PHYSICAL ASTRONOMY.

THE infinite varieties of motion in the heavens, and on the earth, obey a few laws, so universal in their application, that they regulate the curve traced by an atom which seems to be the sport of the winds, with as much certainty as the orbits of the planets. These laws, on which the order of nature depends, remained unknown till the sixteenth century, when Galileo, by investigating the circumstances of falling bodies, laid the foundation of the science of mechanics, which Newton, by the discovery of gravitation, afterwards extended from the earth to the farthest limits of our system.

This original property of matter, by means of which we ascertain the past and anticipate the future, is the link which connects our planet with remote worlds, and enables us to determine distances, and estimate magnitudes, that might seem to be placed beyond the reach of human faculties. To discern and deduce from ordinary and apparently trivial occurrences the universal laws of nature, as Galileo and Newton have done, is a mark of the highest intellectual power.

Simple as the law of gravitation is, its application to the motions of the bodies of the solar system is a problem of great difficulty, but so important and interesting, that the solution of it has engaged the attention and exercised the talents of the most distinguished mathematicians; among whom La Place holds a distinguished place by the brilliancy of his discoveries, as well as from having been the first to trace the influence of this property of matter from the elliptical motions of the planets, to its most remote effects on their mutual perturbations. Such was the object contemplated by him in his splendid work on the Mechanism of the Heavens; a work which may be considered as a great problem of dynamics, wherein it is required to deduce all the phenomena of the solar system from the abstract laws of motion, and to confirm the truth of those laws, by comparing theory with observation.

Tables of the motions of the planets, by which their places may be determined at any instant for thousands of years, are computed from the analytical formulæ of La Place. In a research so profound and complicated, the most abstruse analysis is required, the higher branches of mathematical science are employed from the first, and approximations are made to the most intricate series. Easier methods, and more convergent series, may probably be discovered in process of time, which will supersede those now in use; but the work of La Place, regarded as embodying the results of not only his own researches, but those of so many of his illustrious predecessors and contemporaries, must ever remain, as he himself expressed it to the writer of these pages, a monument to the genius of the age in which it appeared.

Although physical astronomy is now the most perfect of sciences, a wide range is still left for the industry of future astronomers. The whole system of comets is a subject involved in mystery; they obey, indeed, the general law of gravitation, but many generations must be swept from the earth before their paths can be traced through the regions of space, or the periods of their return can be determined. A new and extensive field of investigation has lately been opened in the discovery of thousands of double stars, or, to speak more strictly, of systems of double stars, since many of them revolve round centres in various and long periods. Who can venture to predict when their theories shall be known, or what laws may be revealed by the knowledge of their motions?—but, perhaps, *Veniet tempus, in quo ista quæ nunc latent, in lucem dies extrahat et longioris ævi diligentia: ad inquisitionem tantorum ætas una non sufficit. Veniet tempus, quo posteri nostri tam aperta nos nescisse mirentur.*

It must, however, be acknowledged that many circumstances seem to be placed beyond our reach. The planets are so remote, that observation discloses but little of their structure; and although their similarity to the earth, in the appearance of their surfaces, and in their annual and diurnal revolutions producing the vicissitudes of seasons, and of day and night, may lead us to fancy that they are peopled with inhabitants like ourselves; yet, were it even permitted to form an analogy from the single instance of the earth, the only one known to us, certain it is that the physical nature of the inhabitants of the planets, if such there be, must differ essentially from ours, to enable them to endure every gradation of temperature, from the intensity of heat in Mercury, to the extreme cold that probably reigns in Uranus. Of the use of Comets in the economy of nature it is impossible to form an idea; still less of the Nebulæ, or cloudy appearances that are scattered through the immensity of space; but instead of being surprised that much is unknown, we have reason to be astonished that the successful daring of man has developed so much.

In the following pages it is not intended to limit the account of the *Mécanique Céleste* to a detail of results, but rather to endeavour to explain the methods by which these results are deduced from one general equation of the motion of matter. To accomplish this, without having recourse to the higher branches of mathematics, is impossible; many subjects, indeed, admit of geometrical demonstration; but as the object of this work is rather to give the spirit of La Place's method than to pursue a regular system of demonstration, it would be a deviation from the unity of his plan to adopt it in the present case.

Diagrams are not employed in La Place's works, being unnecessary to those versed in analysis; some, however, will be occasionally introduced for the convenience of the reader.

BOOK I.

CHAPTER I.

DEFINITIONS, AXIOMS, &c.

