundamental heat is not wholly dissipated, it must be immense at great depths, and nevertheless it has no sensible influence at the present time on the mean temperature of the climates. The effects which are observed in them are due to the action of the solar rays. But independently of these two sources of heat, the one fundamental and primitive, proper to the terrestrial globe, the other due to the presence of the sun, is there not a more universal cause, which determines *the temperature of the heavens*, in that part of space which the solar system now occupies? Since the observed facts necessitate this cause, what are the consequences of an exact theory in this entirely new question; how shall we be able to determine that constant value of *the temperature of space*, and deduce from it the temperature which belongs to each planet?

To these questions must be added others which depend on the properties of radiant heat. The physical cause of the reflection of cold, that is to say the reflection of a lesser degree of heat, is very distinctly known; but what is the mathematical expression of this effect?

On what general principles do the atmospheric temperatures depend, whether the thermometer which measures them receives the solar rays directly, on a surface metallic or unpolished, or whether this instrument remains exposed, during the night, under a sky free from clouds, to contact with the air, to radiation from terrestrial bodies, and to that from the most distant and coldest parts of the atmosphere?

The intensity of the rays which escape from a point on the surface of any heated body varying with their inclination according to a law which experiments have indicated, is there not a necessary mathematical relation between this law and the general fact of the equilibrium of heat; and what is the physical cause of this inequality in intensity?

Lastly, when heat penetrates fluid masses, and determines in them internal movements by continual changes of the temperature and density of each molecule, can we still express, by differential

equations, the laws of such a compound effect; and what is the resulting change in the general equations of hydrodynamics?

Such are the chief problems which I have solved, and which have never yet been submitted to calculation. If we consider further the manifold relations of this mathematical theory to civil uses and the technical arts, we shall recognize completely the extent of its applications. It is evident that it includes an entire series of distinct phenomena, and that the study of it cannot be omitted without losing a notable part of the science of nature.

The principles of the theory are derived, as are those of rational mechanics, from a very small number of primary facts, the causes of which are not considered by geometers, but which they admit as the results of common observations confirmed by all experiment.

The differential equations of the propagation of heat express the most general conditions, and reduce the physical questions to problems of pure analysis, and this is the proper object of theory. They are not less rigorously established than the general equations of equilibrium and motion. In order to make this comparison more perceptible, we have always preferred demonstrations analogous to those of the theorems which serve as the foundation of statics and dynamics. These equations still exist, but receive a different form, when they express the distribution of luminous heat in transparent bodies, or the movements which the changes of temperature and density occasion in the interior of fluids. The coefficients which they contain are subject to variations whose exact measure is not yet known; but in all the natural problems which it most concerns us to consider, the limits of temperature differ so little that we may omit the variations of these coefficients.

The equations of the movement of heat, like those which express the vibrations of sonorous bodies, or the ultimate oscillations of liquids, belong to one of the most recently discovered branches of analysis, which it is very important to perfect. After having established these differential equations their integrals must be obtained; this process consists in passing from a common expression to a particular solution subject to all the given conditions. This difficult investigation requires a special analysis

founded on new theorems, whose object we could not in this place make known. The method which is derived from them leaves nothing vague and indeterminate in the solutions, it leads them up to the final numerical applications, a necessary condition of every investigation, without which we should only arrive at useless transformations.

The same theorems which have made known to us the equations of the movement of heat, apply directly to certain problems of general analysis and dynamics whose solution has for a long time been desired.

Profound study of nature is the most fertile source of mathematical discoveries. Not only has this study, in offering a determinate object to investigation, the advantage of excluding vague questions and calculations without issue; it is besides a sure method of forming analysis itself, and of discovering the elements which it concerns us to know, and which natural science ought always to preserve: these are the fundamental elements which are reproduced in all natural effects.

We see, for example, that the same expression whose abstract properties geometers had considered, and which in this respect belongs to general analysis, represents as well the motion of light in the atmosphere, as it determines the laws of diffusion of heat in solid matter, and enters into all the chief problems of the theory of probability.

The analytical equations, unknown to the ancient geometers, which Descartes was the first to introduce into the study of curves and surfaces, are not restricted to the properties of figures, and to those properties which are the object of rational mechanics; they extend to all general phenomena. There cannot be a language more universal and more simple, more free from errors and from obscurities, that is to say more worthy to express the invariable relations of natural things.

Considered from this point of view, mathematical analysis is as extensive as nature itself; it defines all perceptible relations, measures times, spaces, forces, temperatures; this difficult science is formed slowly, but it preserves every principle which it has once acquired; it grows and strengthens itself incessantly in the midst of the many variations and errors of the human mind.

Its chief attribute is clearness; it has no marks to express con-

fused notions. It brings together phenomena the most diverse, and discovers the hidden analogies which unite them. If matter escapes us, as that of air and light, by its extreme tenuity, if bodies are placed far from us in the immensity of space, if man wishes to know the aspect of the heavens at successive epochs separated by a great number of centuries, if the actions of gravity and of heat are exerted in the interior of the earth at depths which will be always inaccessible, mathematical analysis can yet lay hold of the laws of these phenomena. It makes them present and measurable, and seems to be a faculty of the human mind destined to supplement the shortness of life and the imperfection of the senses; and what is still more remarkable, it follows the same course in the study of all phenomena; it interprets them by the same language, as if to attest the unity and simplicity of the plan of the universe, and to make still more evident that unchangeable order which presides over all natural causes.

