CAMBRIDGE LIBRARY COLLECTION

Books of enduring scholarly value

Mathematical Sciences

From its pre-historic roots in simple counting to the algorithms powering modern desktop computers, from the genius of Archimedes to the genius of Einstein, advances in mathematical understanding and numerical techniques have been directly responsible for creating the modern world as we know it. This series will provide a library of the most influential publications and writers on mathematics in its broadest sense. As such, it will show not only the deep roots from which modern science and technology have grown, but also the astonishing breadth of application of mathematical techniques in the humanities and social sciences, and in everyday life.

Mechanism of the Heavens

Published in 1831, this work forms part of a collection of introductory volumes suggested by Henry, Lord Brougham and Vaux, the Lord High Chancellor, for the Society of the Diffusion of Useful Knowledge. Due to the exceptional mathematical ability of its author, however, it outgrew its original plan and has since been seen as a rather more ambitious project. Praised by Somerville's contemporary Sir John Herschel for its presentation of general astronomical theories and the mechanical principles employed in their derivation, the work was a tour de force of scientific and technical exposition. It is especially remarkable both for its author's firm grasp of the subject, especially given her lack of formal mathematical training, and for its clear outline of Newtonian philosophy for a popular audience.

Cambridge University Press has long been a pioneer in the reissuing of out-of-print titles from its own backlist, producing digital reprints of books that are still sought after by scholars and students but could not be reprinted economically using traditional technology. The Cambridge Library Collection extends this activity to a wider range of books which are still of importance to researchers and professionals, either for the source material they contain, or as landmarks in the history of their academic discipline.

Drawing from the world-renowned collections in the Cambridge University Library, and guided by the advice of experts in each subject area, Cambridge University Press is using state-of-the-art scanning machines in its own Printing House to capture the content of each book selected for inclusion. The files are processed to give a consistently clear, crisp image, and the books finished to the high quality standard for which the Press is recognised around the world. The latest print-on-demand technology ensures that the books will remain available indefinitely, and that orders for single or multiple copies can quickly be supplied.

The Cambridge Library Collection will bring back to life books of enduring scholarly value across a wide range of disciplines in the humanities and social sciences and in science and technology.

Mechanism of the Heavens

Mary Somerville
Pierre Simon Marquis de Laplace

Cambridge
University Press

CAMBRIDGE UNIVERSITY PRESS

Cambridge New York Melbourne Madrid Cape Town Singapore São Paolo Delhi

Published in the United States of America by Cambridge University Press, New York

www.cambridge.org
Information on this title: www.cambridge.org/9781108001571

This edition first published 1831
This digitally printed version 2009

ISBN 978-1-108-00157-1

This book reproduces the text of the original edition. The content and language reflect the beliefs, practices and terminology of their time, and have not been updated.

MECHANISM

OF

THE HEAVENS.

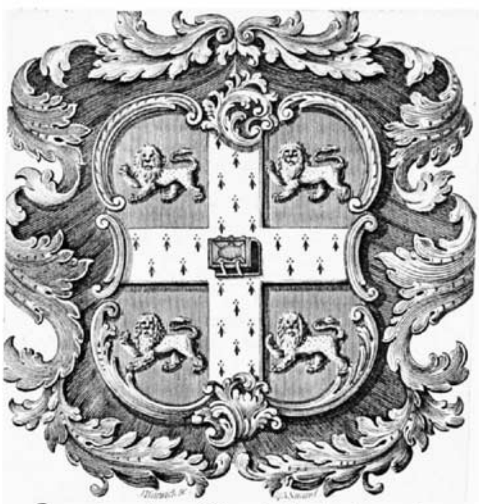
BY

MRS. SOMERVILLE.

LONDON:

JOHN MURRAY, ALBEMARLE-STREET.

MDCCCXXXI



Academiæ Cantabrigiensis
Liber.

TO

HENRY, LORD BROUGHAM AND VAUX,

LORD HIGH CHANCELLOR OF GREAT BRITAIN,

This Work, undertaken at His Lordship's request, is inscribed as a testimony of the Author's esteem and regard.

Although it has unavoidably exceeded the limits of the Publications of the Society for the Diffusion of Useful Knowledge, for which it was originally intended, his Lordship still thinks it may tend to promote the views of the Society in its present form. To concur with that Society in the diffusion of useful knowledge, would be the highest ambition of the Author,

MARY SOMERVILLE.

Royal Hospital, Chelsea,
21*st July*, 1831.

PRELIMINARY DISSERTATION.

In order to convey some idea of the object of this work, it may be useful to offer a few preliminary observations on the nature of the subject which it is intended to investigate, and of the means that have already been adopted with so much success to bring within the reach of our faculties, those truths which might seem to be placed so far beyond them.

All the knowledge we possess of external objects is founded upon experience, which furnishes a knowledge of facts, and the comparison of these facts establishes relations, from which, induction, the intuitive belief that like causes will produce like effects, leads us to general laws. Thus, experience teaches that bodies fall at the surface of the earth with an accelerated velocity, and proportional to their masses. Newton proved, by comparison, that the force which occasions the fall of bodies at the earth's surface, is identical with that which retains the moon in her orbit; and induction led him to conclude that as the moon is kept in her orbit by the attraction of the earth, so the planets might be retained in their orbits by the attraction of the sun. By such steps he was led to the discovery of one of those powers with which the Creator has ordained that matter should reciprocally act upon matter.

Physical astronomy is the science which compares and identifies the laws of motion observed on earth with the motions that take place in the heavens, and which traces, by an uninterrupted chain of deduction from the great principle that governs the universe, the revolutions and rotations of the planets, and the oscillations of the fluids at their surfaces, and which estimates the changes the system has hitherto undergone or may hereafter experience, changes which require millions of years for their accomplishment.