1. The activity of matter seems to be a law of the universe, as we know of no particle that is at rest. Were a body absolutely at rest, we could not prove it to be so, because there are no fixed points to which it could be referred; consequently, if only one particle of matter were in existence, it would be impossible to ascertain whether it were at rest or in motion. Thus, being totally ignorant of absolute motion, relative motion alone forms the subject of investigation: a body is, therefore, said to be in motion, when it changes its position with regard to other bodies which are assumed to be at rest.

2. The cause of motion is unknown, force being only a name given to a certain set of phenomena preceding the motion of a body, known by the experience of its effects alone. Even after experience, we cannot prove that the same consequents will invariably follow certain antecedents; we only believe that they will, and experience tends to confirm this belief.

3. No idea of force can be formed independent of matter; all the forces of which we have any experience are exerted by matter; as gravity, muscular force, electricity, chemical attractions and repulsions, &c. &c., in all which cases, one portion of matter acts upon another.

4. When bodies in a state of motion or rest are not acted upon by matter under any of these circumstances, we know by experience that they will remain in that state: hence a body will continue to move uniformly in the direction of the force which caused its motion, unless in some of the cases enumerated, in which we have ascertained by experience that a change of motion will take place, then a force is said to act.

5. Force is proportional to the differential of the velocity, divided

by the differential of the time, or analytically $F = \frac{dv}{dt}$, which is all we know about it.

6. The direction of a force is the straight line in which it causes a body to move. This is known by experience only.

7. In dynamics, force is proportional to the indefinitely small space caused to be moved over in a given indefinitely small time.

8. Velocity is the space moved over in a given time, how small soever the parts may be into which the interval is divided.

9. The velocity of a body moving uniformly, is the straight line or space over which it moves in a given interval of time; hence if the velocity v be the space moved over in one second or unit of time, vt is the space moved over in t seconds or units of time; or representing the space by s, $s = vt$.

10. Thus it is proved that the space described with a uniform motion is proportional to the product of the time and the velocity.

11. Conversely, v, the space moved over in one second of time, is equal to s, the space moved over in t seconds of time, multiplied by $\frac{1''}{t}$, or $v = s\frac{1''}{t} = \frac{s}{t}$.

12. Hence the velocity varies directly as the space, and inversely as the time; and because $t = \frac{s}{v}$,

13. The time varies directly as the space, and inversely as the velocity.

14. Forces are proportional to the velocities they generate in equal times.

The intensity of forces can only be known by comparing their effects under precisely similar circumstances. Thus two forces are equal, which in a given time will generate equal velocities in bodies of the same magnitude; and one force is said to be double of another which, in a given time, will generate double the velocity in one body that it will do in another body of the same magnitude.

15. The intensity of a force may therefore be expressed by the ratios of numbers, or both its intensity and direction by the ratios of lines, since the direction of a force is the straight line in which it causes the body to move.

16. In general, a line expressing the intensity of a force is taken in the direction of the force, beginning from the point of application.

17. Since motion is the change of rectilinear distance between two points, it appears that force, velocity, and motion are expressed by the ratios of spaces; we are acquainted with the ratios of quantities only.

Uniform Motion.

18. A body is said to move uniformly, when, in equal successive intervals of time, how short soever, it moves over equal intervals of space.

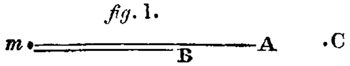
19. Hence in uniform motion the space is proportional to the time.

20. The only uniform motion that comes under our observation is the rotation of the earth upon its axis; all other motions in nature are accelerated or retarded. The rotation of the earth forms the only standard of time to which all recurring periods are referred. To be certain of the uniformity of its rotation is, therefore, of the greatest importance. The descent of materials from a higher to a lower level at its surface, or a change of internal temperature, would alter the length of the radius, and consequently the time of rotation: such causes of disturbance do take place; but it will be shown that their effects are so minute as to be insensible, and that the earth's rotation has suffered no sensible change from the earliest times recorded.

21. The equality of successive intervals of time may be measured by the recurrence of an event under circumstances as precisely similar as possible: for example, from the oscillations of a pendulum. When dissimilarity of circumstances takes place, we rectify our conclusions respecting the presumed equality of the intervals, by introducing an equation, which is a quantity to be added or taken away, in order to obtain the equality.

Composition and Resolution of Forces.

fig. 1.

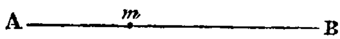


22. Let m be a particle of matter which is free to move in every direction; if two forces, represented both in intensity and direction by the lines mA and mB, be applied to it, and urge it towards C, the particle will move by the combined action of these two forces, and it will require a force equal

to their sum, applied in a contrary direction, to keep it at rest. It is then said to be in a state of equilibrium.