The problems of the theory of heat present so many examples of the simple and constant dispositions which spring from the general laws of nature; and if the order which is established in these phenomena could be grasped by our senses, it would produce in us an impression comparable to the sensation of musical sound.

The forms of bodies are infinitely varied; the distribution of the heat which penetrates them seems to be arbitrary and confused; but all the inequalities are rapidly cancelled and disappear as time passes on. The progress of the phenomenon becomes more regular and simpler, remains finally subject to a definite law which is the same in all cases, and which bears no sensible impress of the initial arrangement.

All observation confirms these consequences. The analysis from which they are derived separates and expresses clearly, 1^o the general conditions, that is to say those which spring from the natural properties of heat, 2^o the effect, accidental but continued, of the form or state of the surfaces; 3^o the effect, not permanent, of the primitive distribution.

In this work we have demonstrated all the principles of the theory of heat, and solved all the fundamental problems. They could have been explained more concisely by omitting the simpler problems, and presenting in the first instance the most general results; but we wished to shew the actual origin of the theory and

its gradual progress. When this knowledge has been acquired and the principles thoroughly fixed, it is preferable to employ at once the most extended analytical methods, as we have done in the later investigations. This is also the course which we shall hereafter follow in the memoirs which will be added to this work, and which will form in some manner its complement¹; and by this means we shall have reconciled, so far as it can depend on ourselves, the necessary development of principles with the precision which becomes the applications of analysis.

The subjects of these memoirs will be, the theory of radiant heat, the problem of the terrestrial temperatures, that of the temperature of dwellings, the comparison of theoretic results with those which we have observed in different experiments, lastly the demonstrations of the differential equations of the movement of heat in fluids.

The work which we now publish has been written a long time since; different circumstances have delayed and often interrupted the printing of it. In this interval, science has been enriched by important observations; the principles of our analysis, which had not at first been grasped, have become better known; the results which we had deduced from them have been discussed and confirmed. We ourselves have applied these principles to new problems, and have changed the form of some of the proofs. The delays of publication will have contributed to make the work clearer and more complete.

The subject of our first analytical investigations on the transfer of heat was its distribution amongst separated masses; these have been preserved in Chapter III., Section II. The problems relative to continuous bodies, which form the theory rightly so called, were solved many years afterwards; this theory was explained for the first time in a manuscript work forwarded to the Institute of France at the end of the year 1807, an extract from which was published in the *Bulletin des Sciences (Société Philomatique, year 1808, page 112)*. We added to this memoir, and successively forwarded very extensive notes, concerning the convergence of series, the diffusion of heat in an infinite prism, its emission in spaces

¹ These memoirs were never collectively published as a sequel or complement to the *Théorie Analytique de la Chaleur*. But, as will be seen presently, the author had written most of them before the publication of that work in 1822. [A. F.]

void of air, the constructions suitable for exhibiting the chief theorems, and the analysis of the periodic movement at the surface of the earth. Our second memoir, on the propagation of heat, was deposited in the archives of the Institute, on the 28th of September, 1811. It was formed out of the preceding memoir and the notes already sent in; the geometrical constructions and those details of analysis which had no necessary relation to the physical problem were omitted, and to it was added the general equation which expresses the state of the surface. This second work was sent to press in the course of 1821, to be inserted in the collection of the Academy of Sciences. It is printed without any change or addition; the text agrees literally with the deposited manuscript, which forms part of the archives of the Institute¹.

In this memoir, and in the writings which preceded it, will be found a first explanation of applications which our actual work

¹ It appears as a memoir and supplement in volumes IV. and V. of the *Mémoires de l'Académie des Sciences*. For convenience of comparison with the table of contents of the *Analytical Theory of Heat*, we subjoin the titles and heads of the chapters of the printed memoir:

THÉORIE DU MOUVEMENT DE LA CHALEUR DANS LES CORPS SOLIDES, PAR M. FOURIER. [*Mémoires de l'Académie Royale des Sciences de l'Institut de France. Tome IV.* (for year 1819). Paris 1824.]

- I. Exposition.
- II. Notions générales et définitions préliminaires.
- III. Equations du mouvement de la chaleur.
- IV. Du mouvement linéaire et varié de la chaleur dans une armoire.
- V. De la propagation de la chaleur dans une lame rectangulaire dont les températures sont constantes.
- VI. De la communication de la chaleur entre des masses disjointes.
- VII. Du mouvement varié de la chaleur dans une sphère solide.
- VIII. Du mouvement varié de la chaleur dans un cylindre solide.
- IX. De la propagation de la chaleur dans un prisme dont l'extrémité est assujettie à une température constante.
- X. Du mouvement varié de la chaleur dans un solide de forme cubique.
- XI. Du mouvement linéaire et varié de la chaleur dans les corps dont une dimension est infinie.

SUITE DU MÉMOIRE INTITULÉ: THÉORIE DU MOUVEMENT DE LA CHALEUR DANS LES CORPS SOLIDES; PAR M. FOURIER. [*Mémoires de l'Académie Royale des Sciences de l'Institut de France. Tome V.* (for year 1820). Paris, 1826.]

- XII. Des températures terrestres, et du mouvement de la chaleur dans l'intérieur d'une sphère solide, dont la surface est assujettie à des changemens périodiques de température.
 - XIII. Des lois mathématiques de l'équilibre de la chaleur rayonnante.
 - XIV. Comparaison des résultats de la théorie avec ceux de diverses expériences.
- [A. F.]