The combined efforts of astronomers, from the earliest dawn of civilization, have been requisite to establish the mechanical theory of astronomy: the courses of the planets have been observed for ages with a degree of perseverance that is astonishing, if we consider the imperfection, and even the want of instruments. The real motions of the earth have been separated from the apparent motions of the planets; the laws of the planetary revolutions have been discovered; and the discovery of these laws has led to the knowledge of the gravitation of matter. On the other hand, descending from the principle of gravitation, every motion in the system of the world has been so completely explained, that no astronomical phenomenon can now be transmitted to posterity of which the laws have not been determined.

Science, regarded as the pursuit of truth, which can only be attained by patient and unprejudiced investigation, wherein nothing is too great to be attempted, nothing so minute as to be justly disregarded, must ever afford occupation of consummate interest and of elevated meditation. The contemplation of the works of creation elevates the mind to the admiration of whatever is great and noble, accomplishing the object of all study, which in the elegant language of Sir James Mackintosh is to inspire the love of truth, of wisdom, of beauty, especially of goodness, the highest beauty, and of that supreme and eternal mind, which contains all truth and wisdom, all beauty and goodness. By the love or delightful contemplation and pursuit of these transcendent aims for their own sake only, the mind of man is raised from low and perishable objects, and prepared for those high destinies which are appointed for all those who are capable of them.

The heavens afford the most sublime subject of study which can be derived from science: the magnitude and splendour of the objects, the inconceivable rapidity with which they move, and the enormous distances between them, impress the mind with some notion of the energy that maintains them in their motions with a durability to which we can see no limits. Equally conspicuous is the goodness of the great First Cause in having endowed man with faculties by which he can not only appreciate the magnificence of his works, but trace, with precision, the operation of his laws, use the globe he inhabits as a base wherewith to measure the magnitude and distance of the sun and planets, and make the diameter of the earth's orbit the first step of a scale by which he may ascend to the starry firmament. Such pursuits, while they ennoble the mind, at the same time inculcate humility, by showing that there is a barrier, which no energy, mental or physical, can ever enable us to pass: that however profoundly we may penetrate the depths of space, there still remain innumerable systems, compared with which those which seem so mighty to us must dwindle into insignificance, or even become invisible; and that not only man, but the globe he inhabits, nay the whole system of which it forms so small a part, might be annihilated, and its extinction be unperceived in the immensity of creation.

A complete acquaintance with Physical Astronomy can only be attained by those who are well versed in the higher branches of mathematical and mechanical science: such alone can appreciate the extreme beauty of the results, and of the means by which these results are obtained. Nevertheless a sufficient skill in analysis to follow the general outline, to see the mutual dependence of the different parts of the system, and to comprehend by what means some of the most extraordinary conclusions have been arrived at, is within the reach of many who shrink from the task, appalled by difficulties, which perhaps are not more formidable than those incident to the study of the elements of every branch of knowledge, and possibly overrating them by not making a sufficient distinction between the degree of mathematical acquirement necessary for making discoveries, and that which is requisite for understanding what others have done. That the study of mathematics and their application to astronomy are full of interest will be allowed by all who have devoted their time and attention to these pursuits, and they only can estimate the delight of arriving at truth, whether it be in the discovery of a world, or of a new property of numbers.

It has been proved by Newton that a particle of matter placed without the surface of a hollow sphere is attracted by it in the same manner as if its mass, or the whole matter it contains, were collected in its centre. The same is therefore true of a solid sphere which may be supposed to consist of an infinite number of concentric hollow spheres. This however is not the case

b 2

with a spheroid, but the celestial bodies are so nearly spherical, and at such remote distances from each other, that they attract and are attracted as if each were a dense point situate in its centre of gravity, a circumstance which greatly facilitates the investigation of their motions.

The attraction of the earth on bodies at its surface in that latitude, the square of whose sine is $\frac{1}{3}$, is the same as if it were a sphere; and experience shows that bodies there fall through 16.0697 feet in a second. The mean distance of the moon from the earth is about sixty times the mean radius of the earth. When the number 16.0697 is diminished in the ratio of 1 to 3600, which is the square of the moon's distance from the earth, it is found to be exactly the space the moon would fall through in the first second of her descent to the earth, were she not prevented by her centrifugal force, arising from the velocity with which she moves in her orbit. So that the moon is retained in her orbit by a force having the same origin and regulated by the same law with that which causes a stone to fall at the earth's surface. The earth may therefore be regarded as the centre of a force which extends to the moon; but as experience shows that the action and reaction of matter are equal and contrary, the moon must attract the earth with an equal and contrary force.

Newton proved that a body projected in space will move in a conic section, if it be attracted by a force directed towards a fixed point, and having an intensity inversely as the square of the distance; but that any deviation from that law will cause it to move in a curve of a different nature. Kepler ascertained by direct observation that the planets describe ellipses round the sun, and later observations show that comets also move in conic sections: it consequently follows that the sun attracts all the planets and comets inversely as the square of their distances from his centre; the sun therefore is the centre of a force extending indefinitely in space, and including all the bodies of the system in its action.

Kepler also deduced from observation, that the squares of the periodic times of the planets, or the times of their revolutions round the sun, are proportional to the cubes of their mean distances from his centre: whence it follows, that the intensity of gravitation of all the bodies towards the sun is the same at equal distances; consequently gravitation is proportional to the masses, for if the planets and comets be supposed to be at equal distances from the sun and left to the effects of gravity, they would arrive at his surface at the same time. The satellites also gravitate to their primaries according to the same law that their primaries do to the sun. Hence, by the law of action and reaction, each body is itself the centre of an attractive force extending indefinitely in space, whence proceed all the mutual disturbances that render the celestial motions so complicated, and their investigation so difficult.