23. If the forces mA, mB, be applied to a particle m in contrary directions, and if mB be greater than mA, the particle m will be put in motion by the difference of these forces, and a force equal to their difference acting in a contrary direction will be required to keep the particle at rest.

fig. 2.



24. When the forces mA, mB are equal, and in contrary directions, the particle will remain at rest.

25. It is usual to determine the position of points, lines, surfaces, and the motions of bodies in space, by means of three plane surfaces, oP, oQ, oR, fig. 3, intersecting at given angles. The intersecting or co-ordinate planes are generally assumed to be perpendicular to each other, so that xoy, xoz, yoz, are right angles. The position of ox, oy, oz, the axes of the co-ordinates, and their origin o, are arbitrary; that is, they may be placed where we please, and are therefore always assumed to be known. Hence the position of a point m in space is determined, if its distance from each co-ordinate plane be given; for by taking oA, oB, oC, fig. 4, respectively equal to the given distances, and drawing three planes through A, B, and C, parallel to the co-ordinate planes, they will intersect in m.

fig. 3.

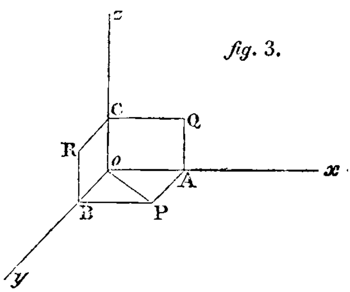
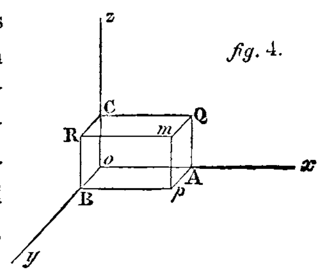
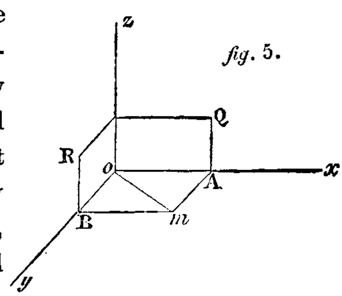


fig. 4.



26. If a force applied to a particle of matter at m, (fig. 5,) make it approach to the plane oQ uniformly by the space mA, in a given time t; and if another force applied to m cause it to approach the plane oR uniformly by the space mB, in the same time t, the particle will move in the diagonal

fig. 5.



mo, by the simultaneous action of these two forces. For, since the forces are proportional to the spaces, if a be the space described in one second, at will be the space described in t seconds; hence if at be equal to the space mA, and bt equal to the space mB, we have $t = \frac{mA}{a} = \frac{mB}{b}$; whence $mA = \frac{a}{b}\,mB$ which is the equation to a straight line *mo*, passing through *o*, the origin of the co-ordinates. If the co-ordinates be rectangular, $\frac{a}{b}$ is the tangent of the angle *mo*A, for $mB = oA$, and oAm is a right angle; hence $oA : Am :: 1 : \tan Aom$; whence $mA = oA \times \tan Aom = mB \,.\, \tan Aom$. As this relation is the same for every point of the straight line *mo*, it is called its equation. Now since forces are proportional to the velocities they generate in equal times, mA, mB are proportional to the forces, and may be taken to represent them. The forces mA, mB are called component or partial forces, and *mo* is called the resulting force. The resulting force being that which, taken in a contrary direction, will keep the component forces in equilibrio.

27. Thus the resulting force is represented in magnitude and direction by the diagonal of a parallelogram, whose sides are mA, mB the partial ones.

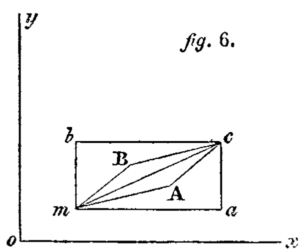


fig. 6.

28. Since the diagonal *cm*, fig. 6, is the resultant of the two forces mA, mB, whatever may be the angle they make with each other, so, conversely these two forces may be used in place of the single force *mc*. But *mc* may be resolved into any two forces whatever which form the sides of a parallelogram of which it is the diagonal; it may, therefore, be resolved into two forces *ma*, *mb*, which are at right angles to each other. Hence it is always possible to resolve a force *mc* into two others which are parallel to two rectangular axes *ox*, *oy*, situate in the same plane with the force; by drawing through *m* the lines *ma*, *mb*, respectively, parallel to *ox*, *oy*, and completing the parallelogram *macb*.

29. If from any point C, fig. 7, of the direction of a resulting force mC, perpendiculars CD, CE, be drawn on the directions of the

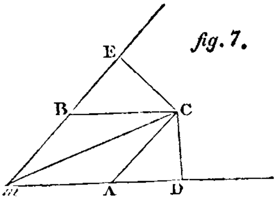


fig. 7.

component forces mA, mB, these perpendiculars are reciprocally as the component forces. That is, CD is to CE as CA to CB, or as their equals mB to mA.