The gravitation of matter directed to a centre, and attracting directly as the mass, and inversely as the square of the distance, does not belong to it when taken in mass; particle acts on particle according to the same law when at sensible distances from each other. If the sun acted on the centre of the earth without attracting each of its particles, the tides would be very much greater than they now are, and in other respects they also would be very different. The gravitation of the earth to the sun results from the gravitation of all its particles, which in their turn attract the sun in the ratio of their respective masses. There is a reciprocal action likewise between the earth and every particle at its surface; were this not the case, and were any portion of the earth, however small, to attract another portion and not be itself attracted, the centre of gravity of the earth would be moved in space, which is impossible.

The form of the planets results from the reciprocal attraction of their component particles. A detached fluid mass, if at rest, would assume the form of a sphere, from the reciprocal attraction of its particles; but if the mass revolves about an axis, it becomes flattened at the poles, and bulges at the equator, in consequence of the centrifugal force arising from the velocity of rotation. For, the centrifugal force diminishes the gravity of the particles at the equator, and equilibrium can only exist when these two forces are balanced by an increase of gravity; therefore, as the attractive force is the same on all particles at equal distances from the centre of a sphere, the equatorial particles would recede from the centre till their increase in number balanced the centrifugal force by their attraction, consequently the sphere would become an oblate spheroid; and a fluid partially or entirely covering a solid, as the ocean and atmosphere cover the earth, must assume that form in order to remain in equilibrio. The surface of the sea is therefore spheroidal, and the surface of the earth only deviates from that figure where it rises above or sinks below the level of the sea; but the deviation is so small that it is unimportant when compared with the magnitude of the earth. Such is the form of the earth and planets, but the compression or flattening at their poles is so small, that even Jupiter, whose rotation is the most rapid, differs but little from a sphere. Although the planets attract each other as if they were spheres on account of their immense distances, yet the satellites are near enough to be sensibly affected in their motions by the forms of their primaries. The moon for example is so near the earth, that the reciprocal attraction between each of her particles and each of the particles in the prominent mass at the terrestrial equator, occasions considerable disturbances in the motions of both bodies. For, the action of the moon on the matter at the earth's equator produces a nutation in the axis of rotation, and the reaction of that matter on the moon is the cause of a corresponding nutation in the lunar orbit.

If a sphere at rest in space receives an impulse passing through its centre of gravity, all its parts will move with an equal velocity in a straight line; but if the impulse does not pass through the centre of gravity, its particles having unequal velocities, will give it a rotatory motion at the same time that it is translated in space. These motions are independent of one another, so that a contrary impulse passing through its centre of gravity will impede its progression, without interfering with its rotation. As the sun rotates about an axis, it seems probable if an impulse in a contrary direction has not been given to his centre of gravity, that he moves in space accompanied by all those bodies which compose the solar system, a circumstance that would in no way interfere with their relative motions; for, in consequence of our experience that force is proportional to velocity, the reciprocal attractions of a system remain the same, whether its centre of gravity be at rest, or moving uniformly in space. It is computed that had the earth received its motion from a single impulse, such impulse must have passed through a point about twenty-five miles from its centre.

Since the motions of the rotation and translation of the planets are independent of each other, though probably communicated by the same impulse, they form separate subjects of investigation.

A planet moves in its elliptical orbit with a velocity varying every instant, in consequence of two forces, one tending to the centre of the sun, and the other in the direction of a tangent to its orbit, arising from the primitive impulse given at the time when it was launched into space: should the force in the tangent cease, the planet would fall to the sun by its gravity; were the sun not to attract it, the planet would fly off in the tangent. Thus, when a planet is in its aphelion or at the point where the orbit is farthest from the sun, his action overcomes its velocity, and brings it towards him with such an accelerated motion, that it at last overcomes the sun's attraction, and shoots past him; then, gradually decreasing in velocity, it arrives at the aphelion where the sun's attraction again prevails. In this motion the radii vectores, or imaginary lines joining the centres of the sun and planets, pass over equal areas in equal times.

If the planets were attracted by the sun only, this would ever be their course; and because his action is proportional to his mass, which is immensely larger than that of all the planets put together, the elliptical is the nearest approximation to their true motions, which are extremely complicated, in consequence of their mutual attraction, so that they do not move in any known or symmetrical curve, but in paths now approaching to, and now receding from the elliptical form, and their radii vectores do not describe areas exactly proportional to the time. Thus the areas become a test of the existence of disturbing forces.

To determine the motion of each body when disturbed by all the rest is beyond the power of analysis; it is therefore necessary to estimate the disturbing action of one planet at a time, whence arises the celebrated problem of the three bodies, which originally was that of the moon, the earth, and the sun, namely,—the masses being given of three bodies projected from three given points, with velocities given both in quantity and direction; and supposing the bodies to gravitate to one another with forces that are directly as their masses, and inversely as the squares of the distances, to find the lines described by these bodies, and their position at any given instant.

By this problem the motions of translation of all the celestial bodies are determined. It is one of extreme difficulty, and would be of infinitely greater difficulty, if the disturbing action were not very small, when compared with the central force. As the disturbing influence of each body may be found separately, it is assumed that the action of the whole system in disturbing any one planet is equal to the sum of all the particular disturbances it experiences, on the general mechanical principle, that the sum of any number of small oscillations is nearly equal to their simultaneous and joint effect.

On account of the reciprocal action of matter, the stability of the system depends on the intensity of the primitive momentum of the planets, and the ratio of their masses to that of the sun: for the nature of the conic sections in which the celestial bodies move, depends on the velocity with which they were first propelled in space; had that velocity been such as to make the planets move in orbits of unstable equilibrium, their mutual attractions might have changed them into parabolas or even hyperbolas; so that the earth and planets might ages ago have been sweeping through the abyss of space: but as the orbits differ very little from circles, the momentum of the planets when projected, must have been exactly sufficient to ensure the permanency and stability of the system. Besides the mass of the sun is immensely greater than those of the planets; and as their inequalities bear the same ratio to their elliptical motions as their masses do to that of the sun, their mutual disturbances only increase or diminish the eccentricities of their orbits by very minute quantities; consequently the magnitude of the sun's mass is the principal cause of the stability of the system. There is not in the physical world a more splendid example of the adaptation of means to the accomplishment of the end, than is exhibited in the nice adjustment of these forces.

The orbits of the planets have a very small inclination to the plane of the ecliptic in which the earth moves; and on that account, astronomers refer their motions to it at a given epoch as a known and fixed position. The paths of the planets, when their mutual disturbances are omitted, are ellipses nearly approaching to circles, whose planes, slightly inclined to the ecliptic, cut it in straight lines passing through the centre of the sun; the points where the orbit intersects the plane of the ecliptic are its nodes.

The orbits of the recently discovered planets deviate more from the ecliptic: that of Pallas has an inclination of 35° to it: on that account it will be more difficult to determine their motions. These little planets have no sensible effect in disturbing the rest, though their own motions are rendered very irregular by the proximity of Jupiter and Saturn.

The planets are subject to disturbances of two distinct kinds, both resulting from the constant operation of their reciprocal attraction, one kind depending upon their positions with regard to each other, begins from zero, increases to a maximum, decreases and becomes zero again, when the planets return to the same relative positions. In consequence of these, the troubled planet is sometimes drawn away from the sun, sometimes brought nearer to him; at one time it is drawn above the plane of its orbit, at another time below it, according to the position of the disturbing body. All such changes, being accomplished in short periods, some in a few months, others in years, or in hundreds of years, are denominated Periodic Inequalities.

The inequalities of the other kind, though occasioned likewise by the disturbing energy of the planets, are entirely independent of their relative positions; they depend on the relative positions of the orbits alone, whose forms and places in space are altered by very minute quantities in immense periods of time, and are therefore called Secular Inequalities.

In consequence of disturbances of this kind, the apsides, or extremities of the major axes of all the orbits, have a direct, but variable motion in space, excepting those of Venus, which are retrograde; and the lines of the nodes move with a variable velocity in the contrary direction. The motions of both are extremely slow; it requires more than 109770 years for the major axis of the earth's orbit to accomplish a sidereal revolution, and 20935 years to complete its tropical motion. The major axis of Jupiter's orbit requires no less than 197561 years to perform its revolution from the disturbing action of Saturn alone. The periods in which the nodes revolve are also very great. Beside these, the inclination and eccentricity of every orbit are in a state of perpetual, but slow change. At the present time, the inclinations of all the orbits are decreasing; but so slowly, that the inclination of Jupiter's orbit is only six minutes less now than it was in the age of Ptolemy. The terrestrial eccentricity is decreasing at the rate of 3914 miles in a century; and if it were to decrease equably, it would be 36300 years before the earth's orbit became a circle. But in the midst of all these vicissitudes, the major axes and mean motions of the planets remain permanently independent of secular changes; they are so connected by Kepler's law of the squares of the periodic times being proportional to the cubes of the mean distances of the planets from the sun, that one cannot vary without affecting the other.

With the exception of these two elements, it appears, that all the bodies are in motion, and every orbit is in a state of perpetual change. Minute as these changes are, they might be supposed liable to accumulate in the course of ages sufficiently to derange the whole order of nature, to alter the relative positions of the planets, to put an end to the vicissitudes of the seasons, and to bring about collisions, which would involve our whole system, now so harmonious, in chaotic confusion. The consequences being so dreadful, it is natural to inquire, what proof exists that creation will be preserved from such a catastrophe? for nothing can be known from observation, since the existence of the human race has occupied but a point in duration, while these vicissitudes embrace myriads of ages. The proof is simple and convincing. All the variations of the solar system, as well secular as periodic, are expressed analytically by the sines and cosines of circular arcs, which increase with the time; and as a sine or cosine never can exceed the radius, but must oscillate between zero and unity, however much the time may increase, it follows, that when the variations have by slow changes accumulated in however long a time to a maximum, they decrease by the same slow degrees, till they arrive at their smallest value, and then begin a new course, thus for ever oscillating about a mean value. This, however, would not be the case if the planets moved in a resisting medium, for then both the eccentricity and the major axes of the orbits would vary with the time, so that the stability of the system would be ultimately destroyed. But if the planets do move in an ethereal medium, it must be of extreme rarity, since its resistance has hitherto been quite insensible.

Three circumstances have generally been supposed necessary to prove the stability of the system: the small eccentricities of the planetary orbits, their small inclinations, and the revolution of all the bodies, as well planets as satellites, in the same direction. These, however, are not necessary conditions: the periodicity of the terms in which the inequalities are expressed is sufficient to assure us, that though we do not know the extent of the limits, nor the period of that grand cycle which probably embraces millions of years, yet they never will exceed what is requisite for the stability and harmony of the whole, for the preservation of which every circumstance is so beautifully and wonderfully adapted.