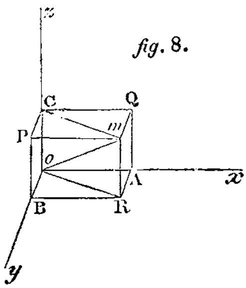
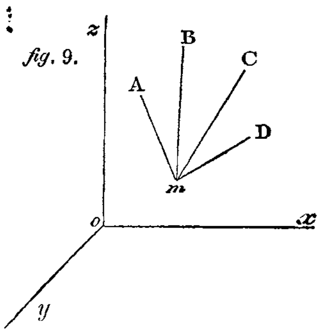


fig. 8.

30. Let BQ, fig. 8, be a figure formed by parallel planes seen in perspective, of which mo is the diagonal. If mo represent any force both in direction and intensity, acting on a material point m, it is evident from what has been said, that this force may be resolved into two other forces, mC, mR, because mo is the diagonal of the parallelogram $mCoR$. Again mC is the diagonal of the parallelogram $mQCP$, therefore it may be resolved into the two forces mQ, mP; and thus the force mo may be resolved into three forces, mP, mQ, and mR; and as this is independent of the angles of the figure, the force mo may be resolved into three forces at right angles to each other. It appears then, that any force mo may be resolved into three other forces parallel to three rectangular axes given in position: and conversely, three forces mP, mQ, mR, acting on a material point m, the resulting force mo may be obtained by constructing the figure BQ with sides proportional to these forces, and drawing the diagonal mo.

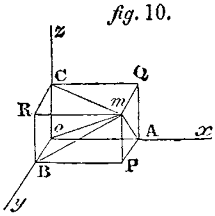
fig. 9.



31. Therefore, if the directions and intensities with which any number of forces urge a material point be given, they may be reduced to one single force whose direction and intensity is known. For example, if there were four forces, mA, mB, mC, mD, fig. 9, acting on m, if the resulting force of mA and mB be found, and then that of mC and mD; these four forces would be reduced to two, and by finding the resulting force of these two, the four forces would be reduced to one.

32. Again, this single resulting force may be resolved into three

fig. 10.



forces parallel to three rectangular axes *ox*, *oy*, *oz*, fig. 10, which would represent the action of the forces *m*A, *m*B, &c., estimated in the direction of the axes; or, which is the same thing, each of the forces *m*A, *m*B, &c. acting on *m*, may be resolved into three other forces parallel to the axes.

33. It is evident that when the partial forces act in the same direction, their sum is the force in that axis; and when some act in one direction, and others in an opposite direction, it is their difference that is to be estimated.

34. Thus any number of forces of any kind are capable of being resolved into other forces, in the direction of two or of three rectangular axes, according as the forces act in the same or in different planes.

35. If a particle of matter remain in a state of equilibrium, though acted upon by any number of forces, and free to move in every direction, the resulting force must be zero.

36. If the material point be in equilibrio on a curved surface, or on a curved line, the resulting force must be perpendicular to the line or surface, otherwise the particle would slide. The line or surface resists the resulting force with an equal and contrary pressure.

37. Let $o\text{A}=\text{X}$, $o\text{B}=\text{Y}$, $o\text{C}=\text{Z}$, fig. 10, be three rectangular component forces, of which $om=\text{F}$ is their resulting force. Then, if $m\text{A}$, $m\text{B}$, $m\text{C}$ be joined, $om=\text{F}$ will be the hypothenuse common to three rectangular triangles, $o\text{A}m$, $o\text{B}m$, and $o\text{C}m$. Let the angles $mo\text{A}=a$, $mo\text{B}=b$, $mo\text{C}=c$; then

$$\text{X}=\text{F}\cos a,\ \text{Y}=\text{F}\cos b,\ \text{Z}=\text{F}\cos c. \qquad (1).$$

Thus the partial forces are proportional to the cosines of the angles which their directions make with their resultant. But BQ being a rectangular parallelopiped

$$\text{F}^2=\text{X}^2+\text{Y}^2+\text{Z}^2. \qquad (2).$$

Hence

$$\frac{\text{X}^2+\text{Y}^2+\text{Z}^2}{\text{F}^2}=\cos^2 a+\cos^2 b+\cos^2 c=1.$$

When the component forces are known, equation (2) will give a value of the resulting force, and equations (1) will determine its direction by the angles a, b, and c; but if the resulting force be given, its resolution into the three component forces X, Y, Z, making