The plane of the ecliptic itself, though assumed to be fixed at a given epoch for the convenience of astronomical computation, is subject to a minute secular variation of 52″.109, occasioned by the reciprocal action of the planets; but as this is also periodical, the terrestrial equator, which is inclined to it at an angle of about 23° 28′, will never coincide with the plane of the ecliptic; so there never can be perpetual spring. The rotation of the earth is uniform; therefore day and night, summer and winter, will continue their vicissitudes while the system endures, or is untroubled by foreign causes.

> Yonder starry sphere
> Of planets, and of fix'd, in all her wheels
> Resembles nearest, mazes intricate,
> Eccentric, intervolved, yet regular
> Then most, when most irregular they seem.

The stability of our system was established by La Grange, 'a discovery,' says Professor Playfair, 'that must render the name for ever memorable in science, and revered by those who delight in the contemplation of whatever is excellent and sublime. After Newton's discovery of the elliptical orbits of the planets, La Grange's discovery of their periodical inequalities is without doubt the noblest truth in physical astronomy; and, in respect of the doctrine of final causes, it may be regarded as the greatest of all.'

Notwithstanding the permanency of our system, the secular variations in the planetary orbits would have been extremely embarrassing to astronomers, when it became necessary to compare observations separated by long periods. This difficulty is obviated by La Place, who has shown that whatever changes time may induce either in the orbits themselves, or in the plane of the ecliptic, there exists an invariable plane passing through the centre of gravity of the sun, about which the whole system oscillates within narrow limits, and which is determined by this property; that if every body in the system be projected on it, and if the mass of each be multiplied by the area described in a given time by its projection on this plane, the sum of all these products will be a maximum. This plane of greatest inertia, by no means peculiar to the solar system, but existing in every system of bodies submitted to their mutual attractions only, always remains parallel to itself, and maintains a fixed position, whence the oscillations of the system may be estimated through unlimited time. It is situate nearly half way between the orbits of Jupiter and Saturn, and is inclined to the ecliptic at an angle of about $1^\circ\ 35'\ 31''$.

All the periodic and secular inequalities deduced from the law of gravitation are so perfectly confirmed by observations, that analysis has become one of the most certain means of discovering the planetary irregularities, either when they are too small, or too long in their periods, to be detected by other methods. Jupiter and Saturn, however, exhibit inequalities which for a long time seemed discordant with that law. All observations, from those of the Chinese and Arabs down to the present day, prove that for ages the mean motions of Jupiter and Saturn have been affected by great inequalities of very long periods, forming what appeared an anomaly in the theory of the planets. It was long known by observation, that five times the mean motion of Saturn is nearly equal to twice that of Jupiter; a relation which the sagacity of La Place perceived to be the cause of a periodic inequality in the mean motion of each of these planets, which completes its period in nearly 929 Julian years, the one being retarded, while the other is accelerated. These inequalities are strictly periodical, since they depend on the configuration of the two planets; and the theory is perfectly confirmed by observation, which shows that in the course of twenty centuries, Jupiter's mean motion has been accelerated by 3° 23′, and Saturn's retarded by 5°.13′.

It might be imagined that the reciprocal action of such planets as have satellites would be different from the influence of those that have none; but the distances of the satellites from their primaries are incomparably less than the distances of the planets from the sun, and from one another, so that the system of a planet and its satellites moves nearly as if all those bodies were united in their common centre of gravity; the action of the sun however disturbs in some degree the motion of the satellites about their primary.

The changes that take place in the planetary system are exhibited on a small scale by Jupiter and his satellites; and as the period requisite for the development of the inequalities of these little moons only extends to a few centuries, it may be regarded as an epitome of that grand cycle which will not be accomplished by the planets in myriads of centuries. The revolutions of the satellites about Jupiter are precisely similar to those of the planets about the sun; it is true they are disturbed by the sun, but his distance is so great, that their motions are nearly the same as if they were not under his influence. The satellites like the planets, were probably projected in elliptical orbits, but the compression of Jupiter's spheroid is very great in consequence of his rapid rotation; and as the masses of the satellites are nearly 100000 times less than that of Jupiter, the immense quantity of prominent matter at his equator must soon have given the circular form observed in the orbits of the first and second satellites, which its superior attraction will always maintain. The third and fourth satellites being further removed from its influence, move in orbits with a very small eccentricity. The same cause occasions the orbits of the satellites to remain nearly in the plane of Jupiter's equator, on account of which they are always seen nearly in the same line; and the powerful action of that quantity of prominent matter is the reason why the motion of the nodes of these little bodies is so much more rapid than those of the planet. The nodes of the fourth satellite accomplish a revolution in 520 years, while those of Jupiter's orbit require no less than 50673 years, a proof of the reciprocal attraction between each particle of Jupiter's equator and of the satellites. Although the two first satellites sensibly move in circles, they acquire a small ellipticity from the disturbances they experience.

The orbits of the satellites do not retain a permanent inclination, either to the plane of Jupiter's equator, or to that of his orbit, but to certain planes passing between the two, and through their intersection; these have a greater inclination to his equator the further the satellite is removed, a circumstance entirely owing to the influence of Jupiter's compression.

A singular law obtains among the mean motions and mean longitudes of the three first satellites. It appears from observation, that the mean motion of the first satellite, plus twice that of the third, is equal to three times that of the second, and that the mean longitude of the first satellite, minus three times that of the second, plus twice that of the third, is always equal to two right angles. It is proved by theory, that if these relations had only been approximate when the satellites were first launched into space, their mutual attractions would have established and maintained them. They extend to the synodic motions of the satellites, consequently they affect their eclipses, and have a very great influence on their whole theory. The satellites move so nearly in the plane of Jupiter's equator, which has a very small inclination to his orbit, that they are frequently eclipsed by the planet. The instant of the beginning or end of an eclipse of a satellite marks the same instant of absolute time to all the inhabitants of the earth; therefore the time of these eclipses observed by a traveller, when compared with the time of the eclipse computed for Greenwich or any other fixed meridian, gives the difference of the meridians in time, and consequently the longitude of the place of observation. It has required all the refinements of modern instruments to render the eclipses of these remote moons available to the mariner; now however, that system of bodies invisible to the naked eye, known to man by the aid of science alone, enables him to traverse the ocean, spreading the light of knowledge and the blessings of civilization over the most remote regions, and to return loaded with the productions of another hemisphere. Nor is this all: the eclipses of Jupiter's satellites have been the means of a discovery, which, though not so immediately applicable to the wants of man, unfolds a property of light, that medium, without whose cheering influence all the beauties of the creation would have been to us a blank. It is observed, that those eclipses of the first satellite which happen when Jupiter is near conjunction, are later by 16′ 26″ than those which take place when the planet is in opposition. But as Jupiter is nearer to us when in opposition by the whole breadth of the earth's orbit than when in conjunction, this circumstance was attributed to the time employed by the rays of light in crossing the earth's orbit, a distance of 192 millions of miles; whence it is estimated, that light travels at the rate of 192000 miles in one second. Such is its velocity, that the earth, moving at the rate of nineteen miles in a second, would take two months to pass through a distance which a ray of light would dart over in eight minutes. The subsequent discovery of the aberration of light confirmed this astonishing result.

Objects appear to be situate in the direction of the rays that proceed from them. Were light propagated instantaneously, every object, whether at rest or in motion, would appear in the direction of these rays; but as light takes some time to travel, when Jupiter is in conjunction, we see him by means of rays that left him 16′ 26″ before; but during that time we have changed our position, in consequence of the motion of the earth in its orbit; we therefore refer Jupiter to a place in which he is not. His true position is in the diagonal of the parallelogram, whose sides are in the ratio of the velocity of light to the velocity of the earth in its orbit, which is as 192000 to 19. In consequence of aberration, none of the heavenly bodies are in the place in which they seem to be. In fact, if the earth were at rest, rays from a star would pass along the axis of a telescope directed to it; but if the earth were to begin to move in its orbit with its usual velocity, these rays would strike against the side of the tube; it would therefore be necessary to incline the telescope a little, in order to see the star. The angle contained between the axis of the telescope and a line drawn to the true place of the star, is its aberration, which varies in quantity and direction in different parts of the earth's orbit; but as it never exceeds twenty seconds, it is insensible in ordinary cases.

The velocity of light deduced from the observed aberration of the fixed stars, perfectly corresponds with that given by the eclipses of the first satellite. The same result obtained from sources so different, leaves not a doubt of its truth. Many such beautiful coincidences, derived from apparently the most unpromising and dissimilar circumstances, occur in physical astronomy, and prove dependences which we might otherwise be unable to trace. The identity of the velocity of light at the distance of Jupiter and on the earth's surface shows that its velocity is uniform; and if light consists in the vibrations of an elastic fluid or ether filling space, which hypothesis accords best with observed phenomena, the uniformity of its velocity shows that the density of the fluid throughout the whole extent of the solar system, must be proportional to its elasticity. Among the fortunate conjectures which have been confirmed by subsequent experience, that of Bacon is not the least remarkable. 'It produces in me,' says the restorer of true philosophy, 'a doubt, whether the face of the serene and starry heavens be seen at the instant it really exists, or not till some time later; and whether there be not, with respect to the heavenly bodies, a true time and an apparent time, no less than a true place and an apparent place, as astronomers say, on account of parallax. For it seems incredible that the species or rays of the celestial bodies can pass through the immense interval between them and us in an instant; or that they do not even require some considerable portion of time.'

As great discoveries generally lead to a variety of conclusions, the aberration of light affords a direct proof of the motion of the earth in its orbit; and its rotation is proved by the theory of falling bodies, since the centrifugal force it induces retards the oscillations of the pendulum in going from the pole to the equator. Thus a high degree of scientific knowledge has been requisite to dispel the errors of the senses.

The little that is known of the theories of the satellites of Saturn and Uranus is in all respects similar to that of Jupiter. The great compression of Saturn occasions its satellites to move nearly in the plane of its equator. Of the situation of the equator of Uranus we know nothing, nor of its compression. The orbits of its satellites are nearly perpendicular to the plane of the ecliptic.

Our constant companion the moon next claims attention. Several circumstances concur to render her motions the most interesting, and at the same time the most difficult to investigate of all the bodies of our system. In the solar system planet troubles planet, but in the lunar theory the sun is the great disturbing cause; his vast distance being compensated by his enormous magnitude, so that the motions of the moon are more irregular than those of the planets; and on account of the great ellipticity of her orbit and the size of the sun, the approximations to her motions are tedious and difficult, beyond what those unaccustomed to such investigations could imagine. Neither the eccentricity of the lunar orbit, nor its inclination to the plane of the ecliptic, have experienced any changes from secular inequalities; but the mean motion, the nodes, and the perigee, are subject to very remarkable variations.

From an eclipse observed at Babylon by the Chaldeans, on the 19th of March, seven hundred and twenty-one years before the Christian era, the place of the moon is known from that of the sun at the instant of opposition; whence her mean longitude may be found; but the comparison of this mean longitude with another mean longitude, computed back for the instant of the eclipse from modern observations, shows that the moon performs her revolution round the earth more rapidly and in a shorter time now, than she did formerly; and that the acceleration in her mean motion has been increasing from age to age as the square of the time; all the ancient and intermediate eclipses confirm this result. As the mean motions of the planets have no secular inequalities, this seemed to be an unaccountable anomaly, and it was at one time attributed to the resistance of an ethereal medium pervading space; at another to the successive transmission of the gravitating force: but as La Place proved that neither of these causes, even if they exist, have any influence on the motions of the lunar perigee or nodes, they could not affect the mean motion, a variation in the latter from such a cause being inseparably connected with

c

variations in the two former of these elements. That great mathematician, however, in studying the theory of Jupiter's satellites, perceived that the secular variations in the elements of Jupiter's orbit, from the action of the planets, occasion corresponding changes in the motions of the satellites: this led him to suspect that the acceleration in the mean motion of the moon might be connected with the secular variation in the eccentricity of the terrestrial orbit; and analysis has proved that he assigned the true cause.

If the eccentricity of the earth's orbit were invariable, the moon would be exposed to a variable disturbance from the action of the sun, in consequence of the earth's annual revolution; but it would be periodic, since it would be the same as often as the sun, the earth, and the moon returned to the same relative positions: on account however of the slow and incessant diminution in the eccentricity of the terrestrial orbit, the revolution of our planet is performed at different distances from the sun every year. The position of the moon with regard to the sun, undergoes a corresponding change; so that the mean action of the sun on the moon varies from one century to another, and occasions the secular increase in the moon's velocity called the acceleration, a name which is very appropriate in the present age, and which will continue to be so for a vast number of ages to come; because, as long as the earth's eccentricity diminishes, the moon's mean motion will be accelerated; but when the eccentricity has passed its minimum and begins to increase, the mean motion will be retarded from age to age. At present the secular acceleration is about 10″, but its effect on the moon's place increases as the square of the time. It is remarkable that the action of the planets thus reflected by the sun to the moon, is much more sensible than their direct action, either on the earth or moon. The secular diminution in the eccentricity, which has not altered the equation of the centre of the sun by eight minutes since the earliest recorded eclipses, has produced a variation of 1° 48′ in the moon's longitude, and of 7° 12′ in her mean anomaly.

The action of the sun occasions a rapid but variable motion in the nodes and perigee of the lunar orbit; the former, though they recede during the greater part of the moon's revolution, and advance during the smaller, perform their sidereal revolutions in $6793^{\text{days}}.4212$, and the latter, though its motion is sometimes retrograde and sometimes direct, in $3232^{\text{days}}.5807$, or a little more than nine years: but such is the difference between the disturbing energy of the sun and that of all the planets put together, that it requires no less than 109770 years for the greater axis of the terrestrial orbit to do the same. It is evident that the same secular variation which changes the sun's distance from the earth, and occasions the acceleration in the moon's mean motion, must affect the motion of the nodes and perigee; and it consequently appears, from theory as well as observation, that both these elements are subject to a secular inequality, arising from the variation in the eccentricity of the earth's orbit, which connects them with the acceleration; so that both are retarded when the mean motion is anticipated. The secular variations in these three elements are in the ratio of the numbers 3, 0.735, and 1; whence the three motions of the moon, with regard to the sun, to her perigee, and to her nodes, are continually accelerated, and their secular equations are as the numbers 1, 4, and 0.265, or according to the most recent investigations as 1, 4, 6776 and 0.391. A comparison of ancient eclipses observed by the Arabs, Greeks, and Chaldeans, imperfect as they are, with modern observations, perfectly confirms these results of analysis.

Future ages will develop these great inequalities, which at some most distant period will amount to many circumferences. They are indeed periodic; but who shall tell their period? Millions of years must elapse before that great cycle is accomplished; but 'such changes, though rare in time, are frequent in eternity.'

The moon is so near, that the excess of matter at the earth's equator occasions periodic variations in her longitude and latitude; and, as the cause must be proportional to the effect, a comparison of these inequalities, computed from theory, with the same given by observation, shows that the compression of the terrestrial spheroid, or the ratio of the difference between the polar and equatorial diameter to the diameter of the equator is $\frac{1}{305\cdot05}$. It is proved analytically, that if a fluid mass of homogeneous matter, whose particles attract each other inversely as the square of the distance, were to revolve about an axis, as the earth, it would assume the form of a spheroid, whose compression is $\frac{1}{230}$. Whence it appears, that the earth is not homogeneous, but decreases in density from its centre to its circumference. Thus the moon's eclipses show the earth to be round, and her inequalities not only determine the form, but the internal structure of our planet; results of analysis which could not have been anticipated. Similar inequalities in Jupiter's satellites prove that his mass is not homogeneous, and that his compression is $\frac{1}{13 \cdot 8}$.

The motions of the moon have now become of more importance to the navigator and geographer than those of any other body, from the precision with which the longitude is determined by the occultations of stars and lunar distances. The lunar theory is brought to such perfection, that the times of these phenomena, observed under any meridian, when compared with that computed for Greenwich in the Nautical Almanack, gives the longitude of the observer within a few miles. The accuracy of that work is obviously of extreme importance to a maritime nation; we have reason to hope that the new Ephemeris, now in preparation, will be by far the most perfect work of the kind that ever has been published.

From the lunar theory, the mean distance of the sun from the earth, and thence the whole dimensions of the solar system are known; for the forces which retain the earth and moon in their orbits, are respectively proportional to the radii vectores of the earth and moon, each being divided by the square of its periodic time; and as the lunar theory gives the ratio of the forces, the ratio of the distance of the sun and moon from the earth is obtained: whence it appears that the sun's distance from the earth is nearly 396 times greater than that of the moon.

The method however of finding the absolute distances of the celestial bodies in miles, is in fact the same with that employed in measuring distances of terrestrial objects. From the extremities of a known base the angles which the visual rays from the object form with it, are measured; their sum subtracted from two right-angles gives the angle opposite the base; therefore by trigonometry, all the angles and sides of the triangle may be computed; consequently the distance of the object is found. The angle under which the base of the triangle is seen from the object, is the parallax of that object; it evidently increases and decreases with the distance; therefore the base must be very great indeed, to be visible at all from the celestial bodies. But the globe itself whose dimensions are ascertained by actual admeasurement, furnishes a standard of measures, with which we compare the distances, masses, densities, and volumes of the sun and planets.

The courses of the great rivers, which are in general navigable to a considerable extent, prove that the curvature of the land differs but little from that of the ocean; and as the heights of the mountains and continents are, at any rate, quite inconsiderable when compared with the magnitude of the earth, its figure is understood to be determined by a surface at every point perpendicular to the direction of gravity, or of the plumb-line, and is the same which the sea would have if it were continued all round the earth beneath the continents. Such is the figure that has been measured in the following manner:—

A terrestrial meridian is a line passing through both poles, all the points of which have contemporaneously the same noon. Were the lengths and curvatures of different meridians known, the figure of the earth might be determined; but the length of one degree is sufficient to give the figure of the earth, if it be measured on different meridians, and in a variety of latitudes; for if the earth were a sphere, all degrees would be of the same length, but if not, the lengths of the degrees will be greatest where the curvature is least; a comparison of the length of the degrees in different parts of the earth's surface will therefore determine its size and form.

An arc of the meridian may be measured by observing the latitude of its extreme points, and then measuring the distance between them in feet or fathoms; the distance thus determined on the surface of the earth, divided by the degrees and parts of a degree contained in the difference of the latitudes, will give the exact length of one degree, the difference of the latitudes being the angle contained between the verticals at the extremities of the arc. This would be easily accomplished were the distance unobstructed, and on a level with the sea; but on account of the innumerable obstacles on the surface of the earth, it is necessary to connect the extreme points of the arc by a series of triangles, the sides and angles of which are either measured or computed, so that the length of the arc is ascertained with much laborious computation. In consequence of the inequalities of the surface, each triangle is in a different plane; they must therefore be reduced by computation to what they would have been, had they been measured on the surface of the sea; and as the earth is spherical, they require a correction to reduce them from plane to spherical triangles.

Arcs of the meridian have been measured in a variety of latitudes, both north and south, as well as arcs perpendicular to the meridian. From these measurements it appears that the length of the degrees increase from the equator to the poles, nearly as the square of the sine of the latitude; consequently, the convexity of the earth diminishes from the equator to the poles. Many discrepancies occur, but the figure that most nearly follows this law is an ellipsoid of revolution, whose equatorial radius is 3962.6 miles, and the polar radius 3949.7; the difference, or 12.9 miles, divided by the equatorial radius, is $\frac{1}{308 \cdot 7}$, or $\frac{1}{309}$ nearly; this fraction is called the compression of the earth, because, according as it is greater or less, the terrestrial ellipsoid is more or less flattened at the poles; it does not differ much from that given by the lunar inequalities. If we assume the earth to be a sphere, the length of a degree of the meridian is $69\frac{1}{22}$ British miles; therefore 360 degrees, or the whole circumference of the globe is 24856, and the diameter, which is something less than a third of the circumference, is 7916 or 8000 miles nearly. Eratosthenes, who died 194 years before the Christian era, was the first to give an approximate value of the earth's circumference, by the mensuration of an arc between Alexandria and Syene.

But there is another method of finding the figure of the earth, totally independent of either of the preceding. If the earth were a homogeneous sphere without rotation, its attraction on bodies at its surface would be everywhere the same; if it be elliptical, the force of gravity theoretically ought to increase, from the equator to the pole, as the square of the sine of the latitude; but for a spheroid in rotation, by the laws of mechanics the centrifugal force varies as the square of the sine of the latitude from the equator where it is greatest, to the pole where it vanishes; and as it tends to make bodies fly off the surface, it diminishes the effects of gravity by a small quantity. Hence by gravitation, which is the difference of these two forces, the fall of bodies ought to be accelerated in going from the equator to the poles, proportionably to the square of the sine of the latitude; and the weight of the same body ought to increase in that ratio. This is directly proved by the oscillations of the pendulum; for if the fall of bodies be accelerated, the oscillations will be more rapid; and that they may always be performed in the same time, the length of the pendulum must be altered. Now, by numerous and very careful experiments, it is proved that a pendulum, which makes 86400 oscillations in a mean day at the equator, will do the same at every point of the earth's surface, if its length be increased in going to the pole, as the square of the sine of the latitude. From the mean of these it appears that the compression of the terrestrial spheroid is about $\frac{1}{342}$, which does not differ much from that given by the lunar inequalities, and from the arcs of the meridian. The near coincidence of these three values, deduced by methods so entirely independent of each other, shows that the mutual tendencies of the centres of the celestial bodies to one another, and the attraction of the earth for bodies at its surface, result from the reciprocal attraction of all their particles. Another proof may be added; the nutation of the earth's axis, and the precession of the equinoxes, are occasioned by the action of the sun and moon on the protuberant matter at the earth's equator; and although these inequalities do not give the absolute value of the terrestrial compression, they show that the fraction expressing it is comprised between the limits $\frac{1}{279}$ and $\frac{1}{578}$.

It might be expected that the same compression should result from each, if the different methods of observation could be made without error. This, however, is not the case; for such discrepancies are found both in the degrees of the me-